Using data assimilation methods to explore the role of longitudinal stress gradients in Greenland outlet glacier flow

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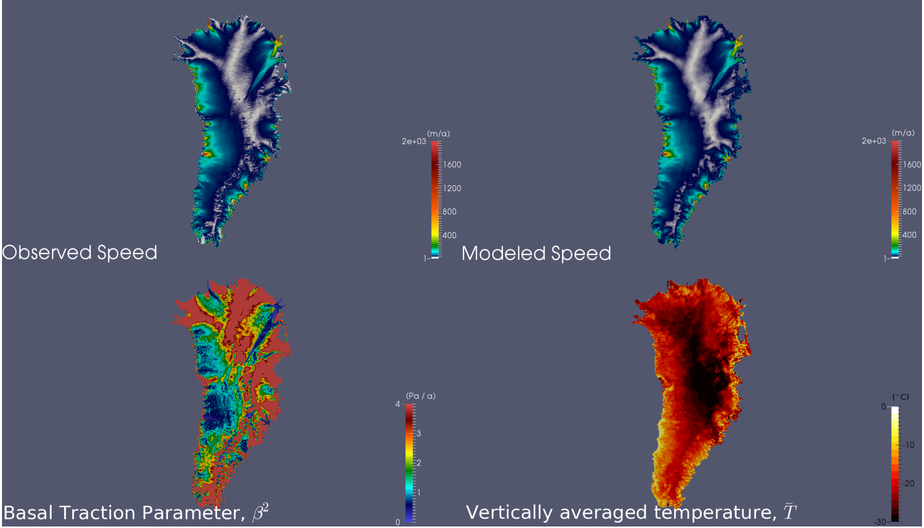
18 June, 2014
Breckenridge, Colorodo

Data assimilation technique

Determine the state of modern Greenland



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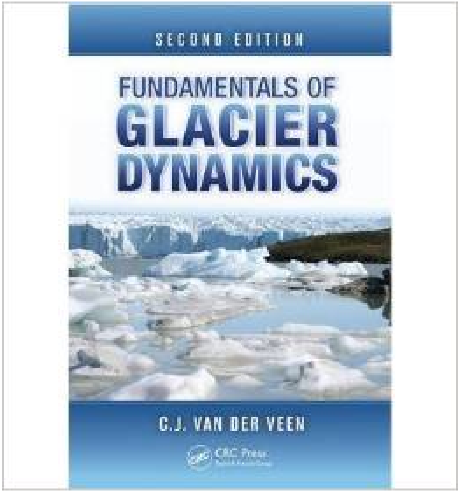
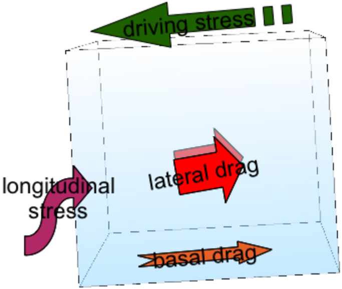
Application of assimilation: Force Balance

After getting a little 'FOGD'



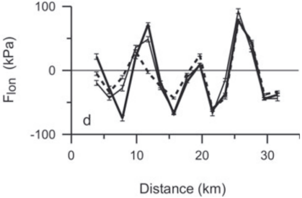


Application of assimilation: Force Balance

770 Journal of Glaciology, Vol. 57, No. 204, 2011

Controls on the recent speed-up of Jakobshavn Isbræ, West Greenland

C.J. VAN DER VEEN,[1,2] J.C. PLUMMER,[1,2] L.A. STEARNS[1,3]



Question

Can data assimilation techniques provide more revealing force budget analyses?

Some reasons they might:

- smoothing of data provided by model, not ad-hoc.
- surface velocity not equal to depth averaged
- temperature dependant rate factors
- no residuals to close budget

Does a new force budget provide insights into glacier dynamics?

Coordinate transformation

Begin from a well known point

"Classic" vertically integrated flow equations (SSA)

Force balance in x direction

$$\frac{\partial}{\partial x}\left(2\nu H\left(2\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right)+\frac{\partial}{\partial y}\left(\nu H\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)=\rho g H\frac{\partial z_s}{\partial x}-\beta^2 u$$

