A new ice sheet model for CCSM part II: First-Order Flow Model

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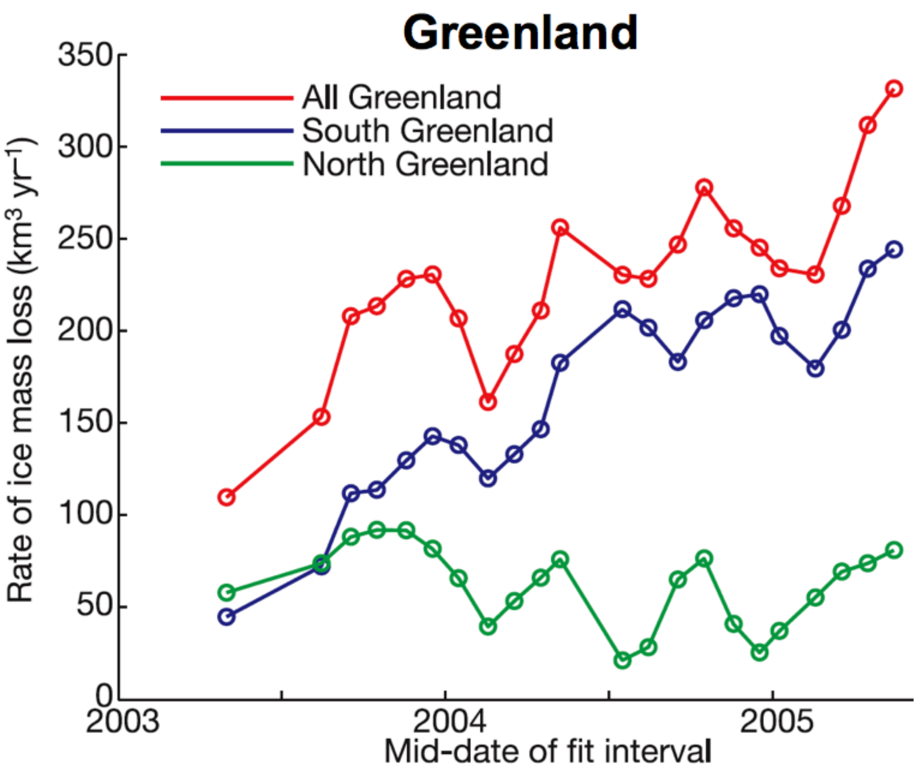
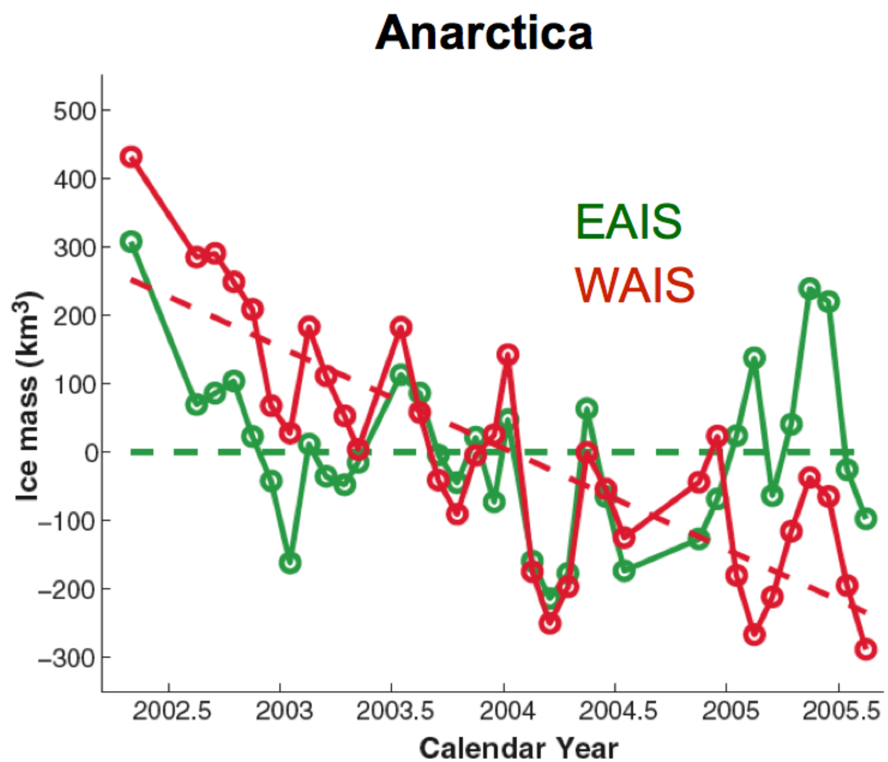
Motivation for Ice Sheet Modeling:

Mass loss to oceans (& sea level rise)





Velicogna & Wahr (*Nature*, **443**, 2006)

Velicogna & Wahr (*Science*, **311**, 2006)

Motivation for *improved* Ice Sheet Models

Current generation models do not capture observed behaviours[1], because:

(1) fundamental physics are lacking (e.g. solving simplified equations, negating realistic simulation of outlet glaciers and ice streams)

(2) processes of fundamental importance are not accounted for (e.g. simplified, static treatment of basal boundary conditions, ignoring interaction with bounding oceans, etc.)

[1] *IPCC* (2007)

1st-order SIA Flow Model

- governing equations
- scaling and reduced equations
- solution method

Model “Validation”

- comparison to analytical / benchmark solutions

Application to Greenland Ice Sheet

- thermomechanical, “diagnostic” velocity field

Current and Future Work

Illulisat, Western Greenland

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[1]Blatter (*J.Glac.*, **41**, 1995)

Illulisat, Western Greenland

Equations of Stress Equilibrium (Cartesian Coordinates)

Assume static balance of forces by ignoring acceleration

$$x: \quad \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial P}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0$$

$$y: \quad \frac{\partial \tau_{yy}}{\partial y} - \frac{\partial P}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} = 0$$

$$z: \quad \frac{\partial \tau_{zz}}{\partial z} - \frac{\partial P}{\partial z} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial x} = \rho g$$

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Equations of Stress Equilibrium (scaled)

$$\lambda = \frac{\text{vert. length scale}}{\text{horiz. length scale}} = \frac{H}{L}$$

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… choice of *L* is somewhat arbitrary…

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$\lambda = H \div L \sim (10^3 \text{ x } 10^{-5}) \rightarrow$ **$\lambda \sim 10^{-2}$, $\lambda^2 \sim 10^{-4}$**

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(2) if we take L as a charc. length for horiz. stress transfer,

$L \sim$ 5-10 x H,

$\lambda \sim 10^{-1}$, **$\lambda^2 \sim 10^{-2}$**

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(2) if we take L as a charc. length for horiz. stress transfer,

$L \sim 5\text{-}10 \times H$,

$\lambda \sim 10^{-1}$, $\lambda^2 \sim 10^{-2}$

… terms associated with λ^2 are negligible,
… terms associated with λ are NOT

First Order Approximation (scaled)

$$\hat{x}: \quad \lambda \frac{\partial \hat{\tau}_{xx}}{\partial \hat{x}} - \frac{\partial \hat{P}}{\partial \hat{x}} + \lambda \frac{\partial \hat{\tau}_{xy}}{\partial \hat{y}} + \frac{\partial \hat{\tau}_{xz}}{\partial \hat{z}} = 0$$

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1st-order SIA: **Red** omissions (λ^2)

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First Order Approximation (scaled)

1st-order SIA: **Red** omissions (λ^2)
0-order SIA: **Red** + **Blue** omissions (λ, λ^2)

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First Order Approximation (unscaled)

$$x: \quad \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \frac{\partial P}{\partial x}$$

$$z: \quad \frac{\partial \tau_{zz}}{\partial z} = \rho g + \frac{\partial P}{\partial z}$$

First Order Approximation (unscaled)

$$x: \quad \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \frac{\partial P}{\partial x}$$

… simplify vert. equation so it can be "stuffed into" horiz. equations …

$$z: \quad \frac{\partial \tau_{zz}}{\partial z} = \rho g + \frac{\partial P}{\partial z}$$

First Order Approximation (unscaled)

…integrate in vertical from upper sfc through depth …

$$z: \quad \frac{\partial \tau_{zz}}{\partial z} - \frac{\partial P}{\partial z} = \rho g$$

$$P = \rho g \left(s - z\right) + \tau_{zz}(z)$$

First Order Approximation (unscaled)

…substitute vertical relation for *P* into horizontal balance …

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…use definition of deviatoric stress to eliminate vertical-normal stress deviator in horiz. equations …

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$$x: \quad 2\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \rho g \frac{\partial s}{\partial x}$$

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(strain rate tensor)

$$2\dot{\varepsilon}_e = \dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}$$

(effective strain rate)

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$$\eta \equiv \frac{1}{2}B\dot{\varepsilon}_e^{\frac{1-n}{n}}$$ (effective viscosity)

$$\tau_{ij} = 2\eta\dot{\varepsilon}_{ij}$$

First Order Approximation (unscaled)

…use constiutive relation to write stresses in terms of strain rates and eff. visc., write strain rates in terms of vel. grads. …

$$x: \quad 2\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \rho g \frac{\partial s}{\partial x}$$

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$$x: \quad 4\frac{\partial}{\partial x}\left(\eta\frac{\partial u}{\partial x}\right) + 2\frac{\partial}{\partial x}\left(\eta\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial y}\left[\eta\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left(\eta\frac{\partial u}{\partial z}\right) = \rho g \frac{\partial s}{\partial x}$$

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Solve for *u* by treating *v* terms as known source (and vice versa)

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Iterate on effective viscosity using "unstable manifold correction[1]"

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Surface and basal boundary conditions are fully HO (not 0-order approx.)

