Resolving Grounding Line Dynamics with the BISICLES AMR Ice Sheet Model

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CESM-LIWG Meeting

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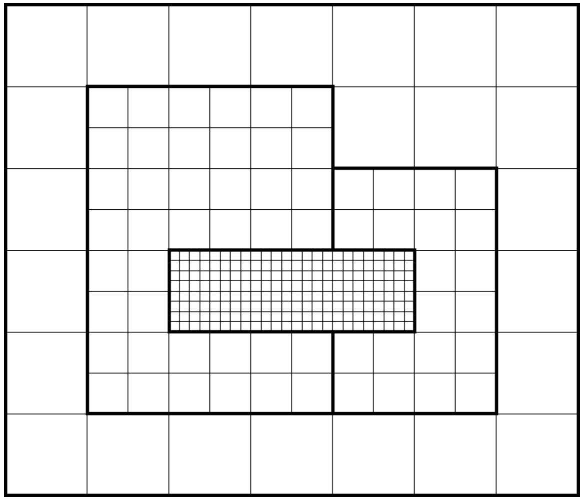
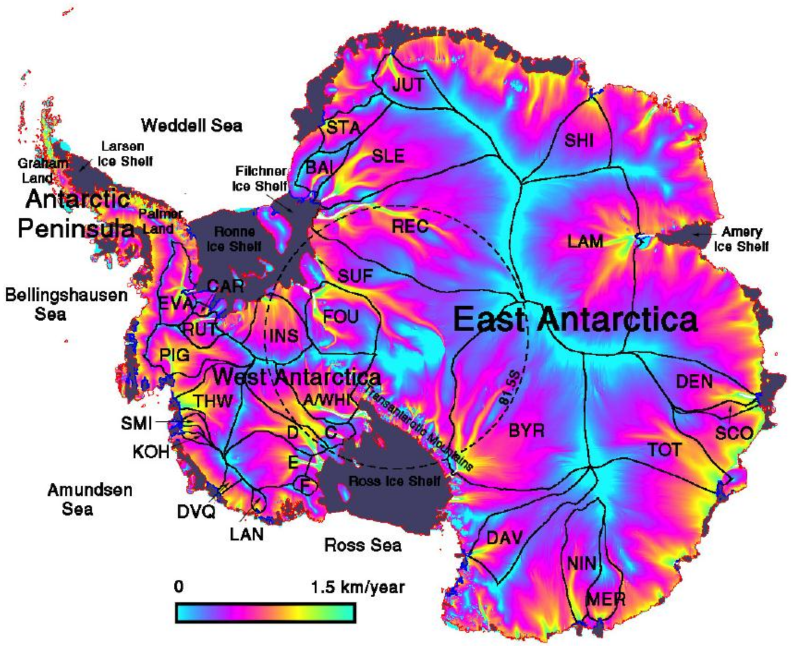
Berkeley-ISICLES (BISICLES)

- DOE ISICLES-funded project to develop a scalable adaptive mesh refinement (AMR) ice sheet model/dycore
  - Local refinement of computational mesh to improve accuracy
- Use Chombo AMR framework to support block-structured AMR
  - Support for AMR discretizations
  - Scalable solvers
  - Developed at LBNL
  - DOE ASCR supported (FASTMath)
- Interface to CISM (and CESM) as an alternate dycore
- Collaboration with LANL and Bristol (U.K.)
- Continuation in SciDAC-funded PISCEES effort



Why is this useful? *(another BISICLE for another fish?)*



[Rignot & Thomas, 2002]

- Ice sheets -- Localized regions where high resolution needed to accurately resolve ice-sheet dynamics (500 m or better at grounding lines)
- Antarctica is really big – too big to resolve at that level of resolution.
- Large regions where such fine resolution is unnecessary (e.g. East Antarctica)
- Well-suited for adaptive mesh refinement (AMR)
- Problems still large: need good parallel efficiency
- Dominated by nonlinear coupled elliptic system for ice velocity solve: good linear and nonlinear solvers

BISICLES: Models and Approximations

Physics: Non-Newtonian viscous flow: $\mu(\dot{\epsilon}^2, T) = A(T)(\dot{\epsilon}^2)^{\frac{(1-n)}{2}}$

- **Full-Stokes**
  - Best fidelity to ice sheet dynamics
  - Computationally expensive (full 3D coupled nonlinear elliptic equations)
- **Approximate Stokes**
  - Use scaling arguments to produce simpler set of equations
  - Common expansion is in ratio of vertical to horizontal length scales $\left(\varepsilon = \frac{[h]}{[l]}\right)$
  - E.g. Blatter-Pattyn (most common "higher-order" model), accurate to $O(\varepsilon^2)$
  - Still 3D, but solve simplified elliptic system (e.g. 2 coupled equations)
- **Depth-integrated**
  - "Shallow Ice" and "Shallow-Shelf" approximations (accurate to $O(\varepsilon)$)
  - Special case of approximate Stokes with 2D equation set
  - Easiest to work with computationally, generally less accurate

"L1L2" Model *(Schoof and Hindmarsh, 2010).*

- Uses asymptotic structure of full Stokes system to construct a higher-order approximation
  - Expansion in ε and $\lambda = \frac{[\tau_{shear}]}{[\tau_{normal}]}$ (ratio of shear & normal stresses)
    - Large λ: shear-dominated flow
    - Small λ: sliding-dominated flow
  - Computing velocity to $O(\varepsilon^2)$ only requires τ to $O(\varepsilon)$
- Computationally **much** less expensive -- enables fully 2D vertically integrated discretizations. (can reconstruct 3d)
- Similar formal accuracy to Blatter-Pattyn $O(\varepsilon^2)$
  - Recovers proper fast- and slow-sliding limits:
    - SIA $(1 \ll \lambda \leq \varepsilon^{-1/n})$ -- accurate to $O(\varepsilon^2 \lambda^{n-2})$
    - SSA $(\varepsilon \leq \lambda \leq 1)$ - accurate to $O(\varepsilon^2)$

"L1L2" Model (Schoof and Hindmarsh, 2010), cont.

- Use this result to construct a computationally efficient scheme:

1. Approximate constitutive relation relating $grad(u)$ and stress field τ with one relating $grad(u|_{z=b})$, vertical shear stresses τ_{xz} and τ_{xz} given by the SIA / lubrication approximation and other components $\tau_{xx}(x, y, z)$, $\tau_{xy}(x, y, z)$, etc

2. leads to an effective viscosity $\mu(x, y, z)$ which depends only on $grad(u|_{z=b})$ and $grad(z_s)$, ice thickness, etc

3. Momentum equation can then be integrated vertically, giving a nonlinear, 2D, elliptic equation for $u|_{z=b}(x, y)$

4. $u(x, y, z)$ can be reconstructed from $u|_{z=b}(x, y)$

Temporal Discretization

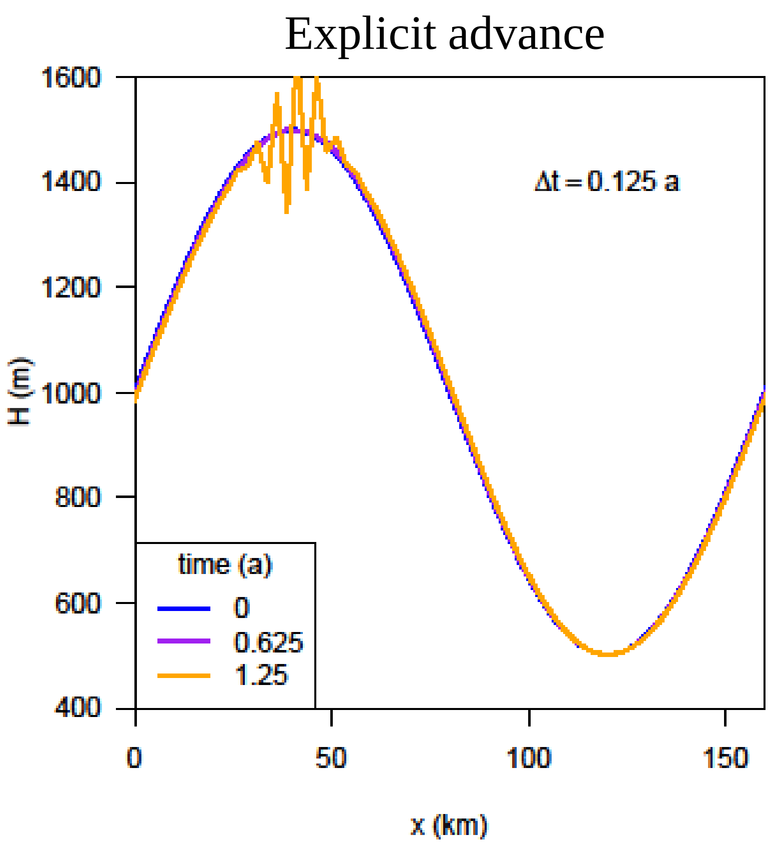
Update equation for H: $\frac{\partial H}{\partial t} + \nabla \cdot (\vec{u}H) = S$

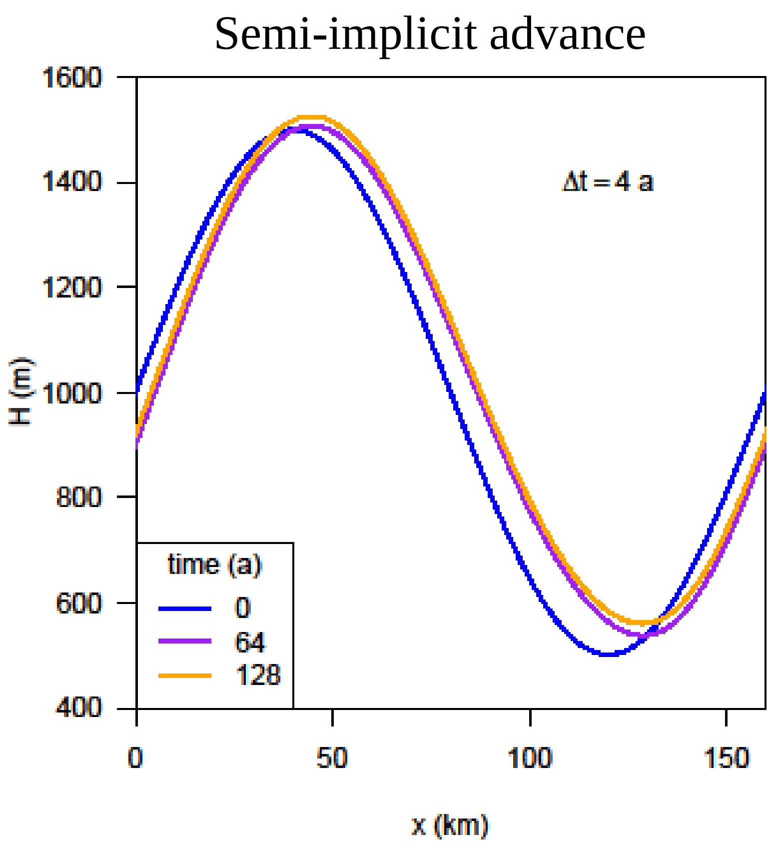
- "looks" like hyperbolic advection equation (explicit scheme, Courant stability -- $\Delta t \propto \Delta x$)
- Velocity field has ∇H piece – diffusion equation for H ($\Delta t \propto \Delta x^2$!)