Force balance in y direction

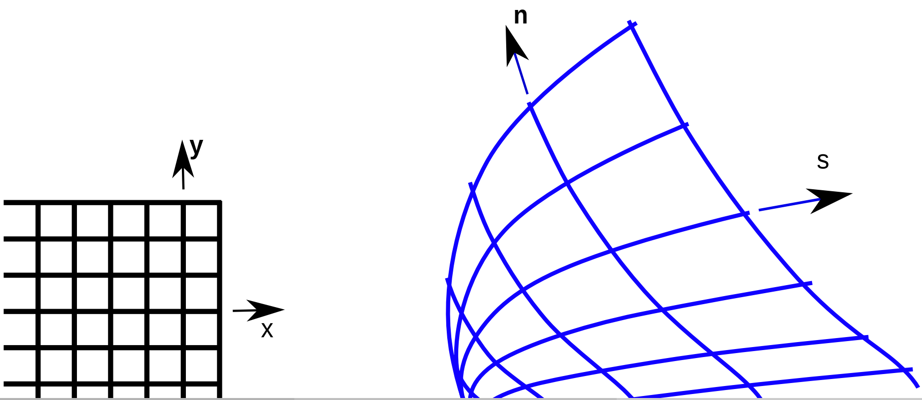
$$\frac{\partial}{\partial y}\left(2\nu H\left(2\frac{\partial v}{\partial y}+\frac{\partial u}{\partial x}\right)\right)+\frac{\partial}{\partial x}\left(\nu H\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)=\rho g H\frac{\partial z_s}{\partial y}-\beta^2 v,$$

Note that the *xy* coordinate system here has nothing to do with flow directions.

Coordinate transformation

Move to a more convenient coordinate system

- Define *sn* coordinate system.
- **s** is along modeled flow direction,
- **n** is across modeled flow.



Coordinate transformation

Along flow force balance

$$\frac{\partial}{\partial s}\left(2\nu H\left(2\frac{\partial \tilde{u}}{\partial s}+\frac{\partial \tilde{v}}{\partial n}\right)\right)+\frac{\partial}{\partial n}\left(\nu H\left(\frac{\partial \tilde{u}}{\partial n}+\frac{\partial \tilde{v}}{\partial s}\right)\right)=\rho g H\frac{\partial z_s}{\partial s}-\beta^2\tilde{u}$$

and

Across flow force balance

$$\frac{\partial}{\partial n}\left(2\nu H\left(2\frac{\partial \tilde{v}}{\partial n}+\frac{\partial \tilde{u}}{\partial s}\right)\right)+\frac{\partial}{\partial s}\left(\nu H\left(\frac{\partial \tilde{u}}{\partial n}+\frac{\partial \tilde{v}}{\partial s}\right)\right)=\rho g H\frac{\partial z_s}{\partial n}-\beta^2\tilde{v}$$

Coordinate transformation

Force balance procedure

1. Assimilate surface velocity by adjusting traction
2. Determine **s** and **n** vectors from assimilated velocity
3. Apply directional derivatives:

$$\frac{\partial \tilde{u}}{\partial s} = \mathbf{s} \cdot \nabla \tilde{u}$$

   And similar for other derivatives; $\frac{\partial \tilde{u}}{\partial \mathbf{n}}$, $\frac{\partial \tilde{v}}{\partial \mathbf{s}}$, $\frac{\partial \tilde{v}}{\partial \mathbf{n}}$

4. Substitute directional derivative for each term in along and across flow stress balance
5. Viscosity, η and β^2 from assimilation
6. Solve coordinate transformed SSA equations for $\tilde{u}$ and $\tilde{v}$

Force budget in rotated coordinate system

Red terms are from assimilation, all partial derivative require directional derivative substitution