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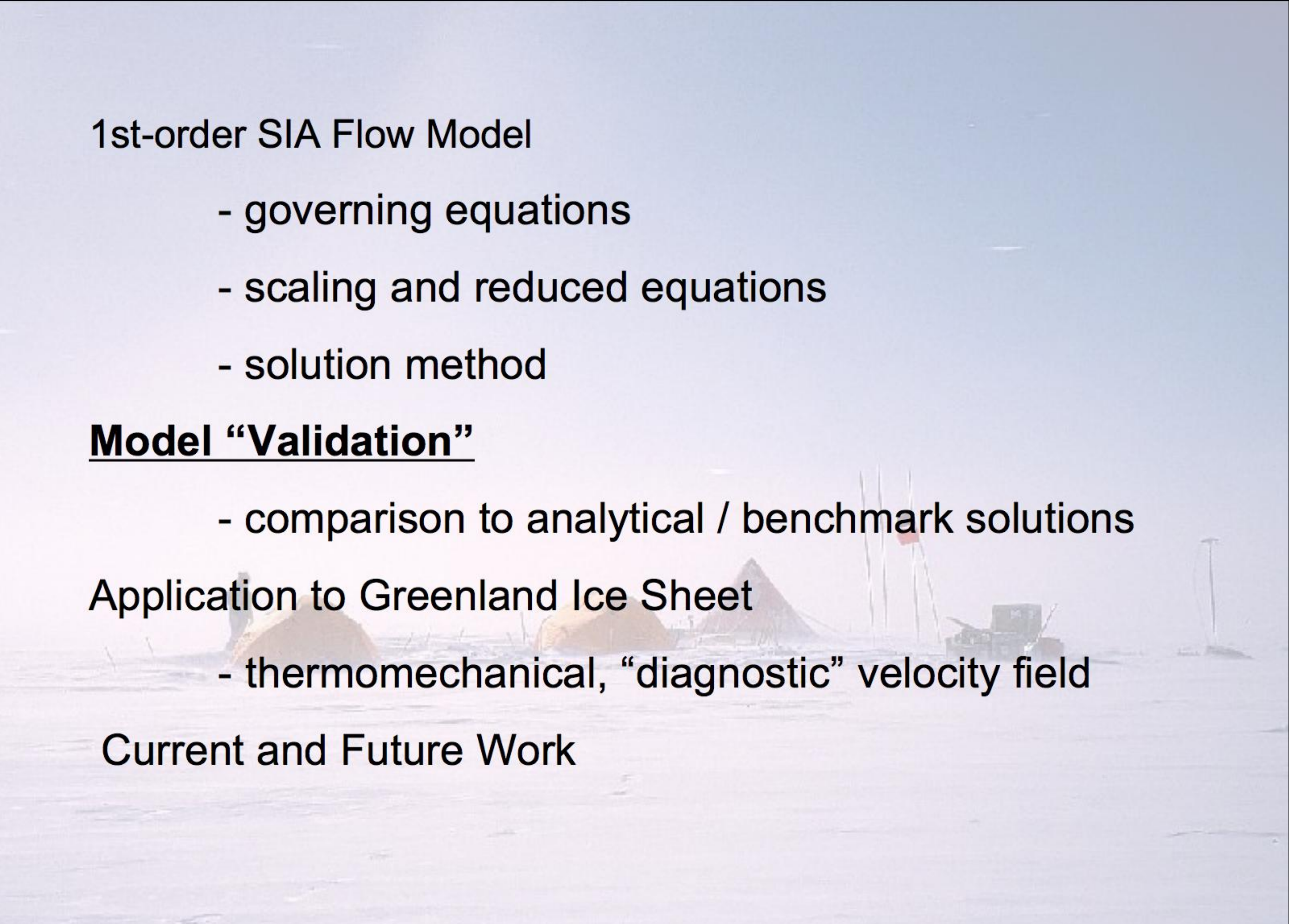

field camp, central Greenland

ISMIP-HOM[1] exp. A (sheet flow)

$s = s(x)$ (constant slope)

$b = b(x,y)$ (periodic bed roughness)

$u(b)=\mathrm{v}(b) = 0$ (no slip)

[1] Pattyn et al. (*EGU*, *AGU*, 2007)

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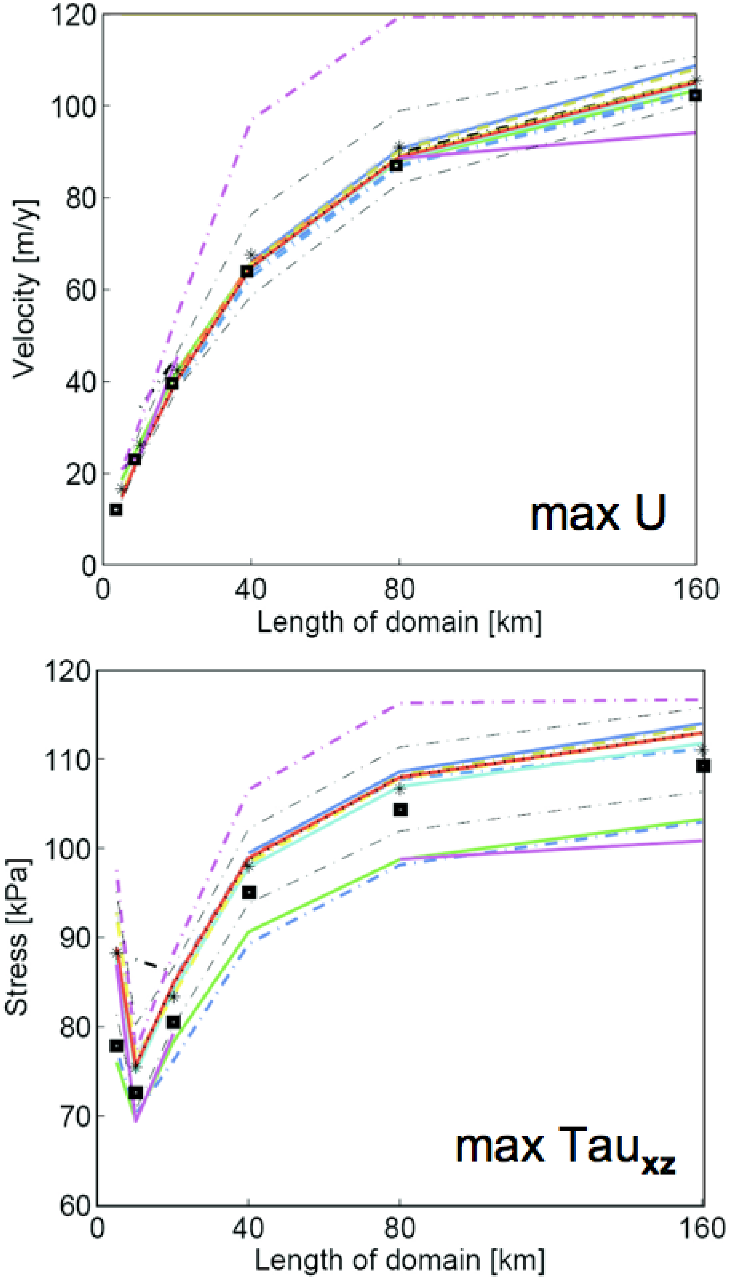

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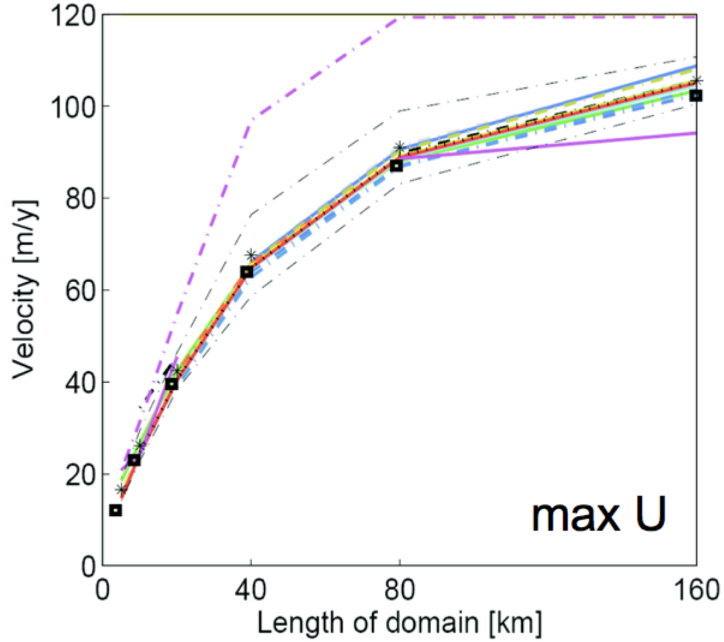