- Strategy (Cornford) – try to factor out diffusive flux and discretize as an advection-diffusion equation:
  - $\vec{F} = \vec{u}H = \vec{F}_{advective} + \vec{F}_{diffusive}$
  - $\vec{F}_{diffusive} = -D\,\nabla H$
  - Now solve: $\frac{\partial H}{\partial t} + \nabla \cdot \vec{F}_{advective} = \nabla \cdot (D\,\nabla H) + S$
    - Advective fluxes: explicit update using unsplit 2nd Order PPM scheme
    - Diffusive fluxes: implicit update (Backward Euler for now)

Temporal Discretization (cont)

- Test case based on ISMIP-HOM A geometry
- $\Delta x = 2.5\ km, \Delta t_{CFL}$ = 5 a





- **Unfortunately, still run into stability issues finer than $\Delta x < 0.5\ km$!**

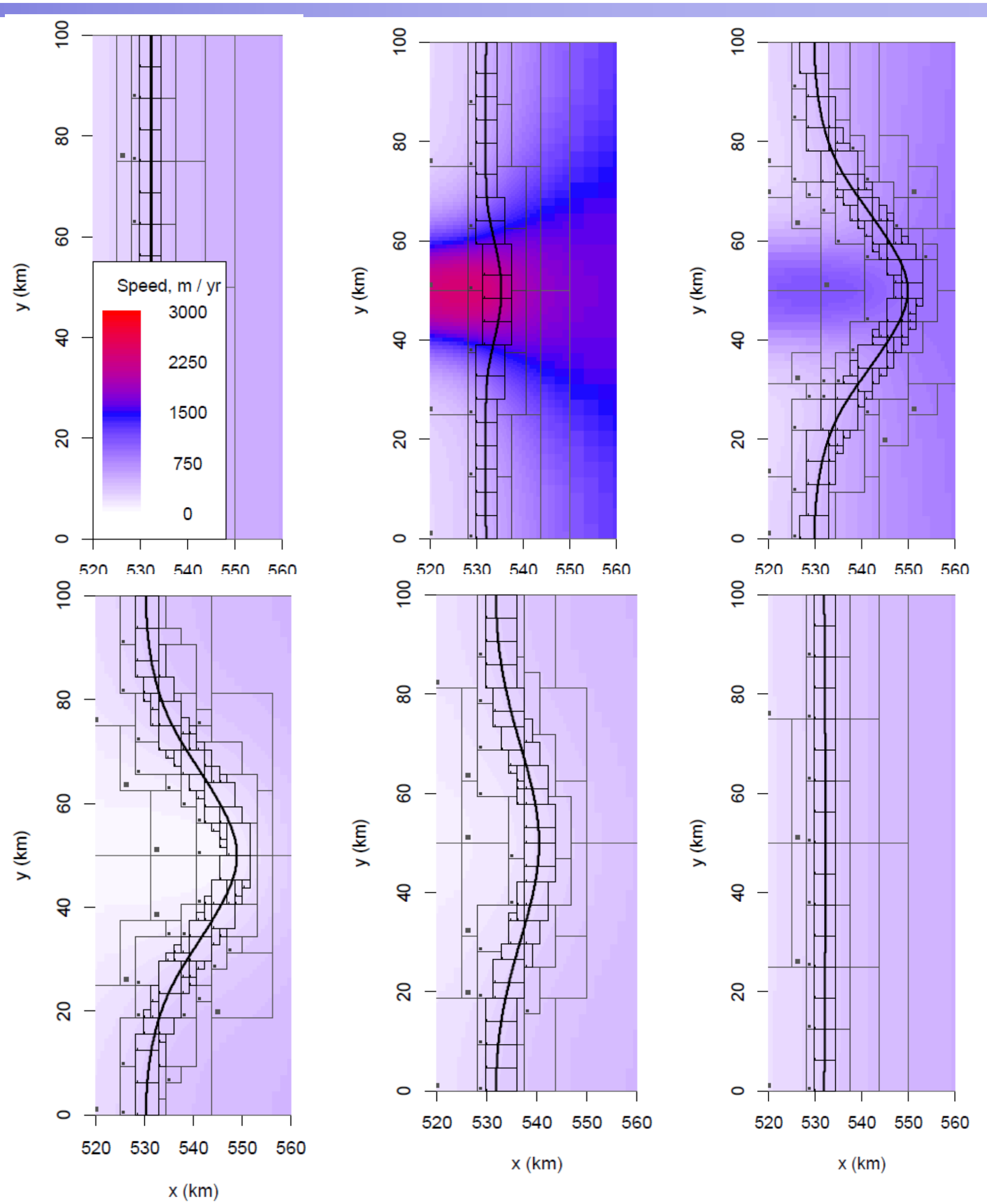
Modified "L1L2" Model (SSA)*

- Use this result to construct a computationally efficient scheme:
  1. Approximate constitutive relation relating $grad(u)$ and stress field τ with one relating $grad(u|_{z=b})$, vertical shear stresses τ_{xz} and τ_{xz} given by the SIA / lubrication approximation and other components $\tau_{xx}(x,y,z)$, $\tau_{xy}(x,y,z)$, etc
  2. leads to an effective viscosity $\mu(x,y,z)$ which depends only on $grad(u|_{z=b})$ and $grad(z_s)$, ice thickness, etc
  3. Momentum equation can then be integrated vertically, giving a nonlinear, 2D, elliptic equation for $u|_{z=b}(x,y)$
  4. ~~$u(x,y,z)$ can be … (x,y)~~
  4. Use $u(x,y,z) = u|_{z=b}(x,y)$ (neglect vertical shear in flux velocity)

BISICLES Results - MISMIP3D

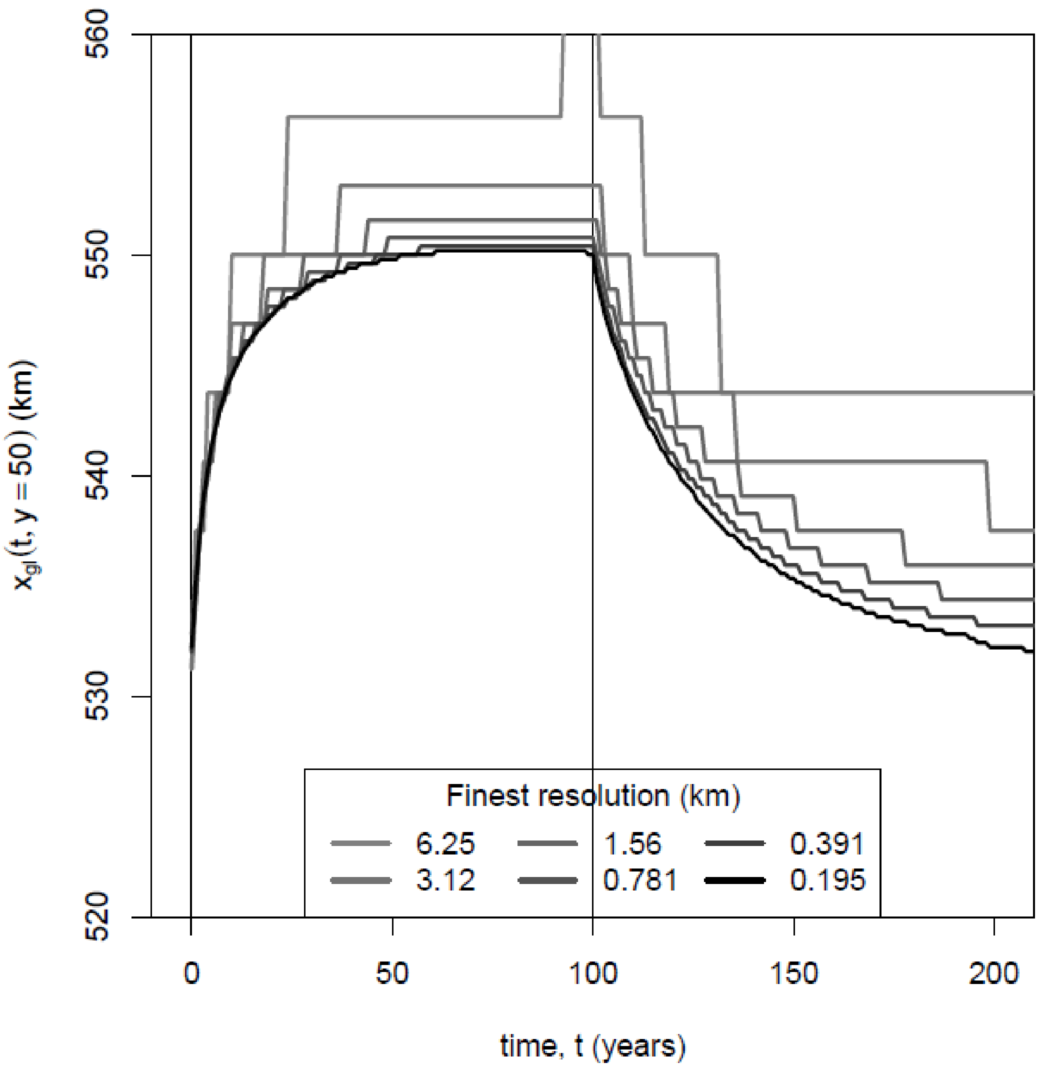
Experiment P75R:
(Pattyn et al (2011)

- Begin with steady-state (equilibrium) grounding line.
- Add Gaussian slippery spot perturbation at center of grounding line
- Ice velocity increases, GL advances.
- After 100 years, remove perturbation.
- Grounding line should return to original steady state.
- Figures show AMR calculation:
  - $\Delta x_0 = 6.5km$ base mesh,
  - 5 levels of refinement
  - Finest mesh $\Delta x_4 = 0.195km$.
  - t = 0, 1, 50, 101, 120, 200 *yr*
- Boxes show patches of refined mesh.
- GL positions match Elmer (full-Stokes)



MISMIP3D (cont)

