Quantifying the Uncertainty in Ice Sheet Model Parameters via Model Calibration

Calibration of the Community Ice Sheet Model

M. T. Pratola[1], S. Price[2], J. Gattiker[1] and D. Higdon[1]

[1]Statistical Sciences Group, Los Alamos National Laboratory

[2]COSIM, Fluid Dynamics and Solid Mechanics Group, Los Alamos National Laboratory

June 22, 2011

Outline

- Introduction: Community Ice Sheet Model (CISM)
- Some idealized experiments using CISM
- A Statistical Framework for Combining CISM with field data
- Early results, future directions

CISM

- Historically, ice sheets thought to respond slowly to short-term climate change
- However, recent observations indicate significant ice sheet volume changes as a result of decadal-scale climate forcing
- Potential changes in discharge from Greenland and/or Antarctic ice sheets are the largest unknown w.r.t. future sea-level rise
- CISM describes ice sheet evolution (velocities, thickness, temperature, etc.) assuming appropriate boundary and initial conditions and atmospheric and oceanic forcing (e.g., from CESM).

Goals

- Our goal is to leverage a statistical model calibration framework to better understand and quantify uncertainties in ice-sheet evolution as simulated by CISM
- Here, we investigate idealized scenarios with uncertainties in:
    - Flow law exponent, n
    - Flow law rate factor activation energy, Q
    - Constant (in $\mathbf{x}, t$) ice-shelf basal melt rate, m

Experimental Setup

- Modified version of the standard "confined shelf" test case:
  - isothermal, rectangular shelf of uniform thickness
  - confined at upstream and lateral margins (zero flux bc)
  - open to the ocean at downstream margin (specified stress bc)
- Constant and steady surface mass balance applied for experiments where n and Q vary
- Constant n, Q, and surface accumulation for experiments where m varied
- All experiments evolve to approximate SS from t=0 to t=1000 yrs

CISM Confined Shelf Example

3 Idealized Experiments

- Represent CISM as $\eta(\theta)$ where θ is some parameter vector of interest
- Ensemble of CISM runs $\{Y_i^c = \eta(\theta_i)\}$ at different θ_i's to get ensemble of outputs
- Choose a "true" value θ_0 to simulate a field observation. The observation, Y^f, is constructed as $\eta(\theta_0)+$ error.
  - Experiment 1: $\theta = n$, select 7 settings for $n \in [1.5, 4.0]$ for model runs. $n_0 =?$
  - Experiment 2: $\theta = (n, Q)$, select 8 settings for $n, Q \in ([1.5, 4.0], [4e4, 8e4])$. $(n_0, Q_0) =?$
  - Experiment 3: $\theta = m$, select 7 settings for $m \in [1.0, 5.0]$. $m_0 =?$

A Statistical Framework for Uncertainty Quantification

- Statiscal computer model calibration experiments - eg. Kennedy & O'Hagan (2001), Higdon et al. (2004), amongst others.
- Useful in situations where
  - model costly to run
  - combine field observations and model output
  - quantify uncertainty in parameters and model predictions

Statistical Calibration Model

Model: $y^f(\mathbf{x}) = \eta(\mathbf{x}, \theta_0) + \epsilon(\mathbf{x})$; $y_i^c(\mathbf{x}, \theta_i) = \eta(\mathbf{x}, \theta_i)$

- We have computer model outputs

$$\mathbf{Y}^c = \left(Y_1^{c\,T}, \ldots, Y_N^{c\,T}\right)^T$$

  and field observations

$$\mathbf{Y}^f = \left(y^f(\mathbf{x}_1), \ldots, y^f(\mathbf{x}_M)\right)^T.$$

- Then a joint model for all the data is:

$$\begin{pmatrix} \mathbf{Y^f} \\ \mathbf{Y^c} \end{pmatrix} \sim N\left(\mu, \sigma^2 \begin{bmatrix} \mathbf{R^f} & \mathbf{R^{fc}} \\ \mathbf{R^{fc}} & \mathbf{R^c} \end{bmatrix} + \begin{bmatrix} \sigma_\epsilon^2 \mathbf{I_n} & 0 \\ 0 & 0 \end{bmatrix}\right),$$

$$corr(y_k^c(x_i, \theta_k), y^f(x_j, \theta_0)) = e^{-\sum_{p=1}^{P} \gamma_p (x_{ip} - x_{jp})^2 - \sum_{q=1}^{Q} \phi_q (\theta_{kq} - \theta_{0q})^2}$$