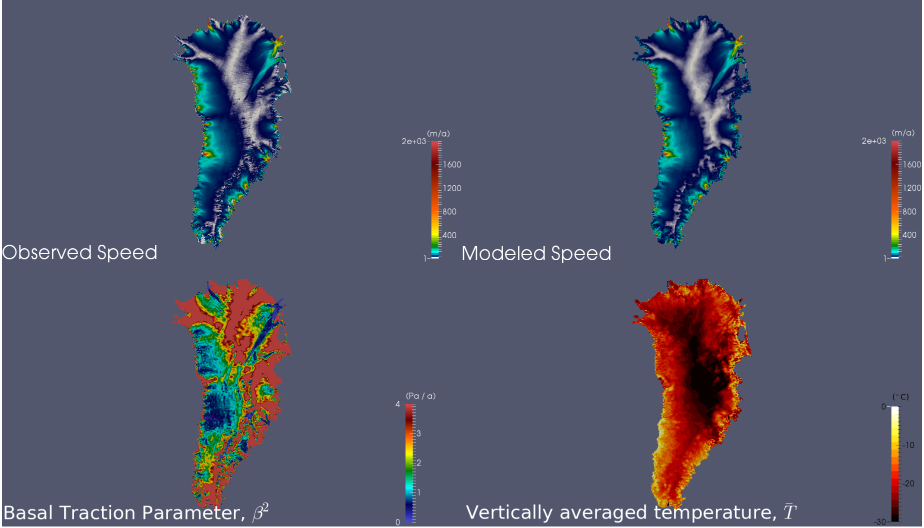
Along flow force balance

$$\underbrace{\frac{\partial}{\partial s}\left(2\nu H\left(2\frac{\partial \tilde{u}}{\partial s}+\frac{\partial \tilde{v}}{\partial n}\right)\right)}_{\text{Longitudinal stress gradient}}+\underbrace{\frac{\partial}{\partial n}\left(\nu H\left(\frac{\partial \tilde{u}}{\partial n}+\frac{\partial \tilde{v}}{\partial s}\right)\right)}_{\text{Lateral stress gradient}}=\underbrace{\rho g H\frac{\partial z_s}{\partial s}}_{\text{Driving stress}}-\underbrace{\beta^2\tilde{u}}_{\text{Basal drag}}$$

Across flow force balance

$$\frac{\partial}{\partial n}\left(2\nu H\left(2\frac{\partial \tilde{v}}{\partial n}+\frac{\partial \tilde{u}}{\partial s}\right)\right)+\frac{\partial}{\partial s}\left(\nu H\left(\frac{\partial \tilde{u}}{\partial n}+\frac{\partial \tilde{v}}{\partial s}\right)\right)=\rho g H\frac{\partial z_s}{\partial n}-\beta^2\tilde{v}$$

Across flow force balance is challenging to interpret!



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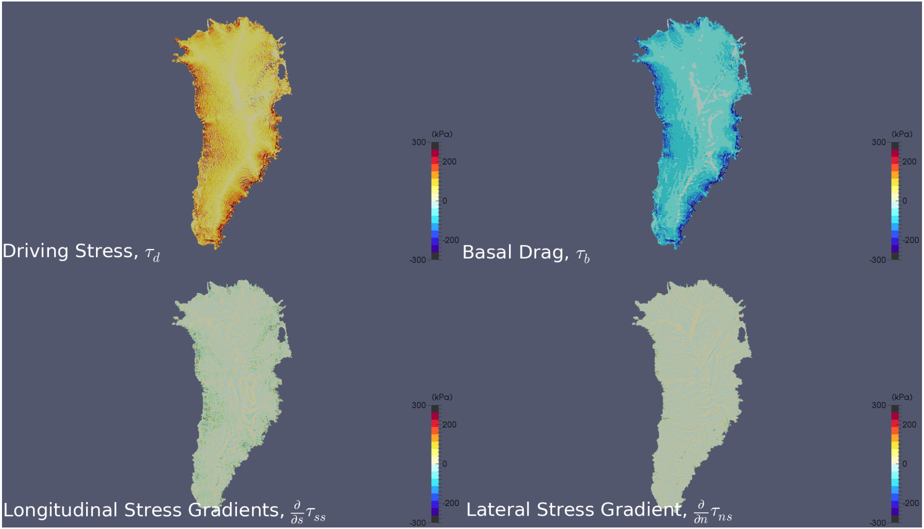
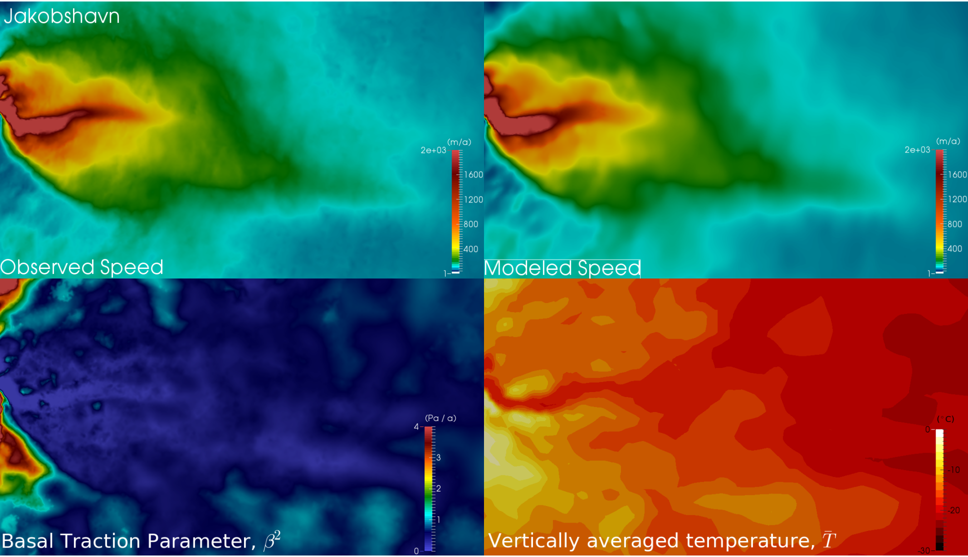
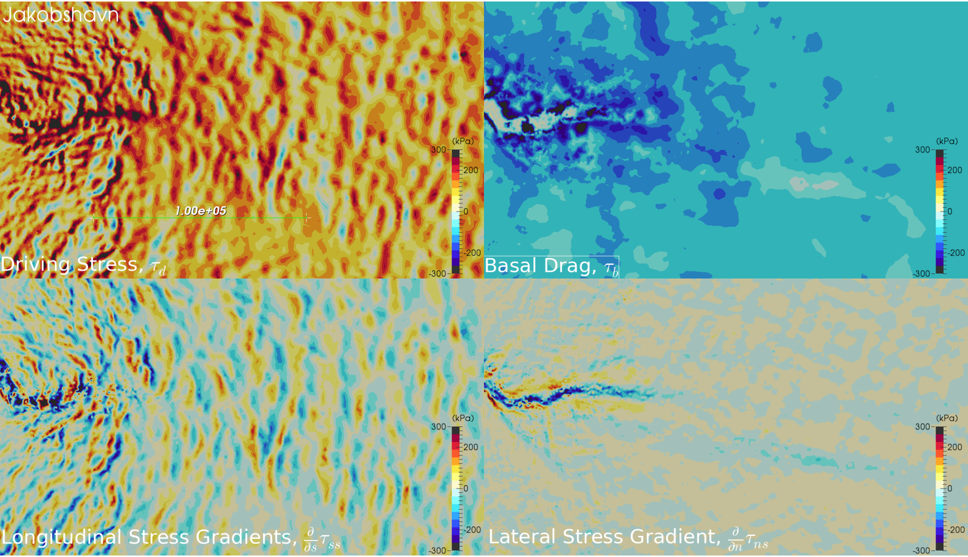