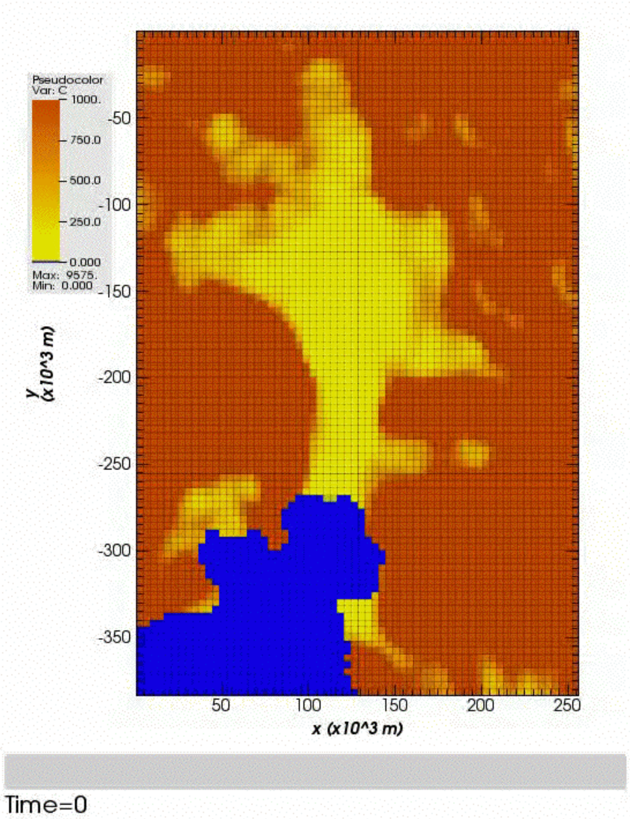
- Plot shows grounding line position x_{GL} at $y = 50km$ vs. time for different spatial resolutions.
- $\Delta x = \mathbf{0.195}\boldsymbol{km} \rightarrow \mathbf{6.25}\ \boldsymbol{km}$
- Appears to require finer than 1 km mesh to resolve dynamics
- Converges as O(Δx) (as expected)



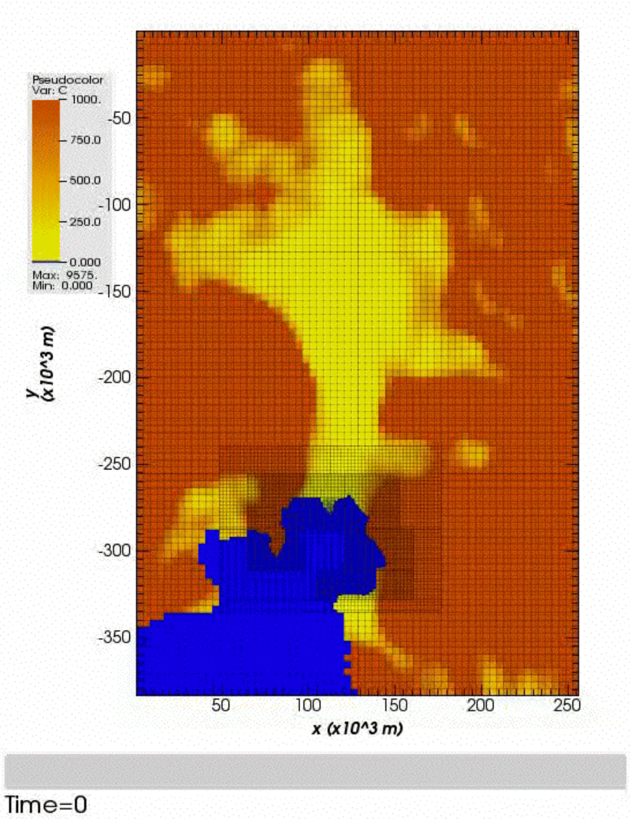
BISICLES Results – Pine Island Glacier

- Cornford, et al, JCP (2011, submitted)
- PIG configuration from LeBrocq:
  - Bathymetry: combined Timmerman (2010), Jenkins (2010), Nitsche (2007)
  - AGASEA thickness
  - Isothermal ice, A=4.0$\times 10^{-17}\ Pa^{-\frac{1}{3}} m^{-1/3} a$
  - Basal friction chosen to roughly agree with Joughin (2010) velocities
- Specify melt rate under shelf:
  - $M_s = \begin{cases} 0 & H < 50m \\ \frac{1}{9}(H - 50) & 50 \le H \le 500m \\ 50 & H > 500\, m \end{cases}$ m/a
- Constant surface flux = 0.3 m/a
- Evolve problem – refined meshes follow the grounding line.
- Calving model and marine boundary condition at calving front

PIG (cont)



Pseudocolor
Var: C
1000.
750.0
500.0
250.0
0.000
Max: 9575.
Min: 0.000
y (x10^3 m)
x (x10^3 m)
Time=0



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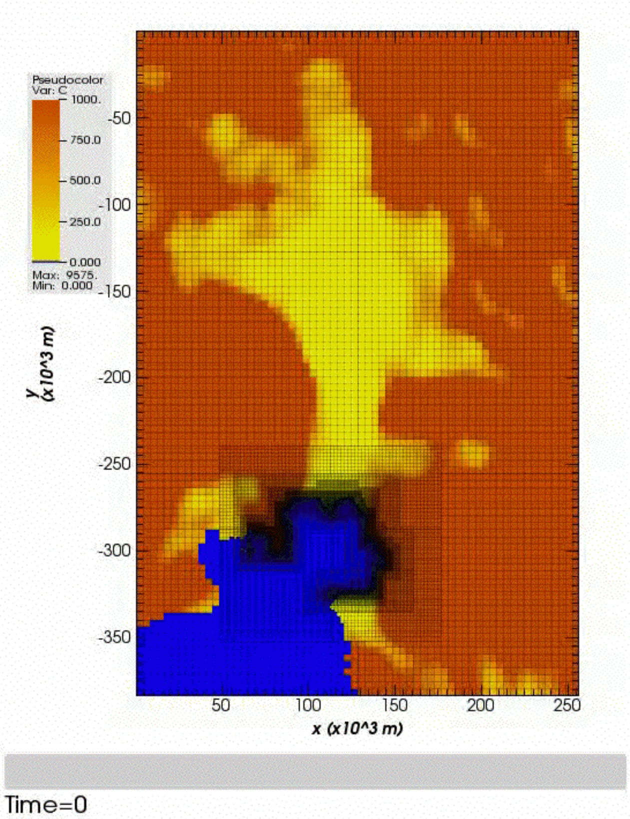

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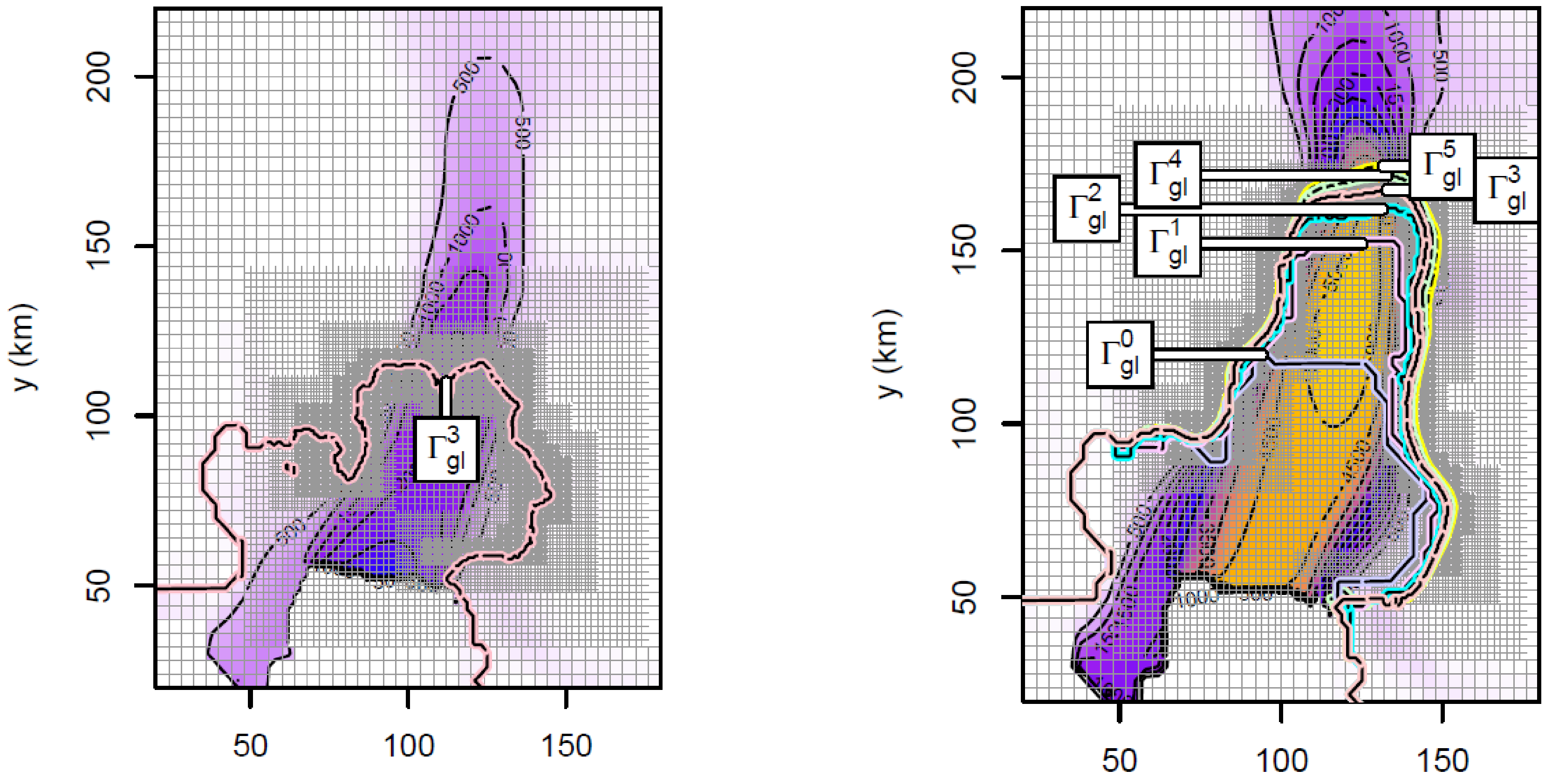
Max: 9575.
Min: 0.000
y (x10^3 m)
x (x10^3 m)
Time=0