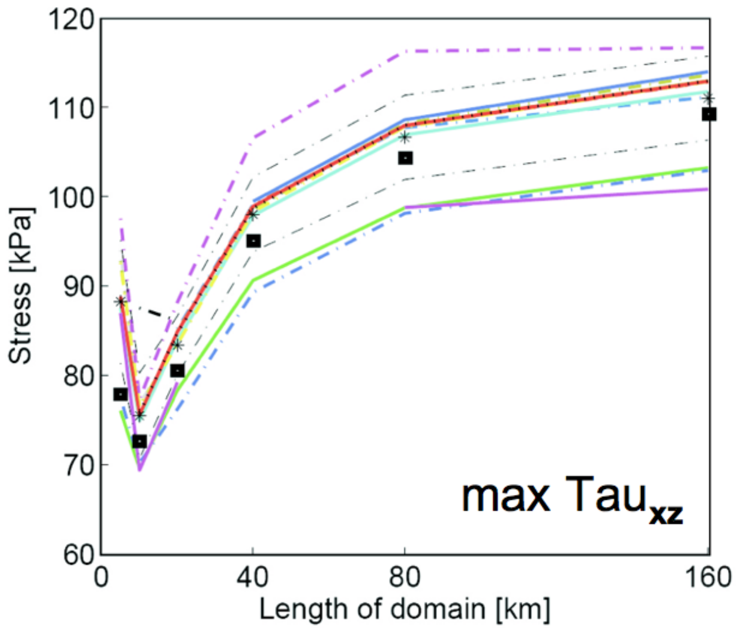

ISMIP-HOM exp. C (stream flow)

$s = s(x)$ (constant slope)

$b = b(x)$ (constant slope)

$u(b) = \beta\tau_b$ (sliding law)

$\beta = \beta(x,y)$ (periodic traction)

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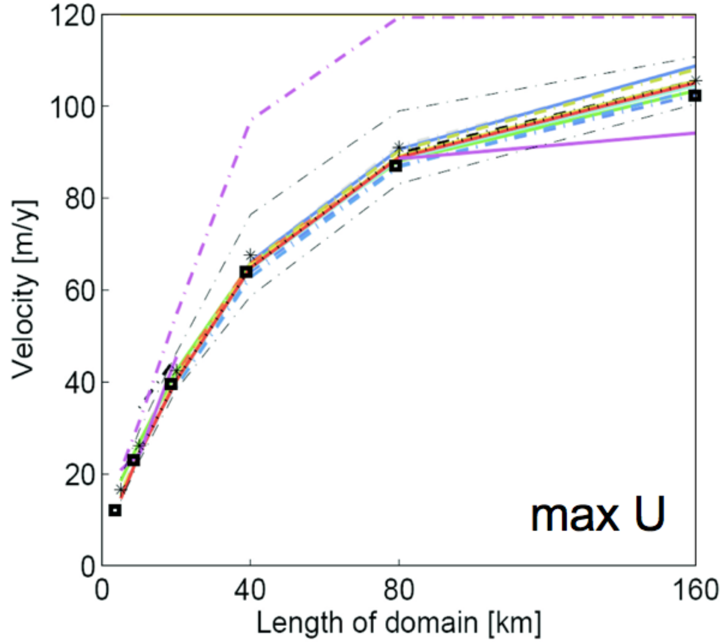

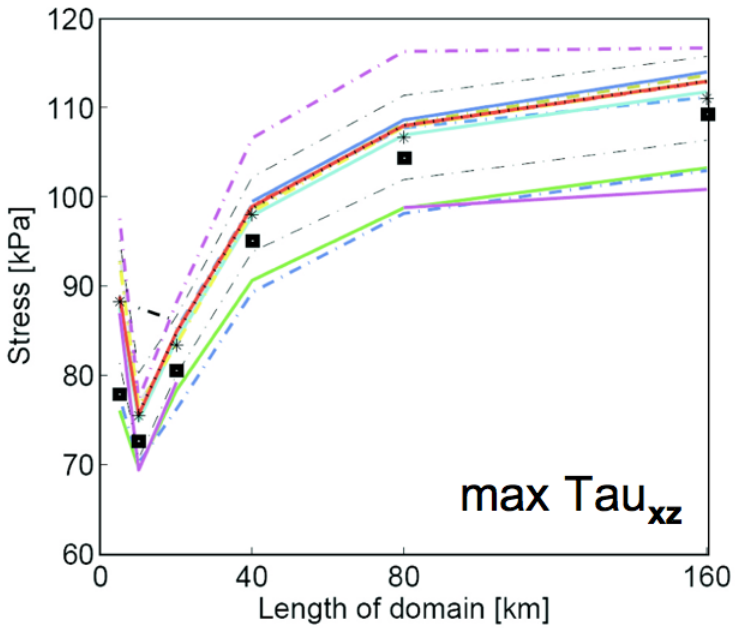

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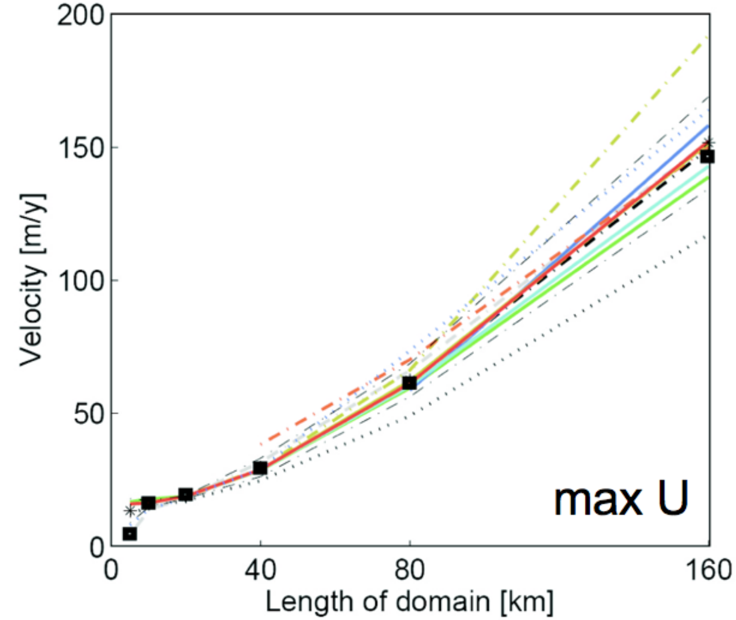

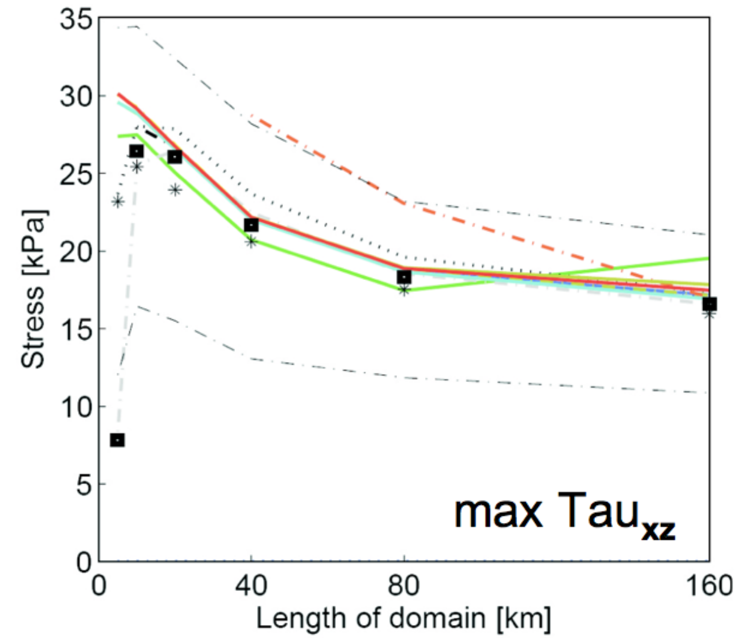

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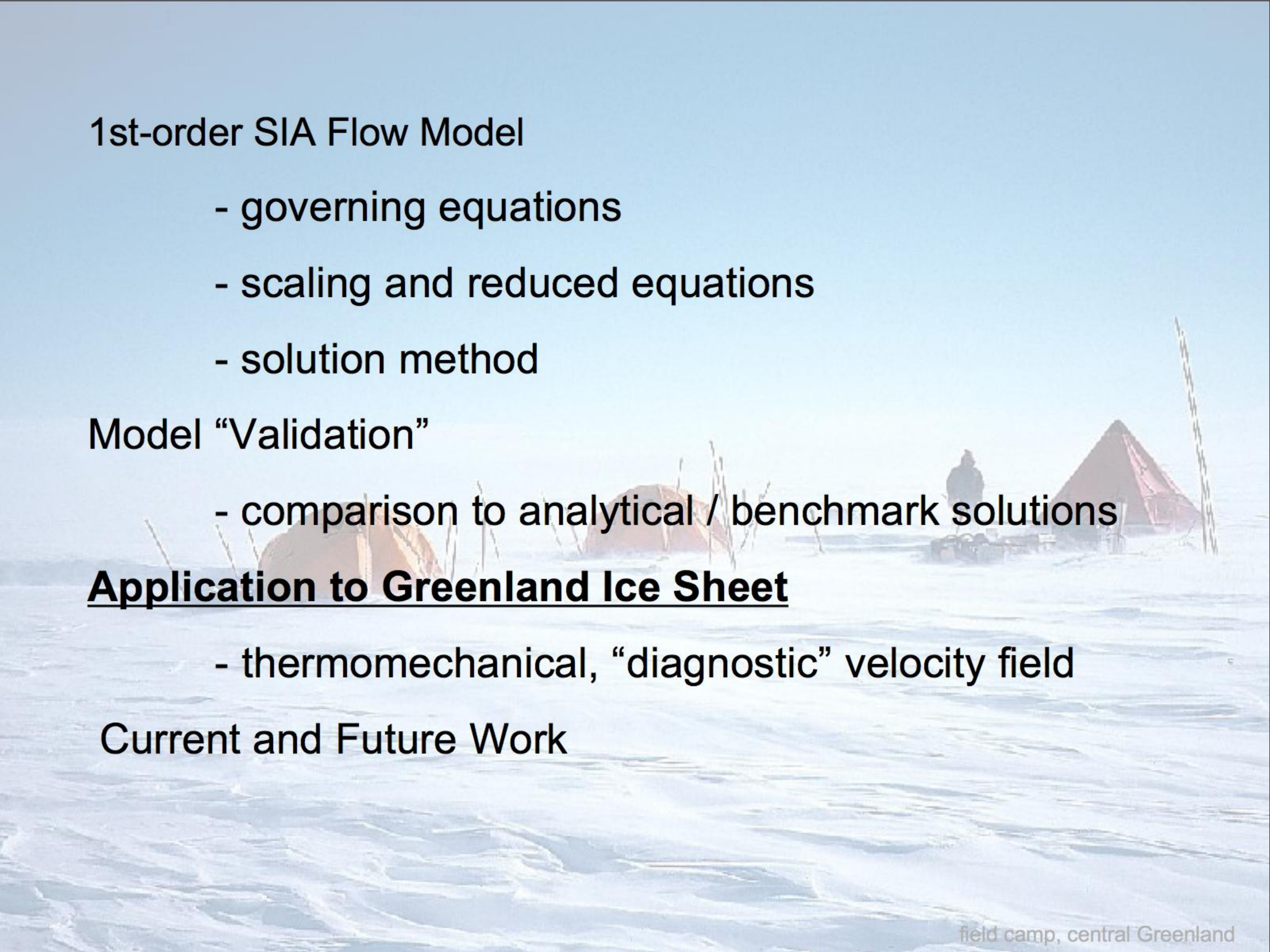