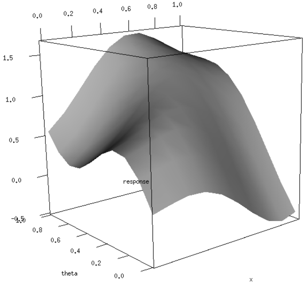
Statistical Calibration Model

- Posterior of parameters is $\theta_0, \mu, \sigma^2, \sigma_\epsilon^2, \gamma, \phi | \mathbf{Y^f}, \mathbf{Y^c} \propto L(\mathbf{Y^f}, \mathbf{Y^c} | \cdot) \pi(\theta_0, \mu, \sigma^2, \sigma_\epsilon^2, \gamma, \phi)$ which we can sample using MCMC to get our parameter estimates.
- Can also sample the posterior predictive distribution, $y^c(\mathbf{x}, \theta_0) | \mathbf{Y^f}, \mathbf{Y^c}$ for making predictive inference.
- In particular,

$$E[y^c(\mathbf{x}, \theta_0) | \mathbf{Y^f}, \mathbf{Y^c}, \cdot] = \mathbf{w}^T \left(\begin{pmatrix} \mathbf{Y^f} \\ \mathbf{Y^c} \end{pmatrix} - \mu \right)$$

where $\mathbf{w}^T = \mathbf{c}^T \mathbf{\Sigma}^{-1}$, with $\mathbf{\Sigma}$ as before, and $\mathbf{c}^T = \left[cov\left(y^c(\mathbf{x}, \theta_0), y^f(\mathbf{x}_1, \theta_0) \right), \ldots \right]$.

The Idea (in pictures)



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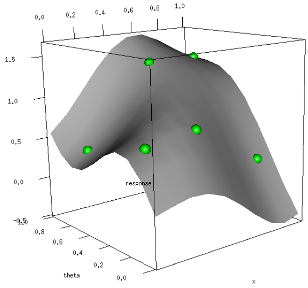

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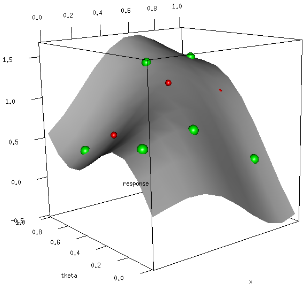

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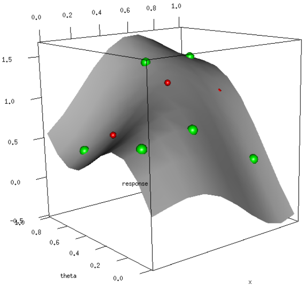

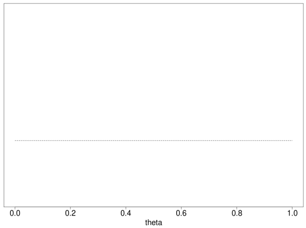

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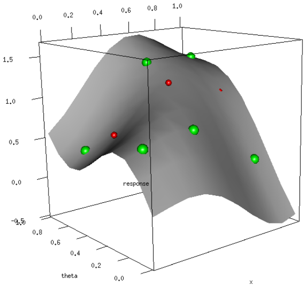

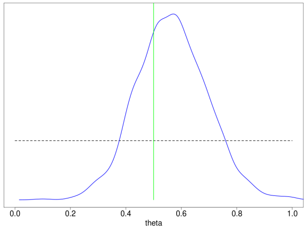

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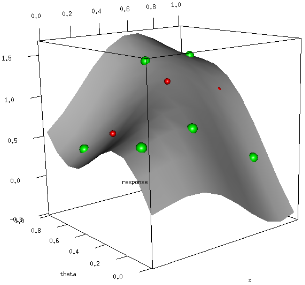