PIG, cont

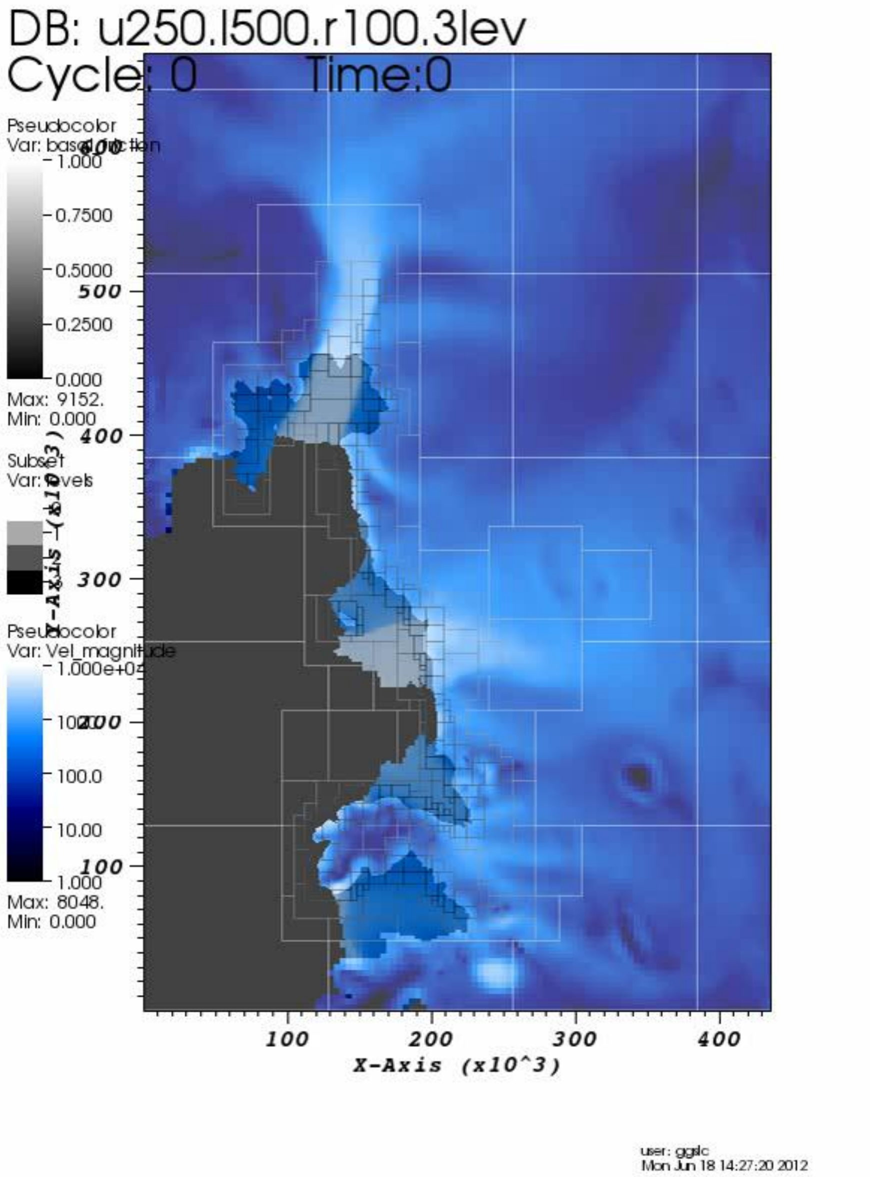


Initial Condition **Solution after 30 years**

Coloring is ice velocity, Γ_{gl} is the grounding line. Superscripts denote number of refinements. Note resolution-dependence of Γ_{gl}

Amundsen Sea Sector

- Regional Model
- Heavy subshelf melting drives retreat (up to 100 m/a)
- Melt rate function of depth (strongest melting near GL)
- 4 km base mesh
- 3 levels of refinement (2km, 1km, 500m)
- Courtesy of Steph Cornford



Filchner-Ronne/Ross

- Light melting (< 5 m/a)
- 5 km base resolution
- 2 refinement levels (2.5km, 1.25km)
- “few hours” for 32 processors to evolve for 50 yrs
- Courtesy of Steph Cornford
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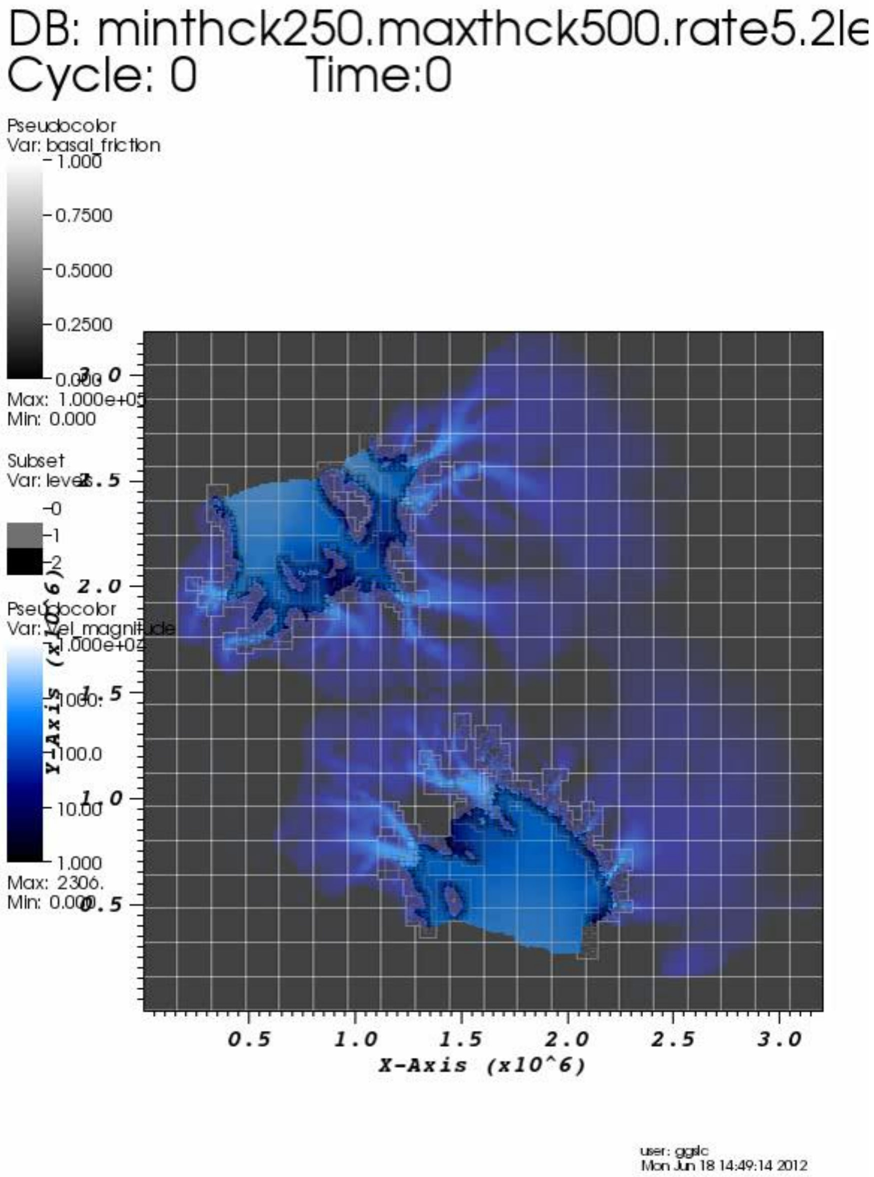
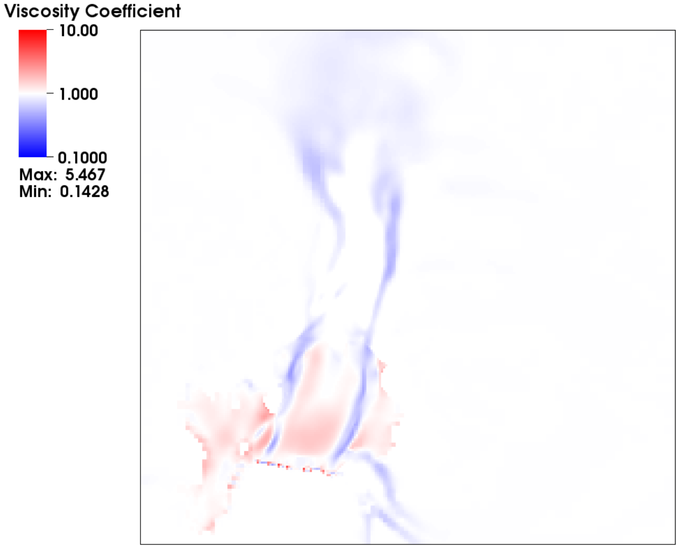
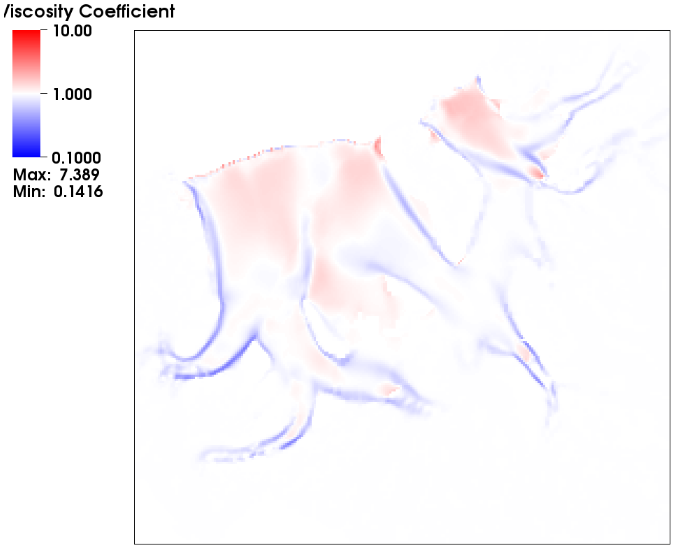