field camp, central Greenland

Greenland Ice Sheet: diagnostic velocity field

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Momentum Balance BCs:

surface:	free surface	
*bed:	$u=v=0$	
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Energy Balance BCs:

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sides:	no lateral diffusion	

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Calculation:

… hold geometry, T_{surf}, Q_{geo} steady …
… allow $B(T)$, $\boldsymbol{u}$, and η_{eff} to evolve to steady state …

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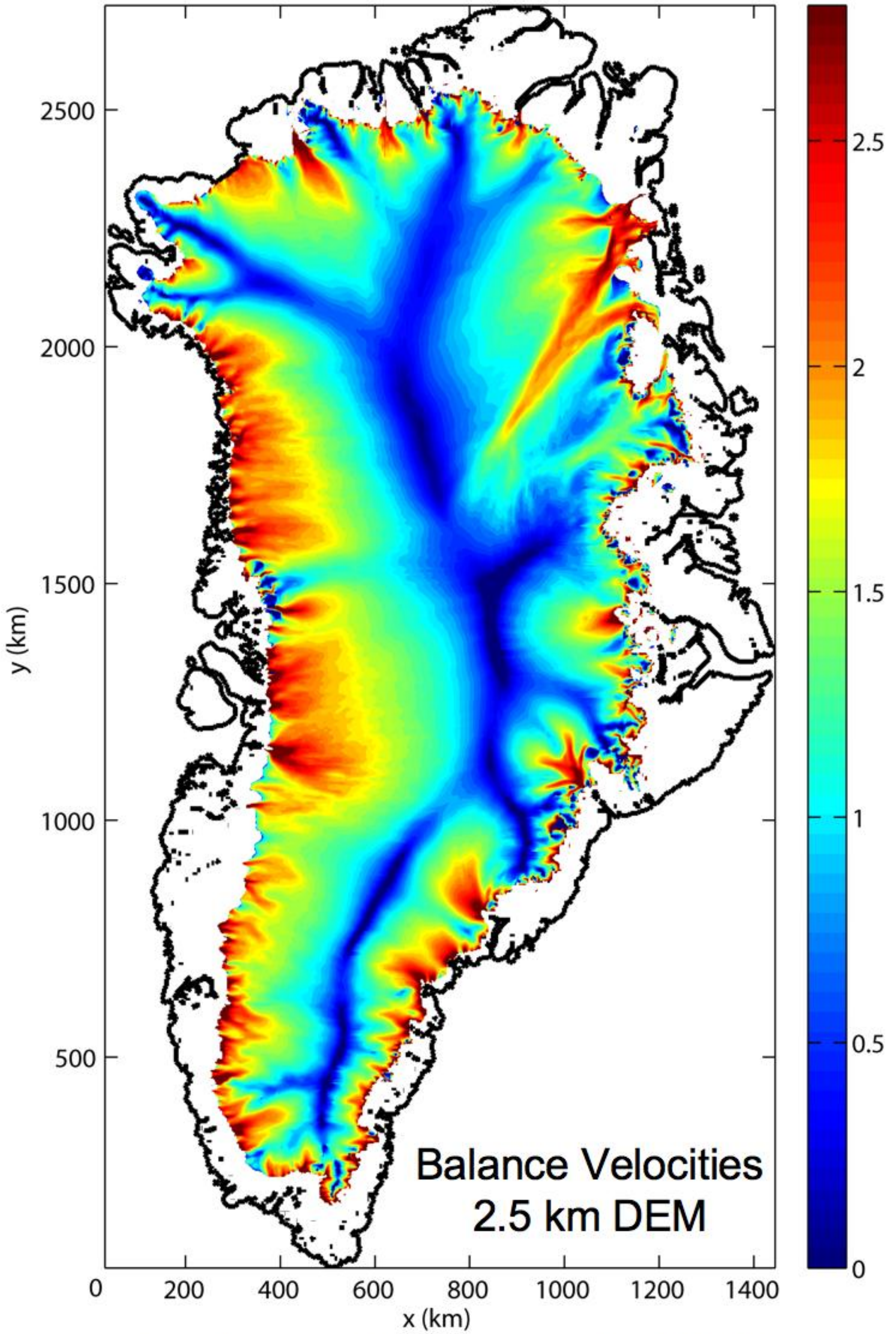
How do modeled and observed[1] flow fields compare spatially?

[1]Bamber et al. (*J.Glac.*, v.46, 2000)



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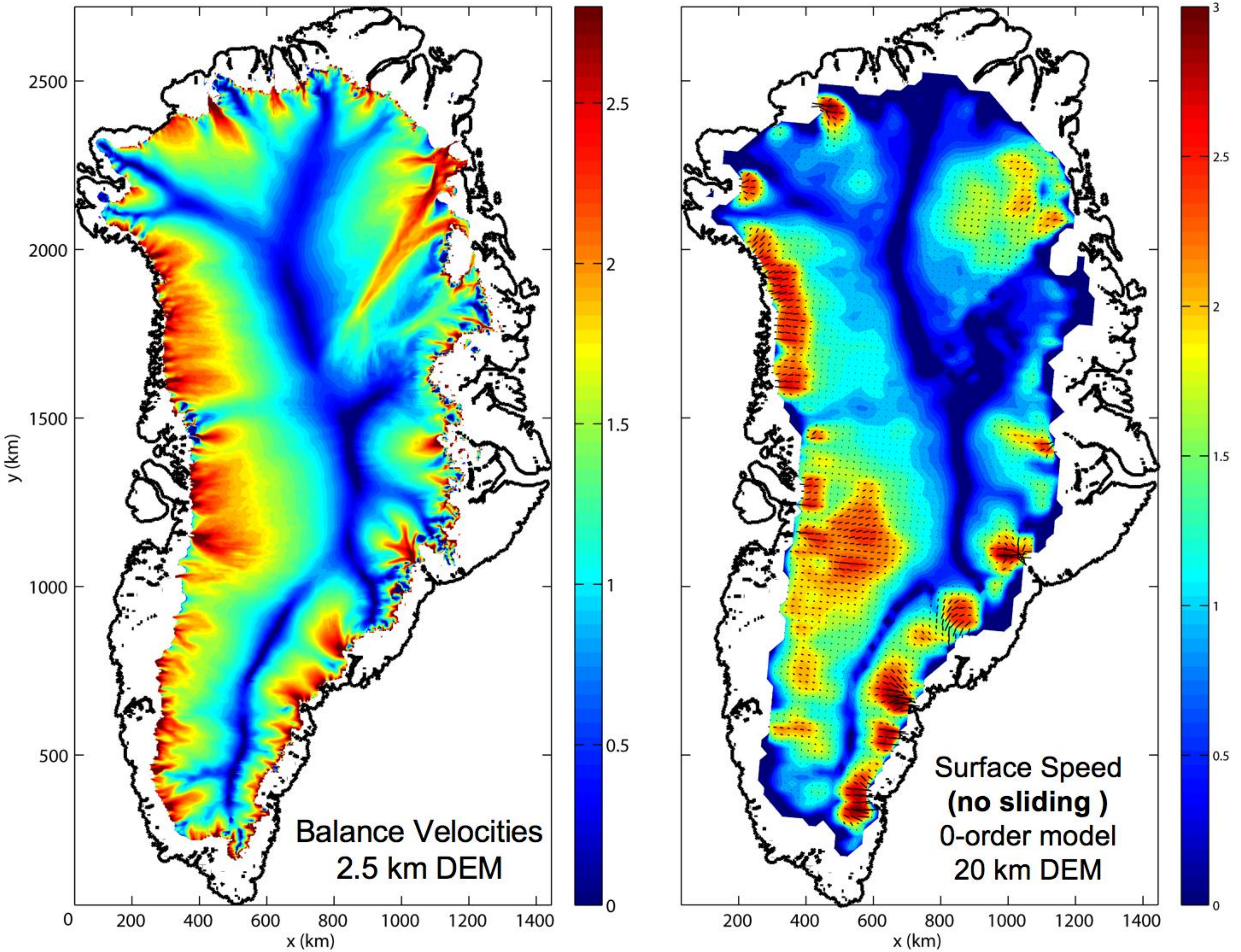