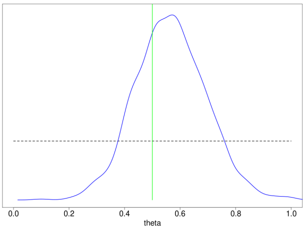

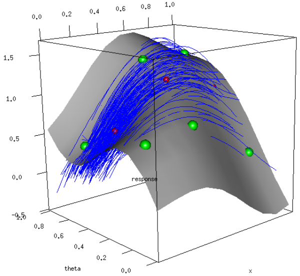

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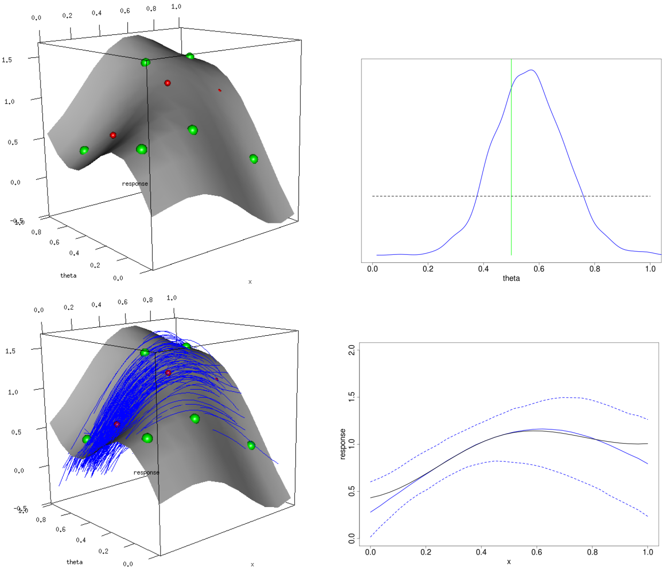
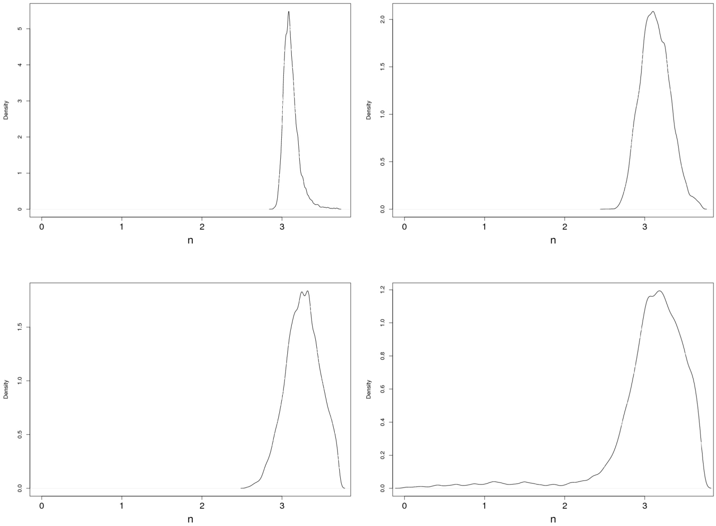
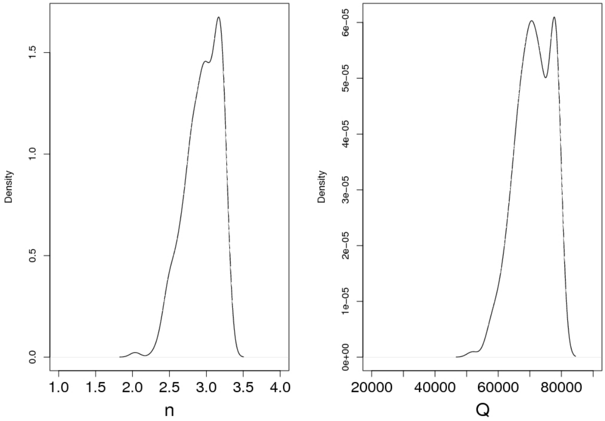