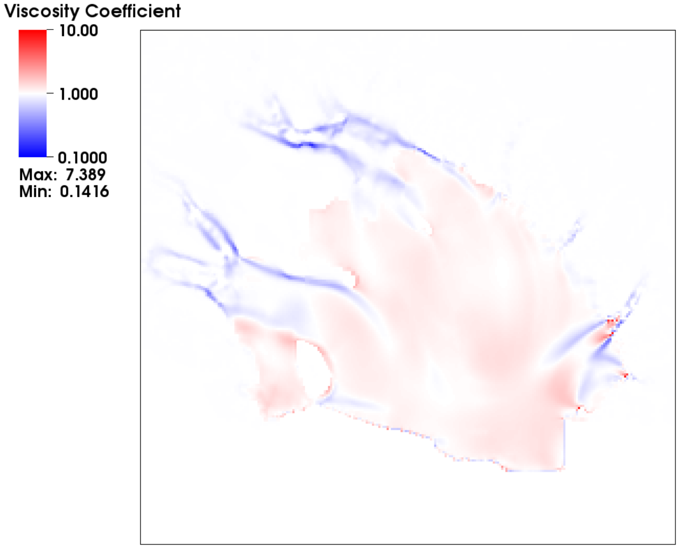
Simple Rheology/Damage model



Pine Island Glacier



Filchner-Ronne Ice Shelf

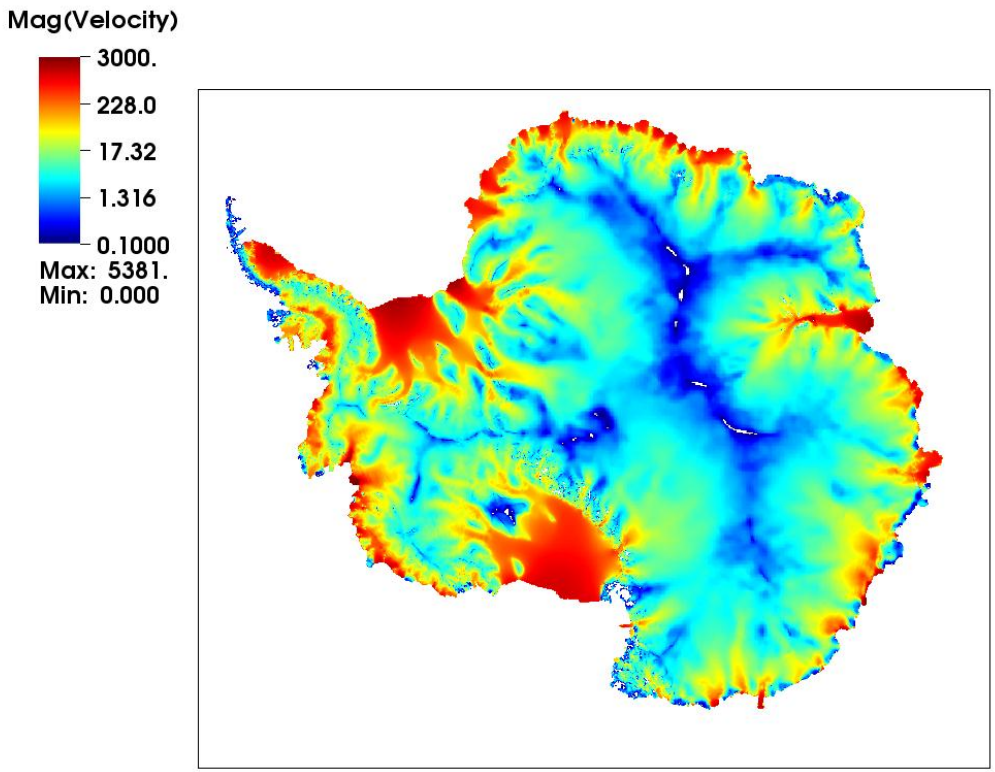


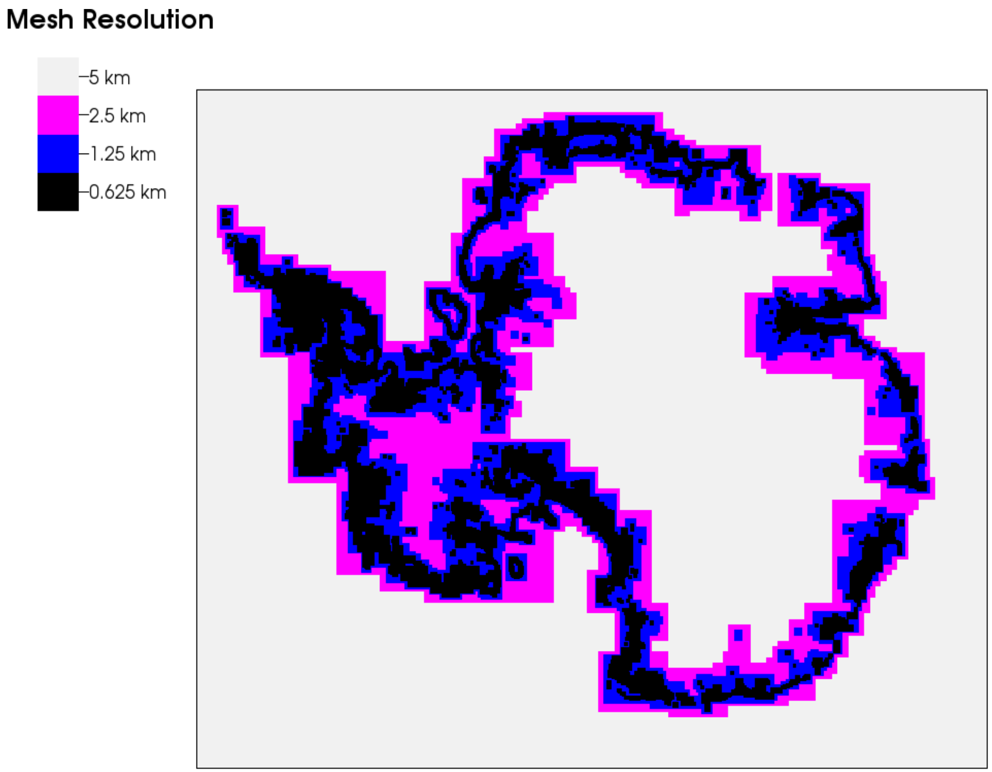
Ross Ice Shelf

- Solve control problem for ice initial condition
- Include new parameter φ which multiplies viscosity
- $\varphi < 1$ (blue) = softening
- $\varphi > 1$ (red) = hardening

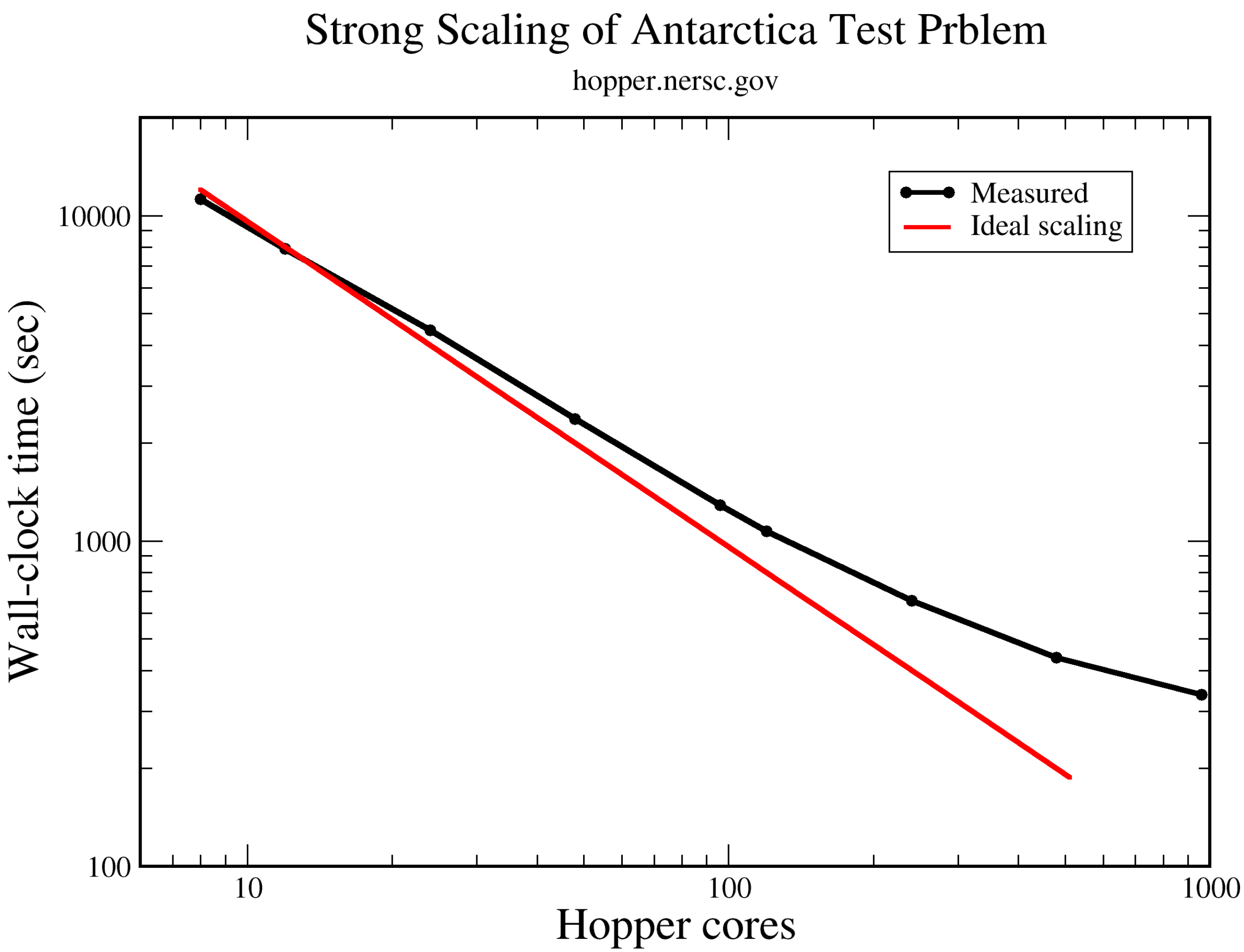
Antarctica (Ice2Sea)

- Refinement based on Laplacian(velocity), grounding lines
- 5 km base mesh with 3 levels of refinement
  - base level (5 km): 409,600 cells (100% of domain)
  - level 1 (2.5 km): 370,112 cells (22.5% of domain)
  - Level 2 (1.25 km): 955,072 cells (14.6% of domain)
  - Level 3 (625 m): 2,065,536 cells (7.88% of domain)