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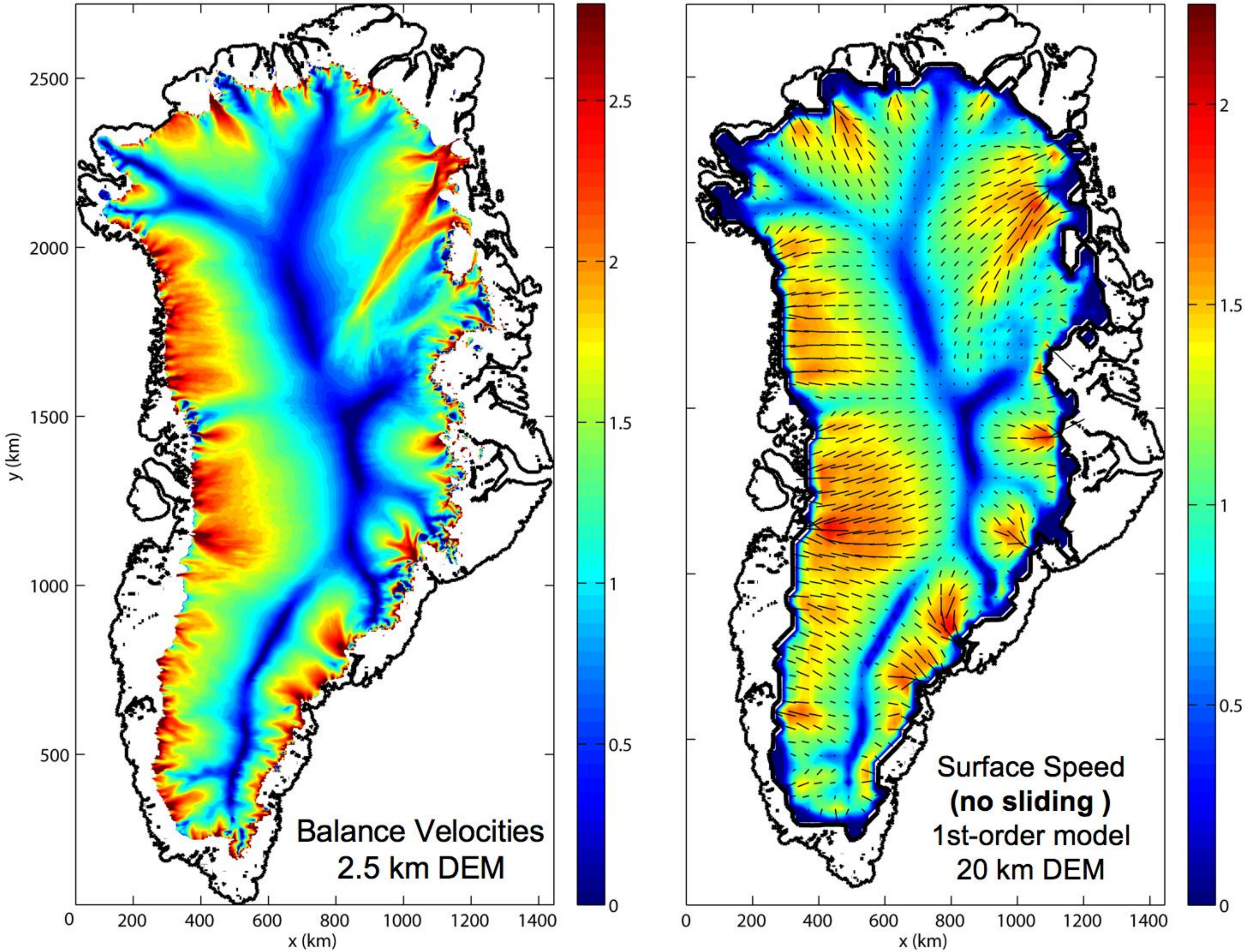

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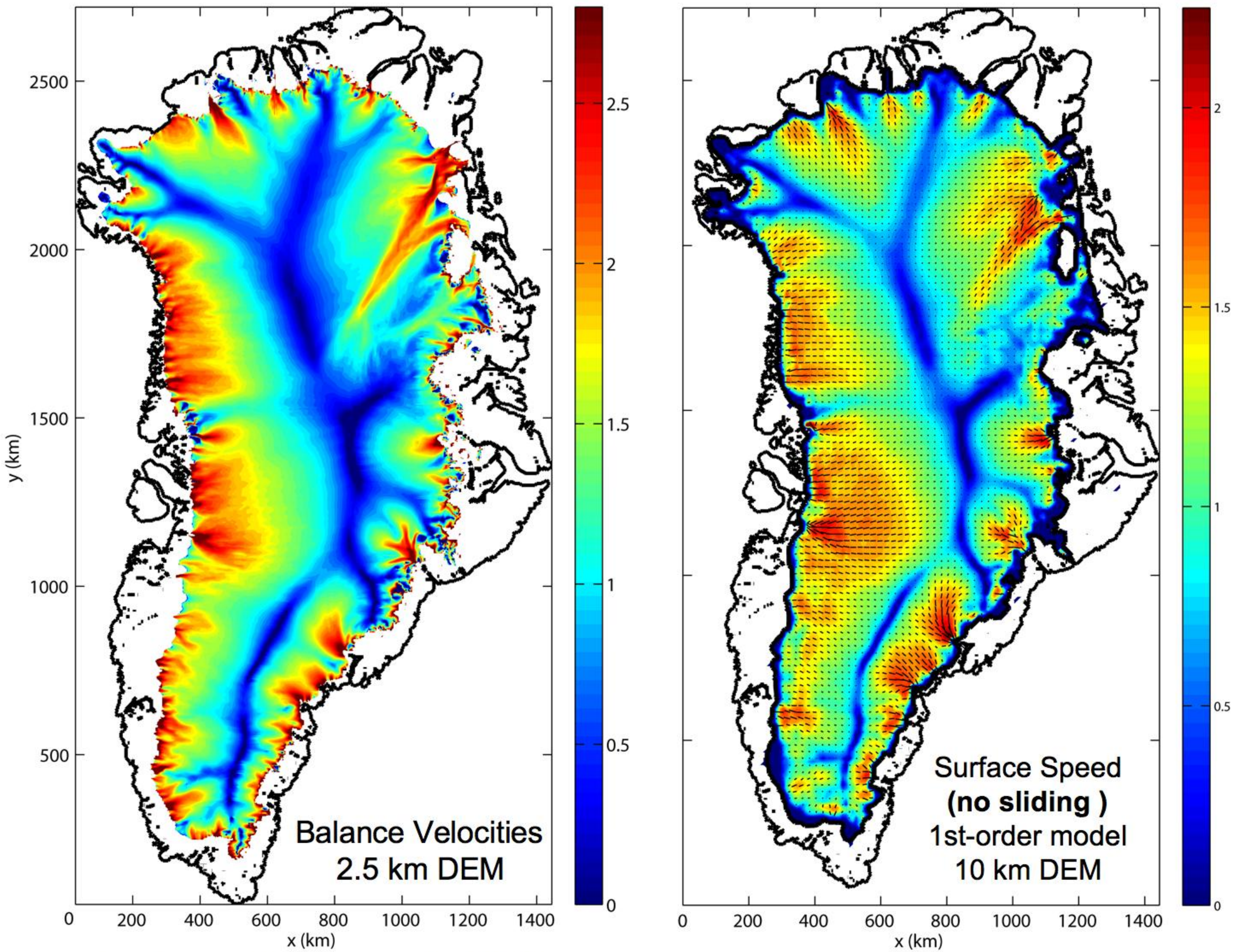