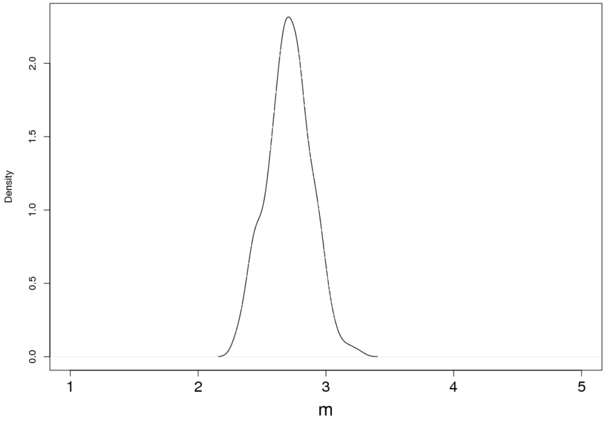
Results: Experiment 1 ($n_0 = 3$)



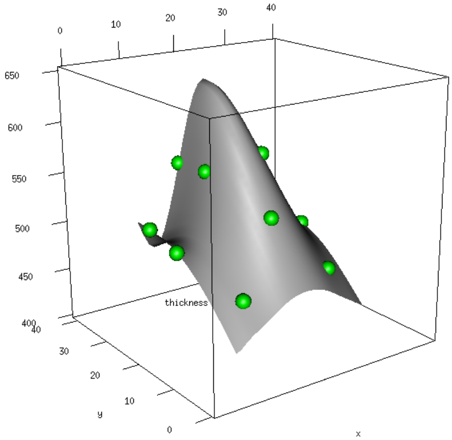
Results: Experiment 2 ($n_0 = 3$, $Q_0 = 60 \times 10^3$)



Results: Experiment 3 ($m_0 = 2.72$)



Calibrated Prediction (eg: melt experiment)



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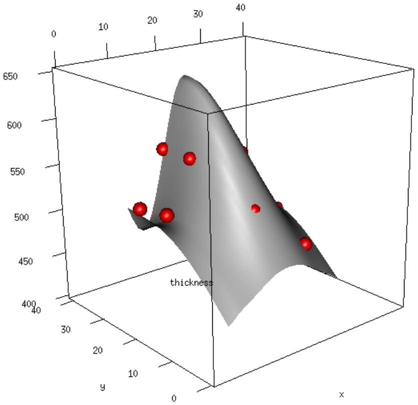
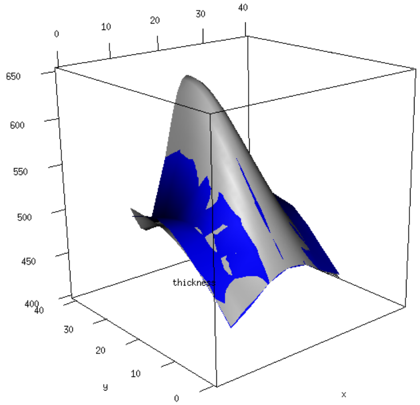


Calibrated Prediction (eg: melt experiment)



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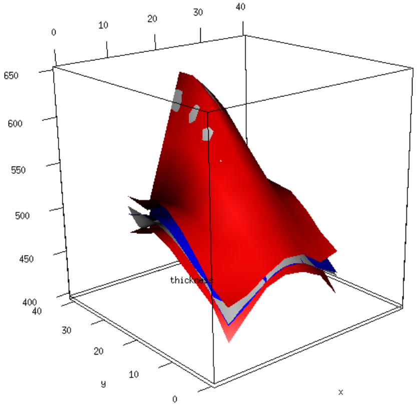

Conclusions & Future Directions

- Outlined a typical approach to statistical uncertainty quantification in model calibration
- Method gave reasonable results, but improvements needed to analyze simulated examples that are closer to the real-world problem
- Scientific goal is to perform uncertainty quantification for an ice-shelf where, for example, n, Q and m (or other parameters of interest) are unknown (we're not there yet).