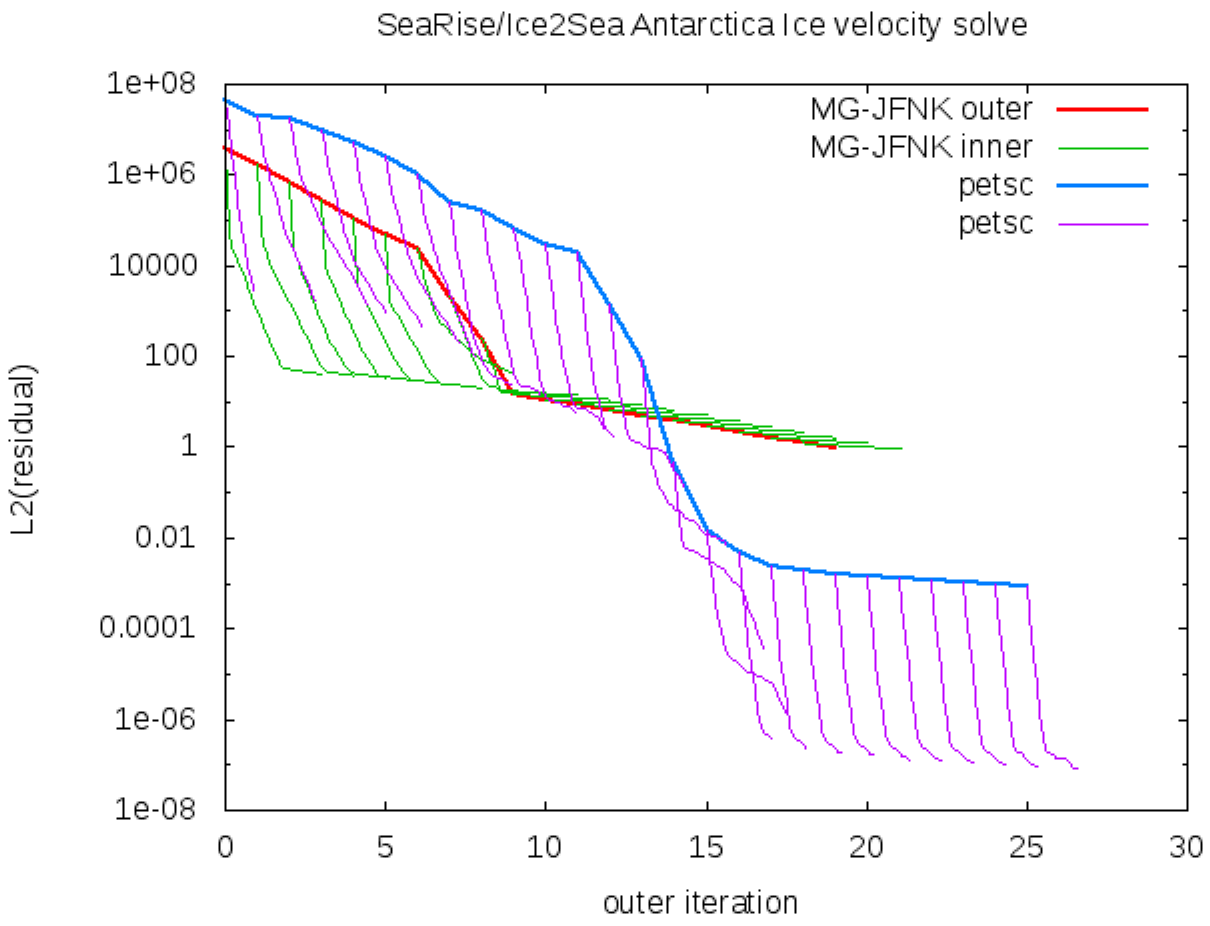


Parallel scaling, Antarctica benchmark



(Preliminary scaling result – includes I/O and serialized initialization)

Linear Solvers – GAMG vs. Geometric MG



Conclusions

- Fine (sub 1-km) resolution required to get grounding lines right
- AMR is a natural fit for this problem
- Split advective/diffusive approach to temporal evolution looked promising, but was eventually insufficient.
- "SSA*" modified L1L2 approach improves stability, appears to be "good enough" for grounding lines and fast-flowing ice streams and shelves.

BISICLES – Next steps

- More work with linear and nonlinear velocity solves.
  - PETSc/AMG linear solvers look promising (in progress)
- Revisit semi-implicit time-discretization for stability, accuracy.
- Finish coupling with existing Glimmer-CISM code and CESM
- Full-Stokes for grounding lines?
- Embedded-boundary discretizations for GL's and margins.
- Performance/scaling optimization and autotuning.
- Refinement in time?

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- US Department of Energy Office of Science (ASCR) funded BISICLES project
- US Department of Energy Office of Science (ASCR/BER) SciDAC applications program (PISCEES)
- Steph Cornford, Tony Payne at the University of Bristol
- Bill Lipscomb, Doug Ranken, Stephen Price (LANL)
- Mark Adams (Columbia University)

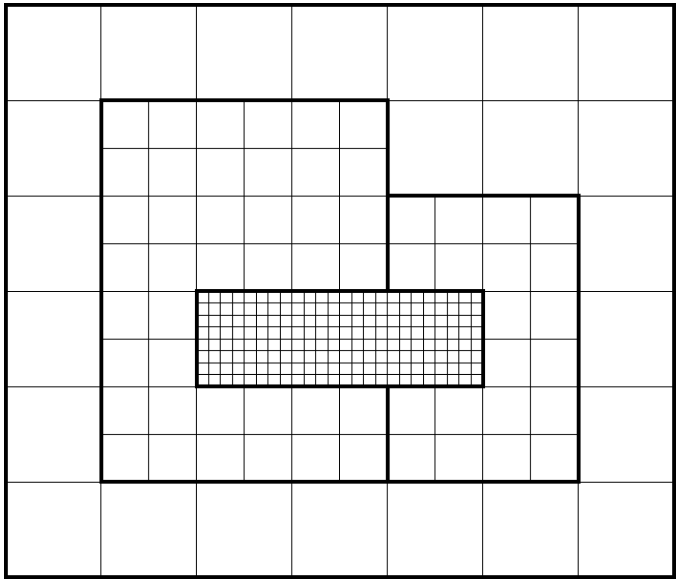
Extras

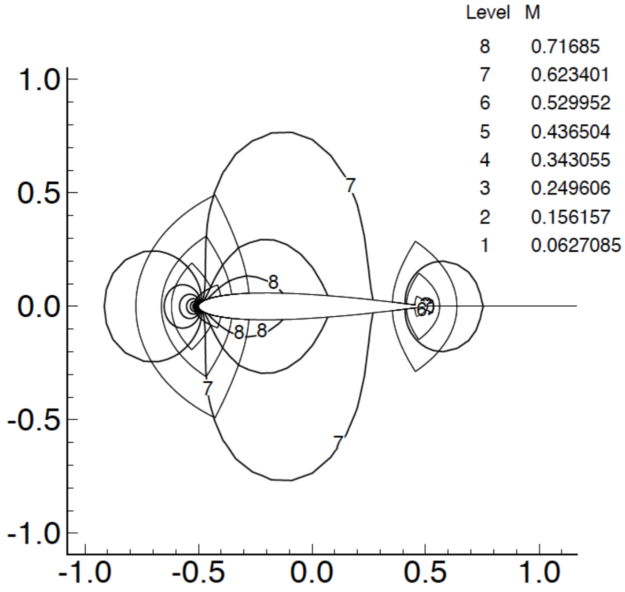
Interface with Glimmer-CISM

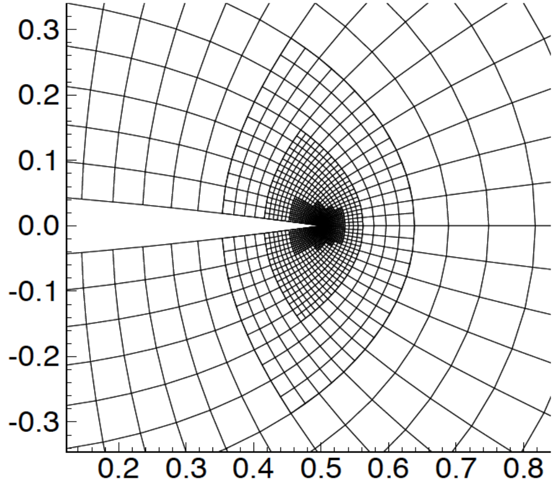
- Glimmer-CISM has coupler to CESM, additional physics
  - Well-documented and widely accepted
- Our approach – couple to Glimmer-CISM code as an alternate "dynamical core"
  - Allows leveraging existing Glimmer-CISM capabilities
  - Use the same coupler to CESM
  - BISICLES code sets up within Glimmer-CISM and maintains its own storage, etc.
  - Communicates through defined interface layer
  - Instant access to a wide variety of test problems
  - Interface development almost complete
  - Part of larger alternative "dycore" discussion for Glimmer-CISM

Block-Structured Local Refinement

- Refined regions are organized into rectangular patches.







- *Algorithmic advantages:*
  - *Build on mature structured-grid discretization methods.*
  - *Low overhead due to irregular data structures, relative to single structured-grid algorithm.*

Models and Approximations

- Full-Stokes
  - Best fidelity to ice sheet dynamics
  - Computationally expensive (full 3D coupled nonlinear elliptic equations)
- Approximate Stokes
  - Use scaling arguments to produce simpler set of equations
  - Common expansion is in ratio of vertical to horizontal length scales ($\varepsilon = \frac{[h]}{[l]}$)
  - E.g. Blatter-Pattyn (most common "higher-order" model), accurate to $O(\varepsilon^2)$
  - Still 3D, but solve simplified elliptic system (e.g. 2 coupled equations)
- Depth-integrated
  - Special case of approximate Stokes with 2D equation set ("Shelfy-stream")
  - Easiest to work with computationally
  - Generally less accurate

"L1L2" Model *(Schoof and Hindmarsh, 2010)*

- Asymptotic expansion in 2 flow parameters:
  - ε -- ratio of length scales $\frac{[h]}{[x]}$
  - λ – ratio of shear to normal stresses $\frac{[\tau_{shear}]}{[\tau_{normal}]}$
    - Large λ: shear-dominated flow
    - Small λ: sliding-dominated flow
- Blatter-Pattyn approximates full-Stokes to $O(\varepsilon^2)$ for all λ regimes

- Asymptotic expansion: (e.g. $u(x,z) = u_0 + \varepsilon u_1 + O(\varepsilon^2)$)
  - Leading order velocity term: $u_0 = u_0(x)$ **(no vertical dependence)**
  - Don't need shear stresses to $O(\varepsilon^2)$ to compute velocity to $O(\varepsilon^2)$
  - Provides basis for depth-integrated approach

"L1L2" Model (Schoof and Hindmarsh, 2010).

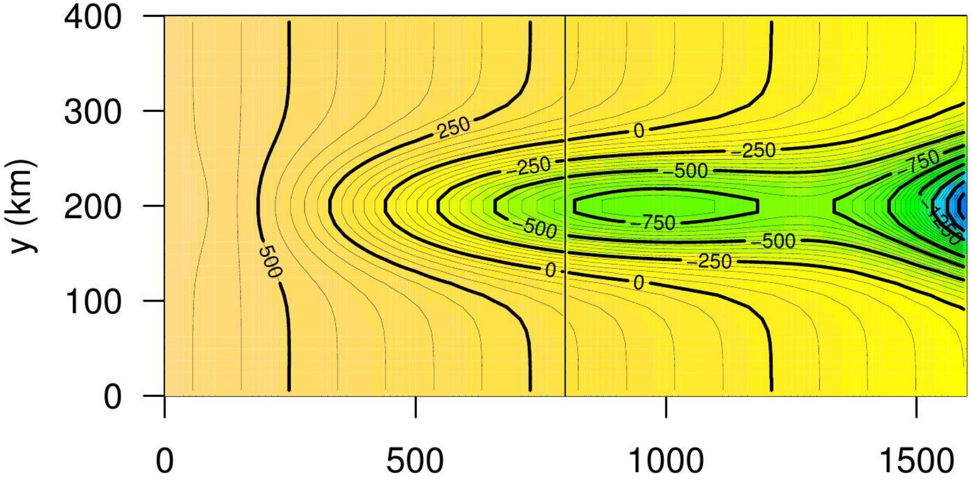
- Uses asymptotic structure of full Stokes system to construct a higher-order approximation
  - Expansion in ε -- ratio of length scales $\frac{[h]}{[x]}$
  - Computing velocity to $O(\varepsilon^2)$ only requires τ to $O(\varepsilon)$