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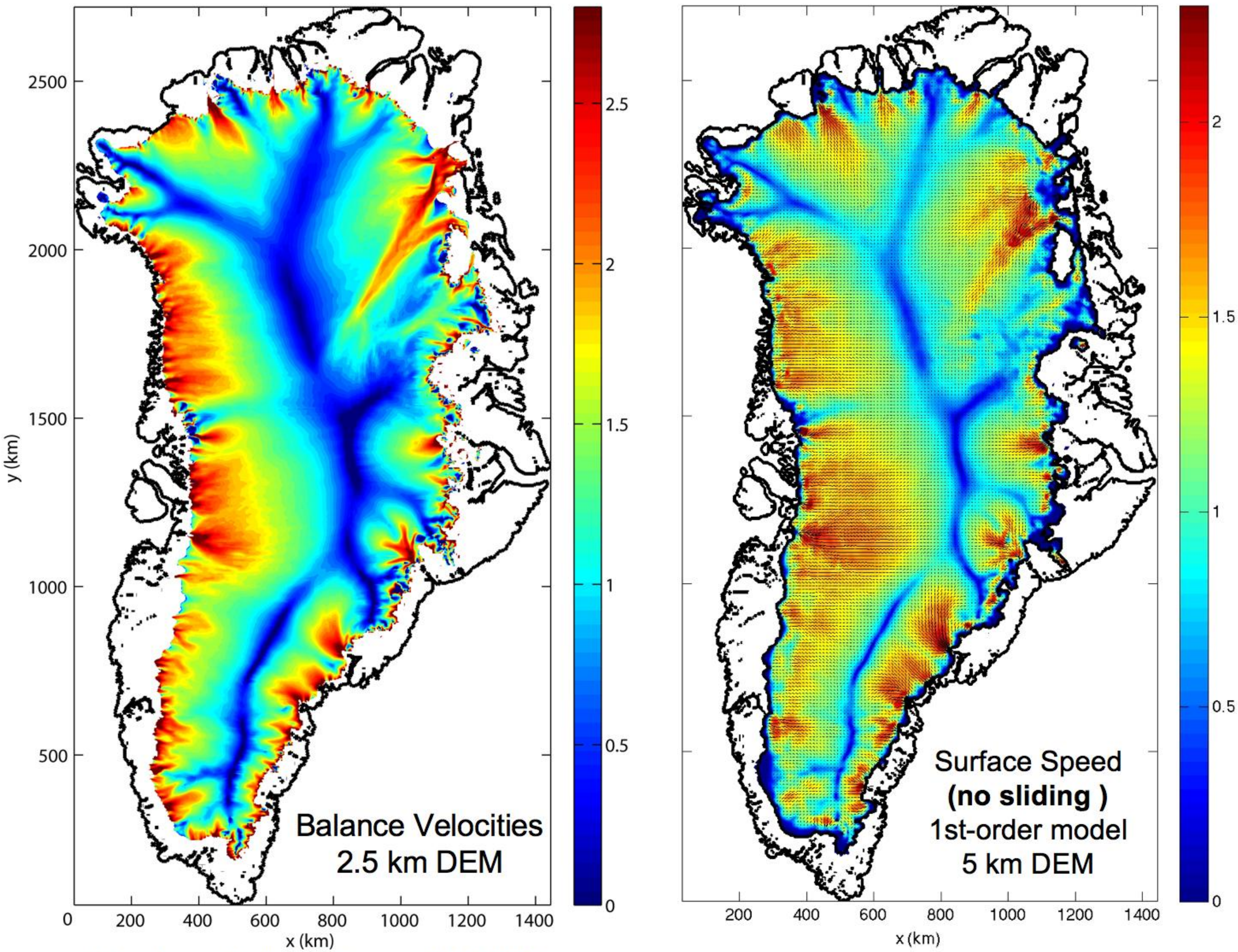

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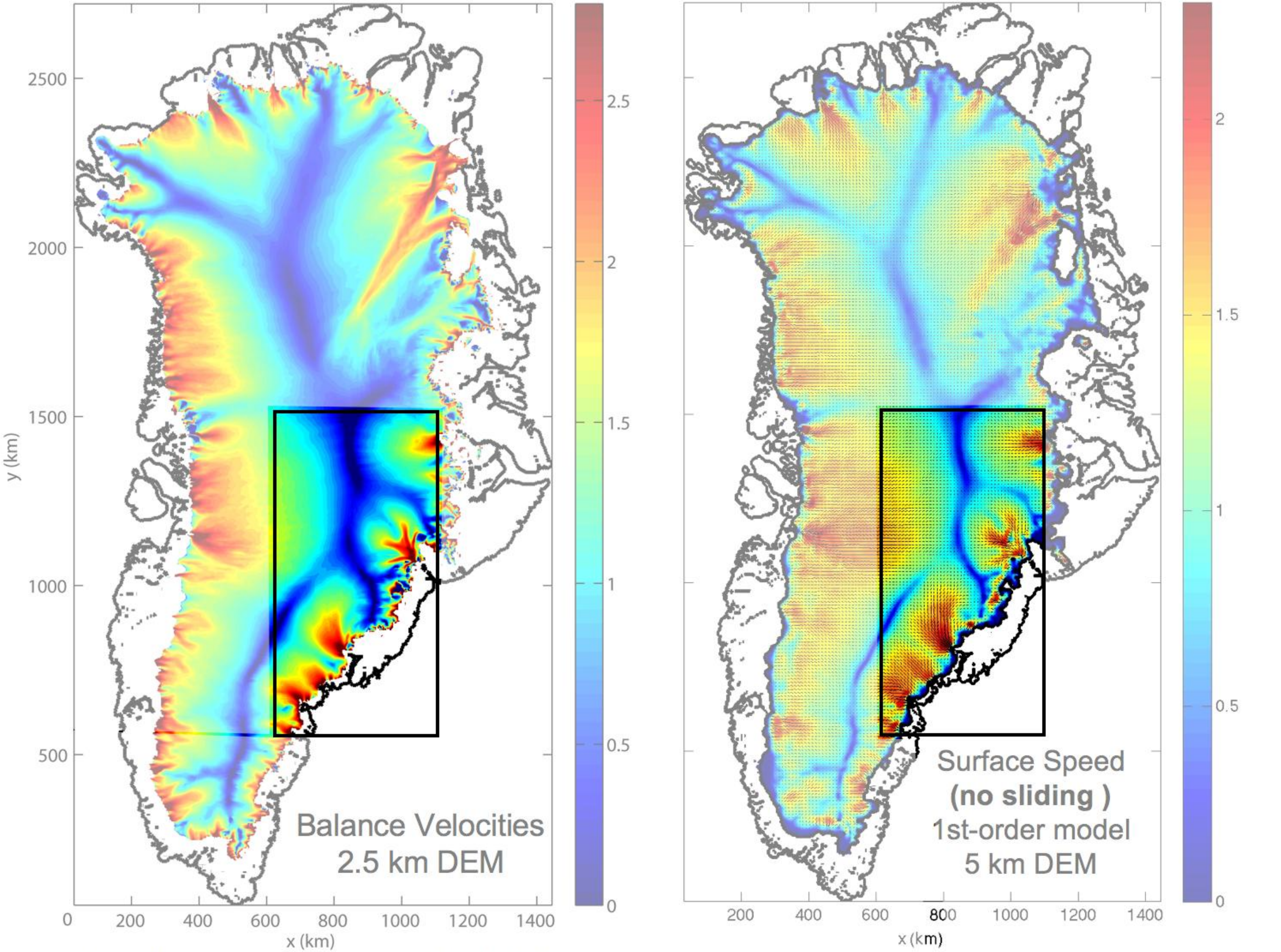

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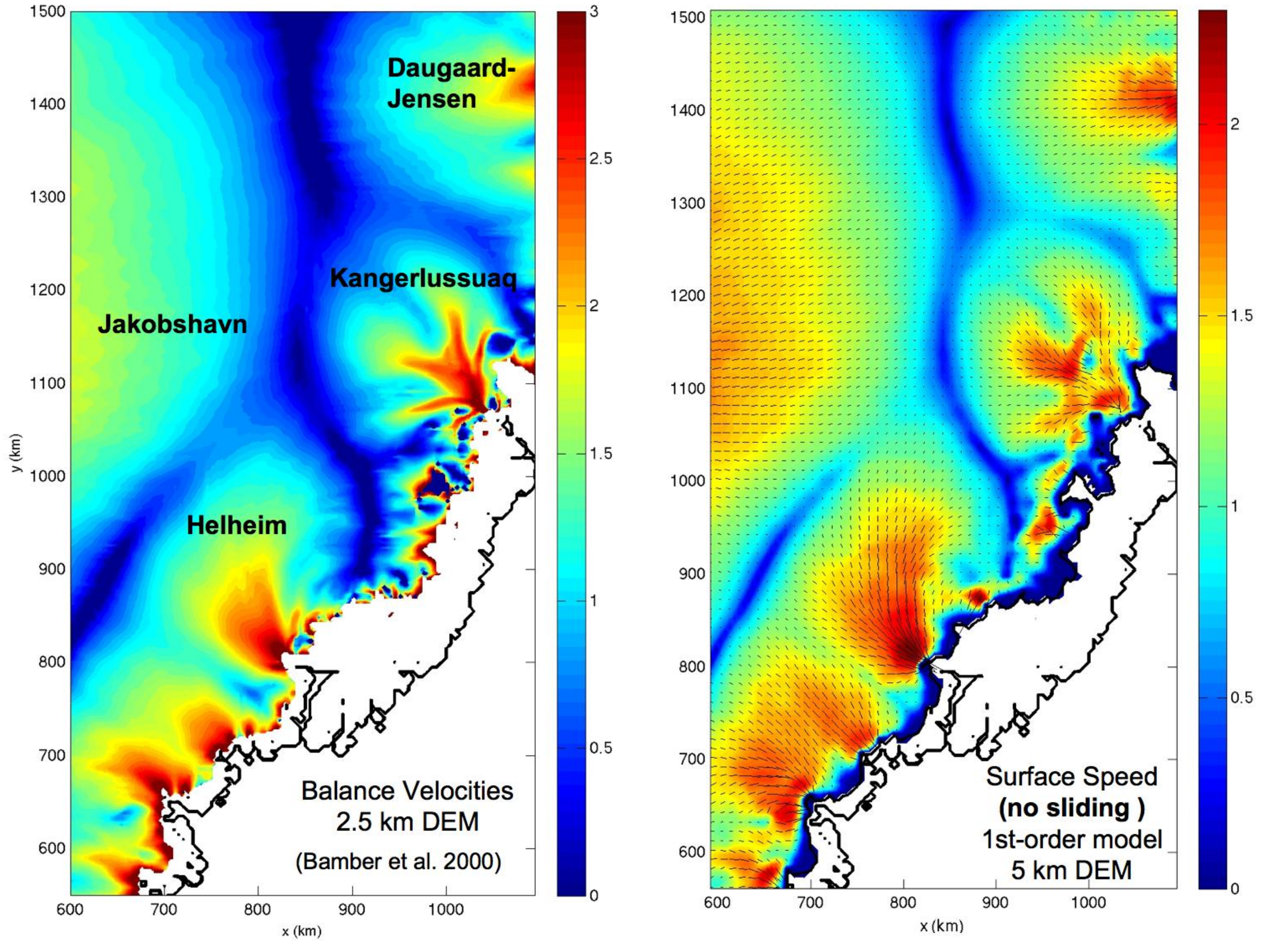
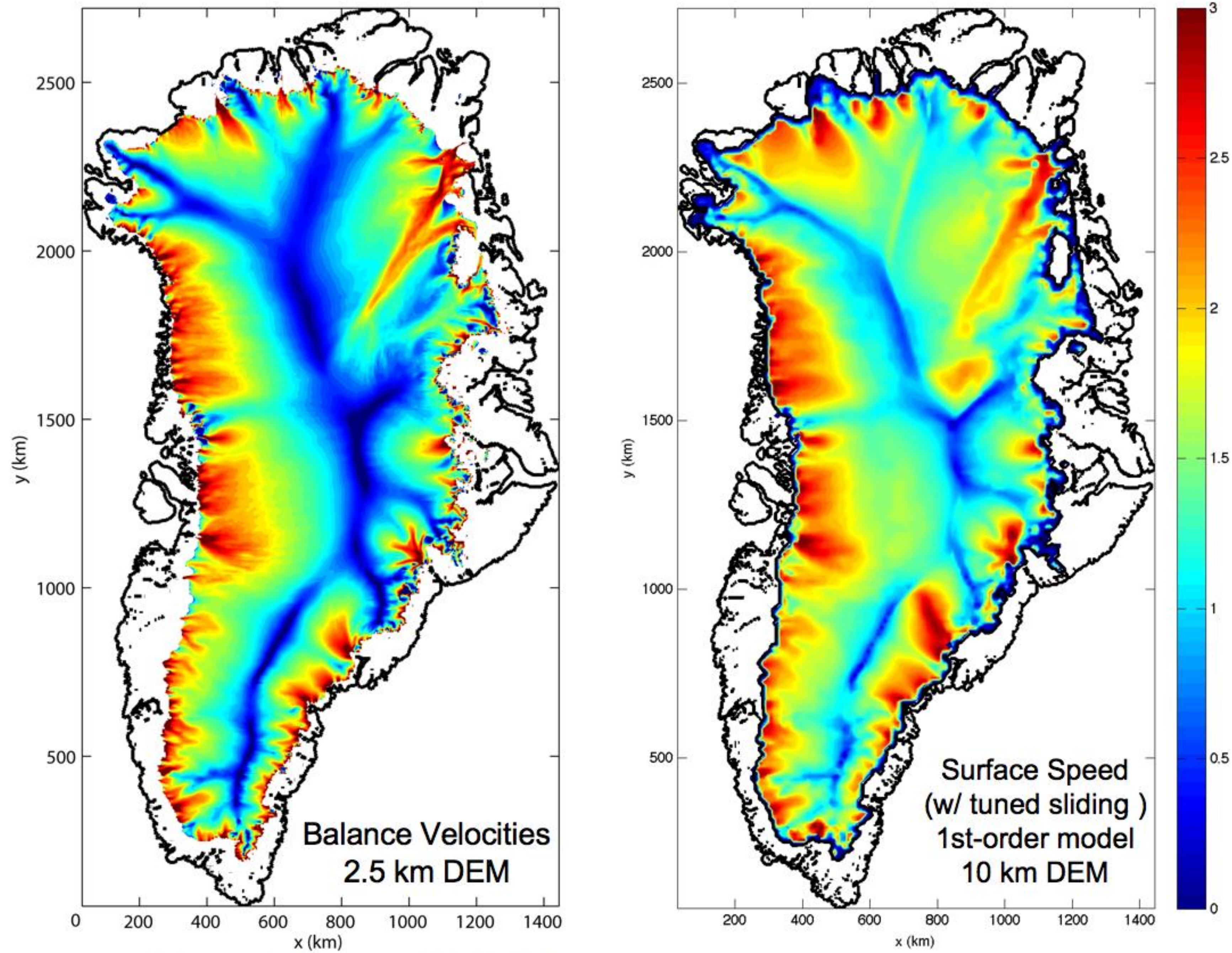


Daugaard-
Jensen
Kangerlussuaq
Jakobshavn
Helheim
Balance Velocities
2.5 km DEM
(Bamber et al. 2000)
y (km)
x (km)
Surface Speed
(no sliding)
1st-order model
5 km DEM
x (km)



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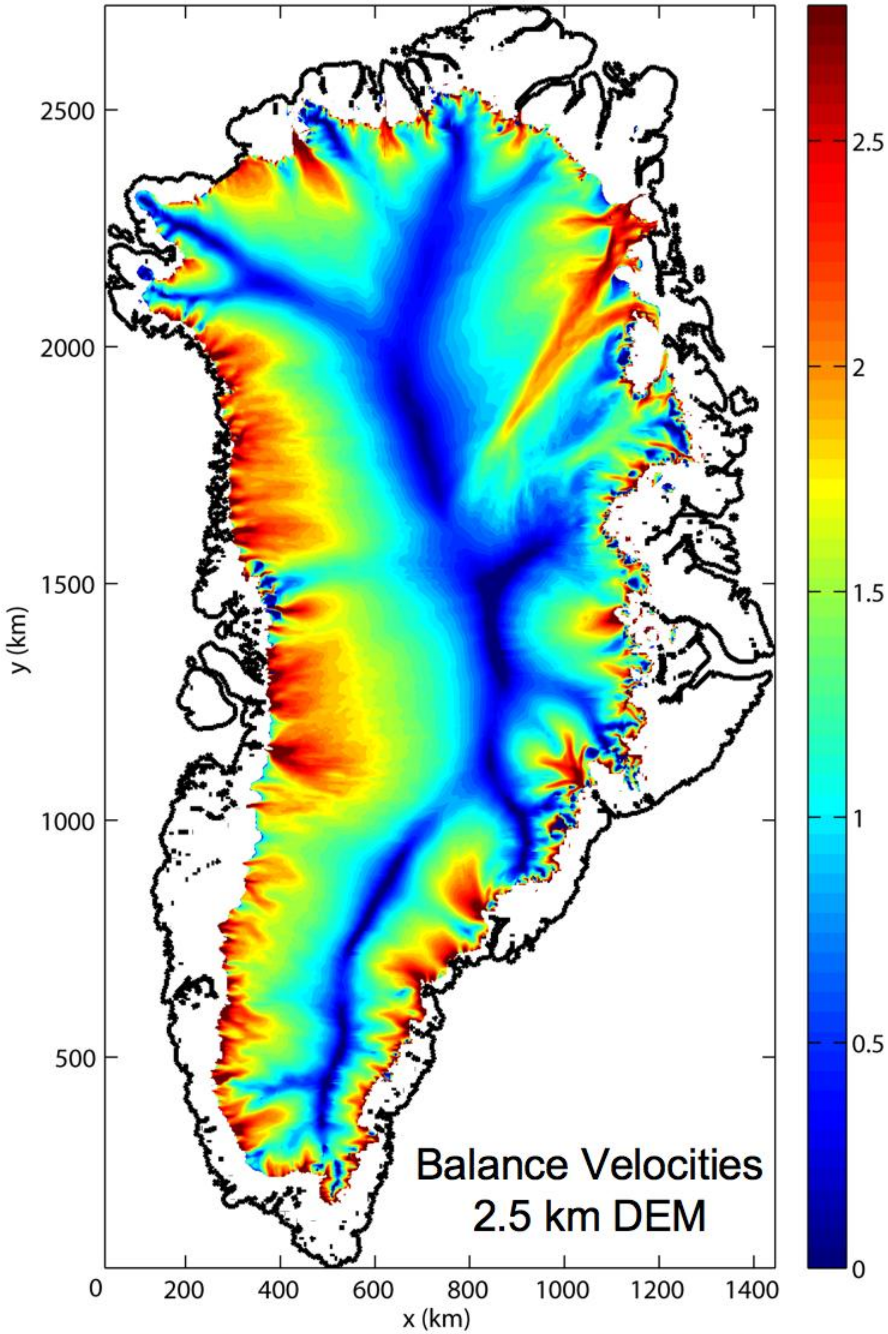