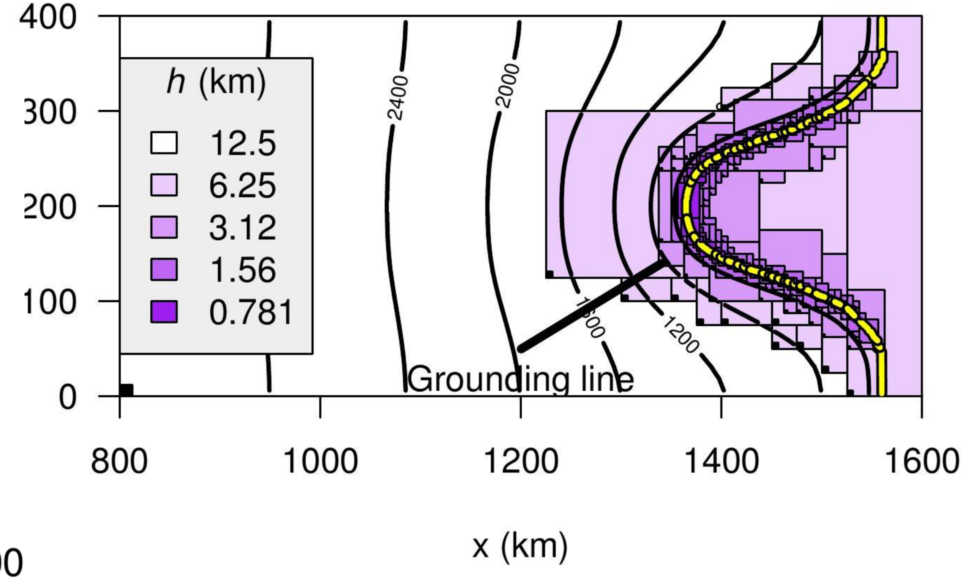
- Computationally **much** less expensive -- enables fully 2D vertically integrated discretizations. (can reconstruct 3d)

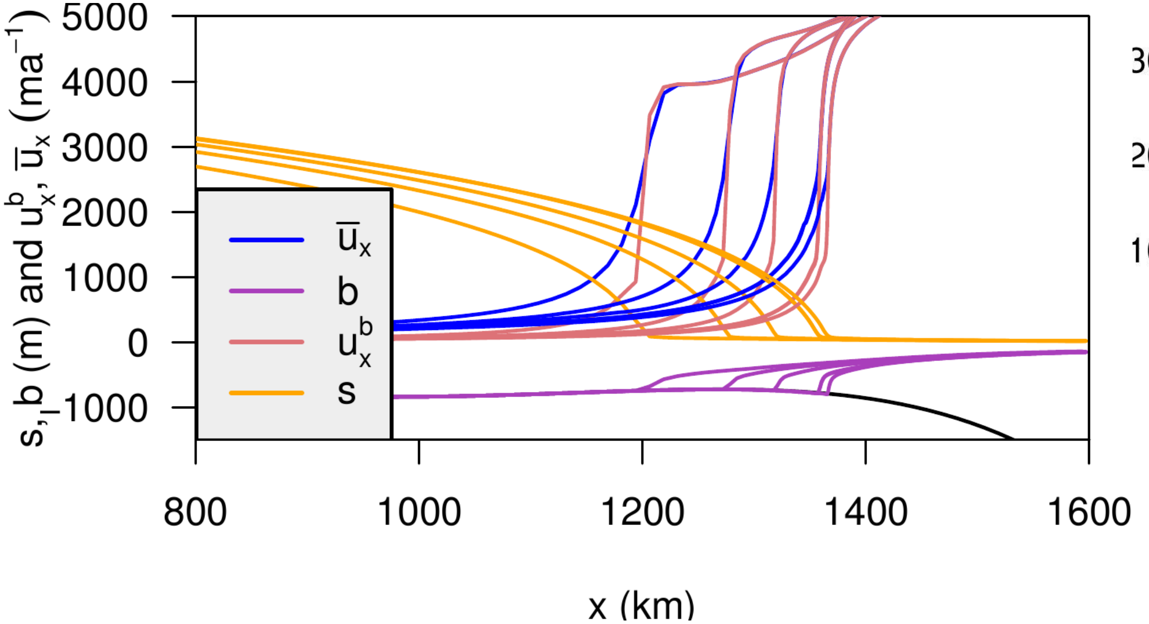
- Similar formal accuracy to Blatter-Pattyn $O(\varepsilon^2)$
  - Recovers proper fast- and slow-sliding limits:
    - SIA $(1 \ll \lambda \leq \varepsilon^{-1/n})$ -- accurate to $O(\varepsilon^2 \lambda^{n-2})$
    - SSA $(\varepsilon \leq \lambda \leq 1)$ - accurate to $O(\varepsilon^2)$

BISICLES results – Grounding line study

- Bedrock topography based on Katz and Worster (2010)
- Evolve initially uniform-thickness ice to steady state
- Repeatedly add refinement and evolve to steady state
- G.L. advances with finer resolution
- Appear to need better than 1 km

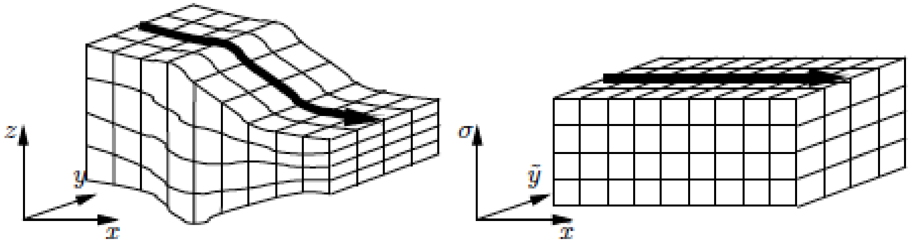






Discretizations

- Baseline model is the one used in Glimmer-CISM:
  - Logically-rectangular grid, obtained from a time-dependent uniform mapping.
  - 2D equation for ice thickness, coupled with 2D steady elliptic equation for the horizontal velocity components. The vertical velocity is obtained from the assumption of incompressibility.
  - Advection-diffusion equation for temperature.



$$\frac{\partial H}{\partial t} = b - \nabla \cdot H\bar{\mathbf{u}}$$

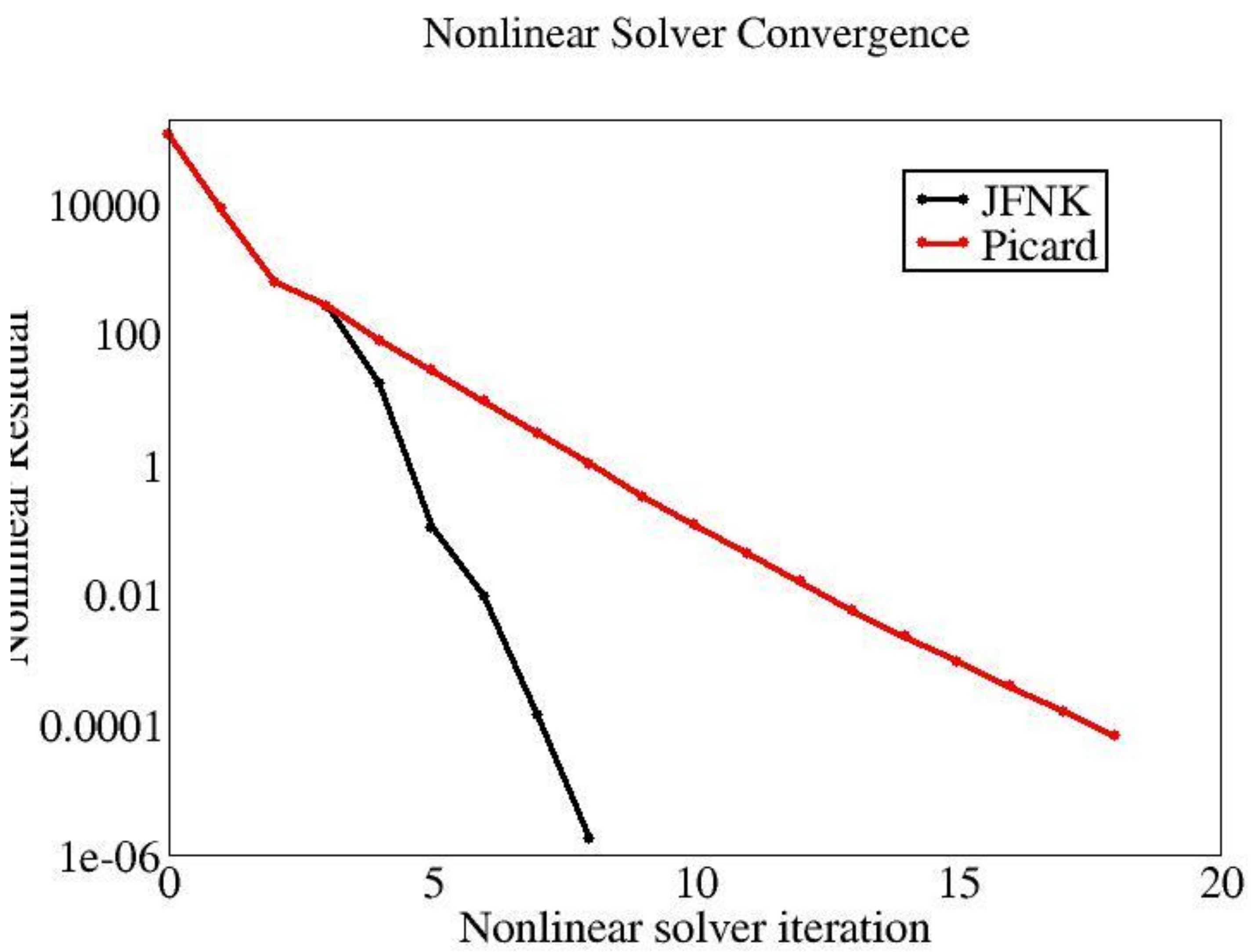
$$\frac{\partial T}{\partial t} = \frac{k}{\rho c}\nabla^2 T - \mathbf{u}\cdot\nabla T + \frac{\Phi}{\rho c} - w\frac{\partial T}{\partial z}$$

- Use of Finite-volume discretizations (vs. Finite-difference discretizations) simplifies implementation of local refinement.
- Software implementation based on constructing and extending existing solvers using the Chombo libraries.

Nonlinear Solvers

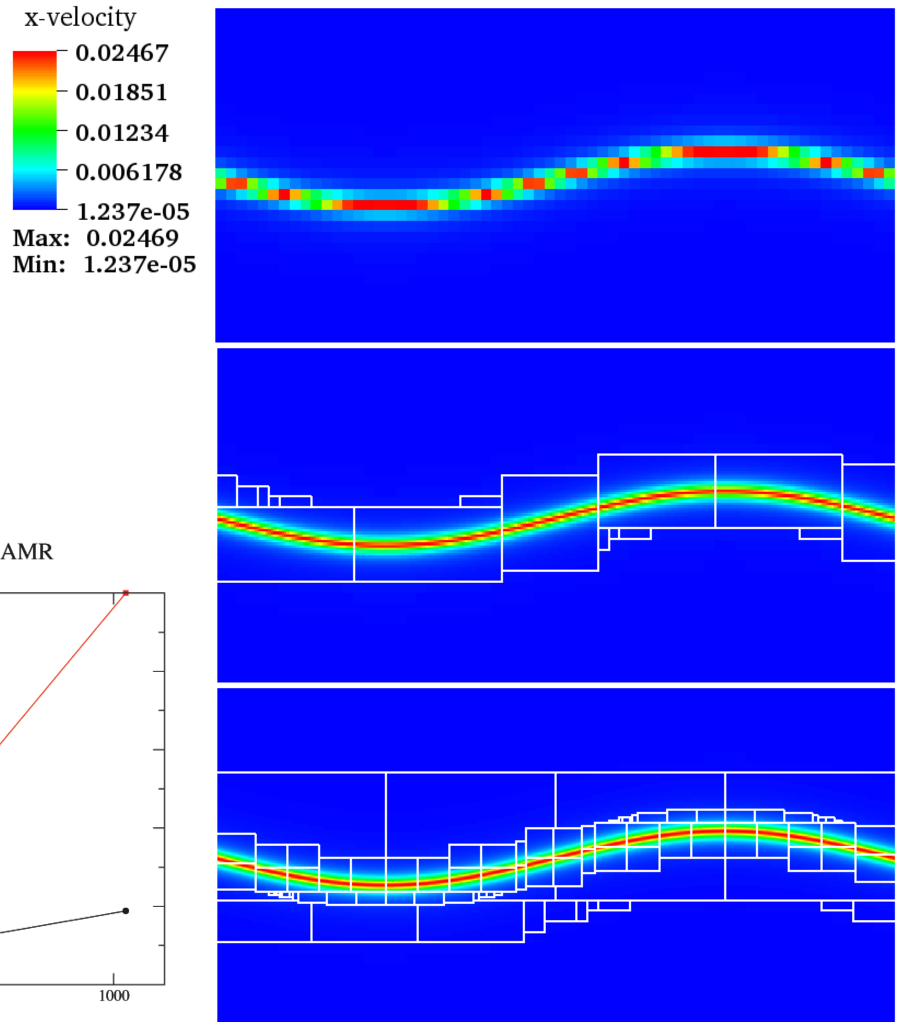
- Most computational effort spent in nonlinear ice velocity solve.
- Picard iteration:
  - Robust
  - Simple to implement
  - Slow (but steady) convergence
- Jacobian-free Newton-Krylov (JFNK):
  - More complex to implement
  - Works best with decent initial guess
  - Rapid convergence
  - Well-suited for Chombo AMR elliptic solvers
- Approach – use Picard iteration initially, then switch to JFNK when convergence slows

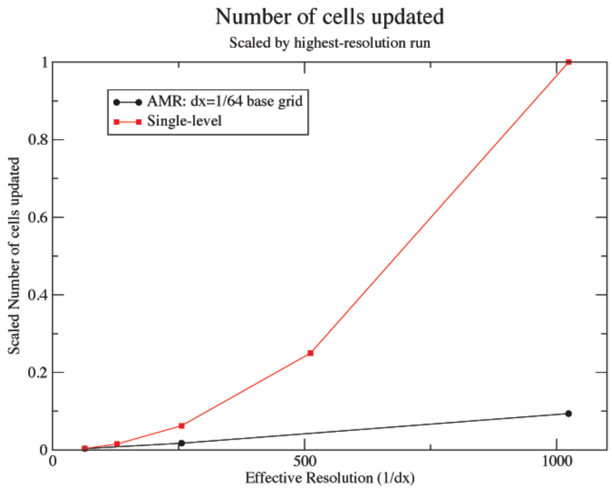
Nonlinear Solvers (cont)

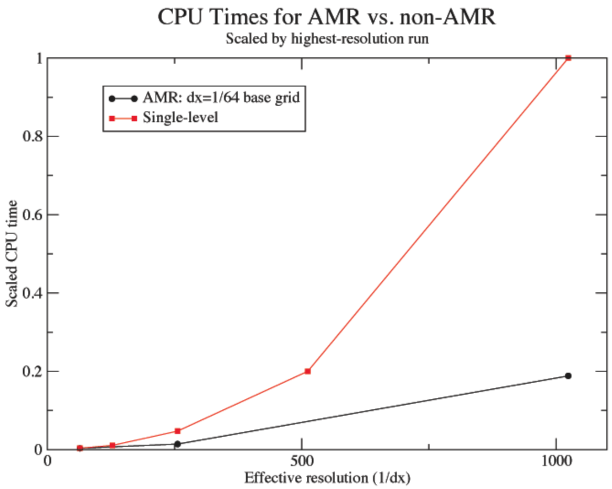


BISICLES Results

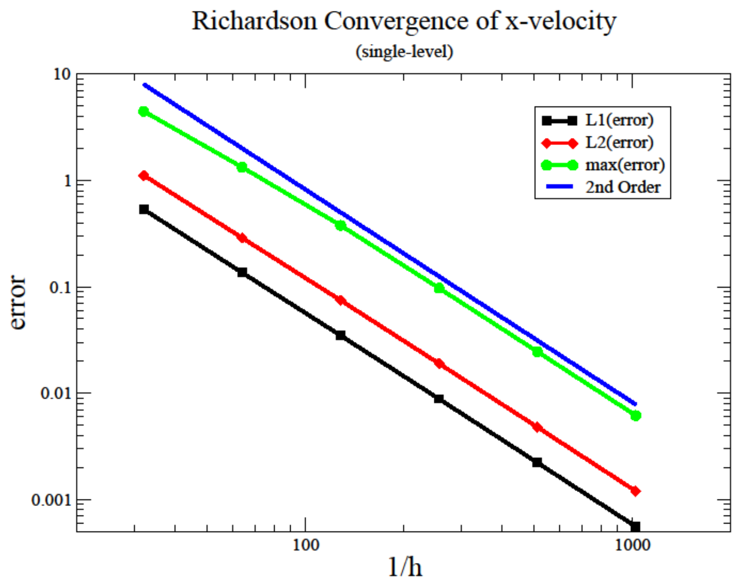
- Ice-stream Simulation
  [based on Pattyn et al (2008)]:
  - High resolution is required to accurately resolve the ice stream.
  - AMR simulation allows high resolution around the ice stream at a fraction of the cost of a uniformly refined mesh.

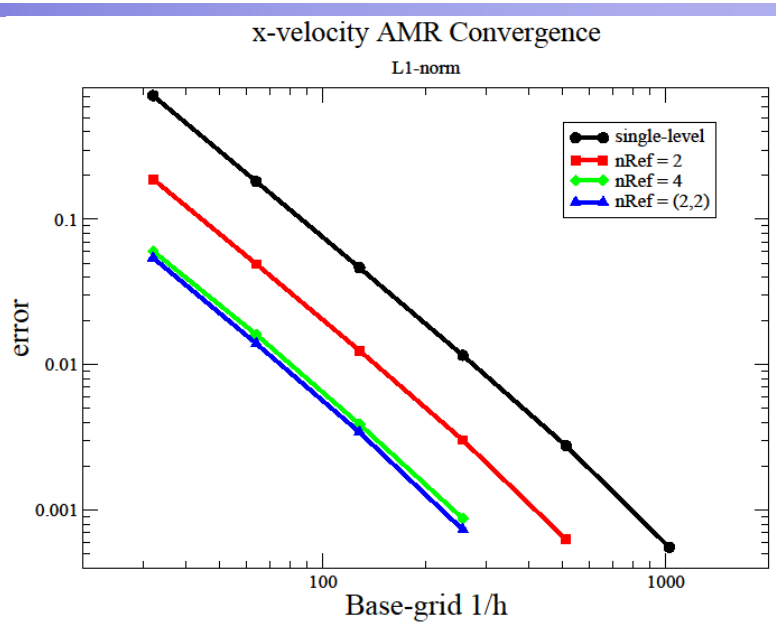


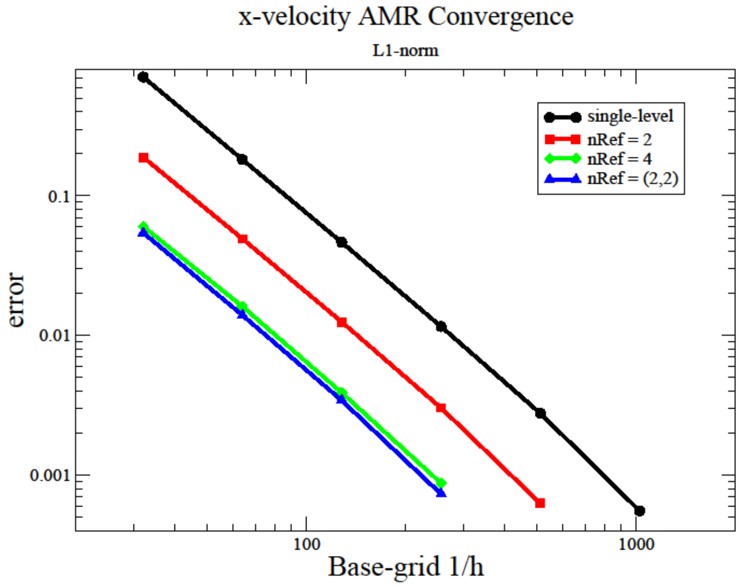


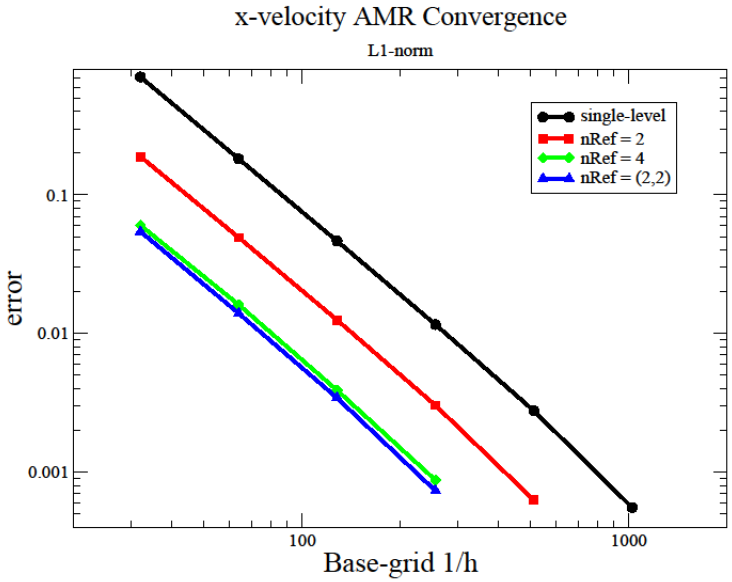


Numerical Accuracy and Convergence









Continental-scale: Antarctica

- Ice2sea geometry
- Temperature field from Pattyn and Gladstone