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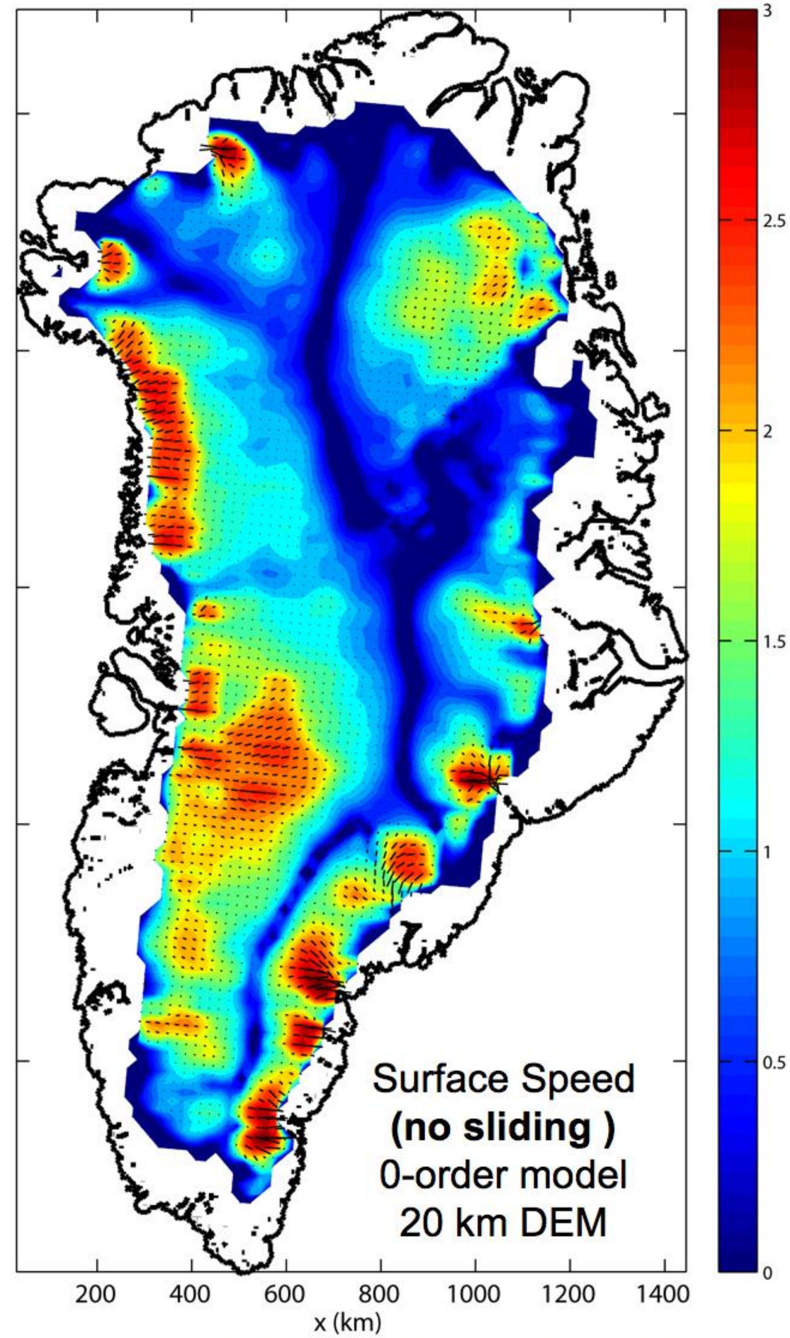

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Model "Validation"

- comparison to analytical / benchmark solutions

Application to Greenland Ice Sheet

- thermomechanical, "diagnostic" velocities

Current and Future Work

field camp, Southern Greenland

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"Prognostic" mode

- improved methods of thickness evolution

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Basal hydrology

- need time-dependent, conservative model of basal water flow with reasonable time step

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END

First Order Approximation (solution)

… put all terms containing *u* on LHS and all terms containing *v* on RHS. Solve for *u* by treating RHS as known source using *v* from previous iteration …

$$x: \quad 4\frac{\partial}{\partial x}\left(\eta\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\eta\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\eta\frac{\partial u}{\partial z}\right)=\ldots$$

$$\ldots-2\frac{\partial}{\partial x}\left(\eta\frac{\partial v}{\partial y}\right)-\frac{\partial}{\partial y}\left(\eta\frac{\partial v}{\partial x}\right)+\rho g\frac{\partial s}{\partial x}$$

First Order Approximation (neglected terms)

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$$\dot{\varepsilon}_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

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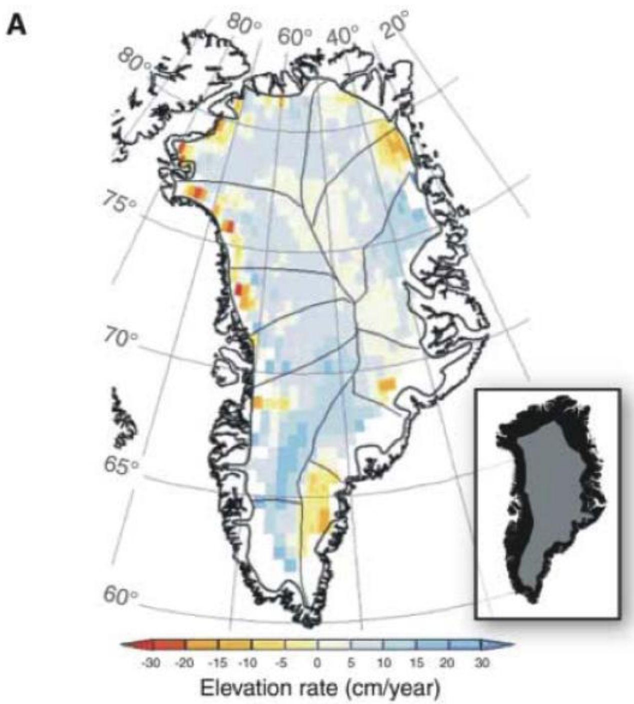
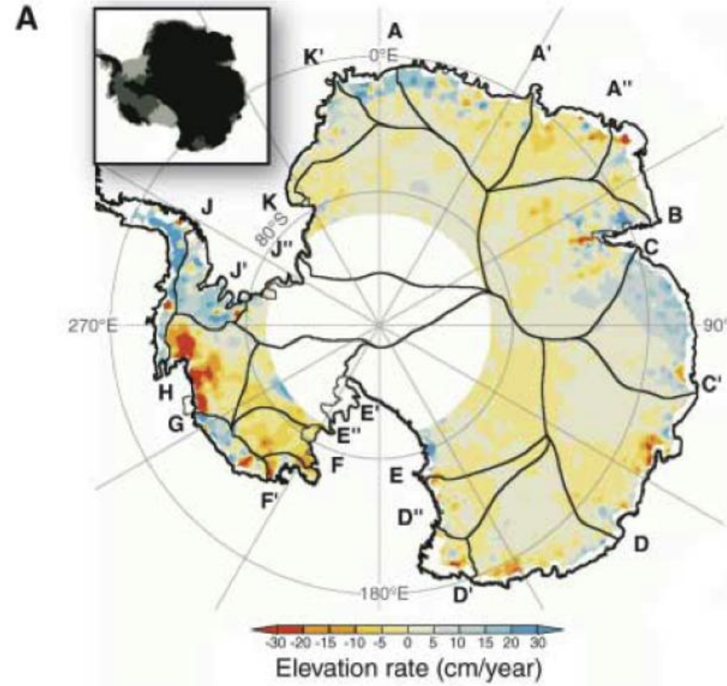
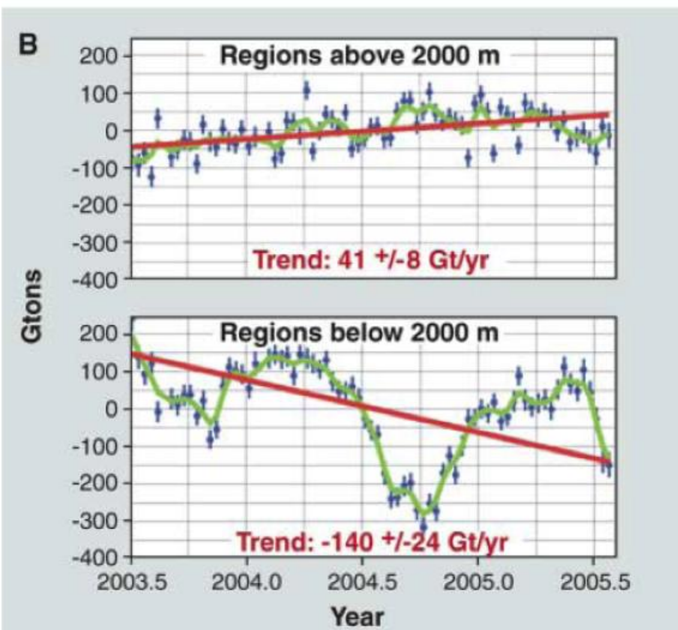
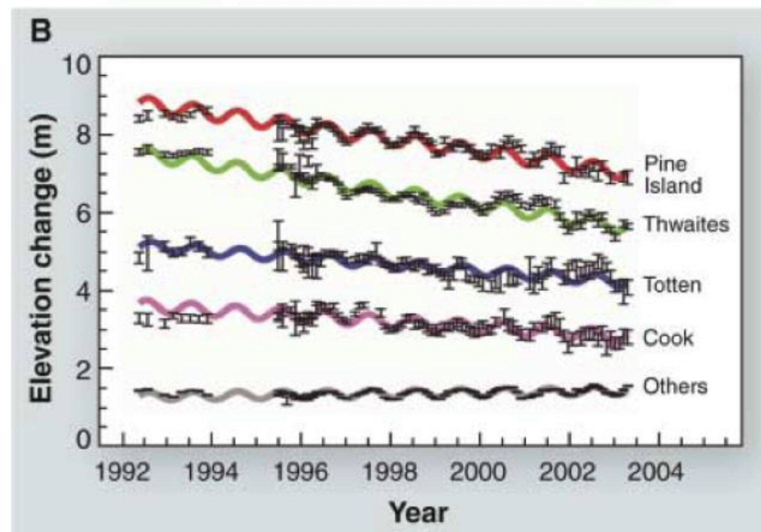
$$\dot{\varepsilon}_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \quad \rightarrow \quad \dot{\varepsilon}_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z}\right)$$









Shepherd & Wingham (*Science*, **315**, 2007)