Driving Stress, τ_d
Basal Drag, τ_b
Longitudinal Stress Gradients, $\frac{\partial}{\partial s}\tau_{ss}$
Lateral Stress Gradient, $\frac{\partial}{\partial n}\tau_{ns}$
(kPa)
300
200
0
-200
-300



Jakobshavn
(m/a)
2e+03
1600
1200
800
400
1
Observed Speed
(m/a)
2e+03
1600
1200
800
400
1
Modeled Speed
(Pa / a)
4
3
2
1
0
Basal Traction Parameter, β^2
(°C)
0
-10
-20
-30
Vertically averaged temperature, $\bar{T}$



Jakobshavn
1.00e+05
Driving Stress, τ_d
(kPa)
300
200
0
-200
-300
Basal Drag, τ_b
(kPa)
300
200
0
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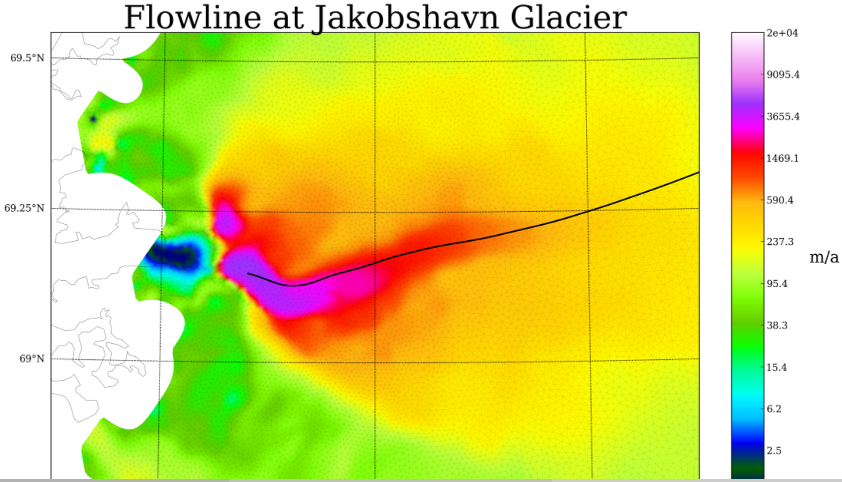
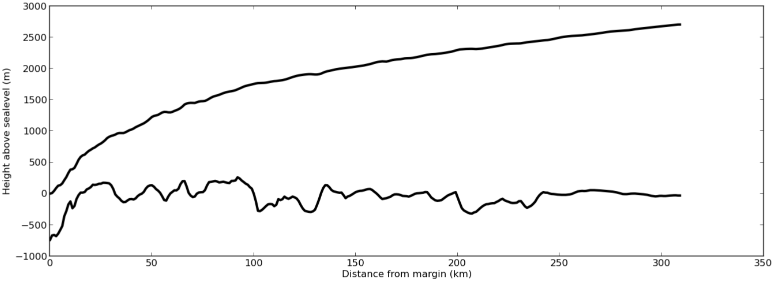
Evaluation along a flow line

Simplify interpretation



Evaluation along a flow line
Simplify interpretation



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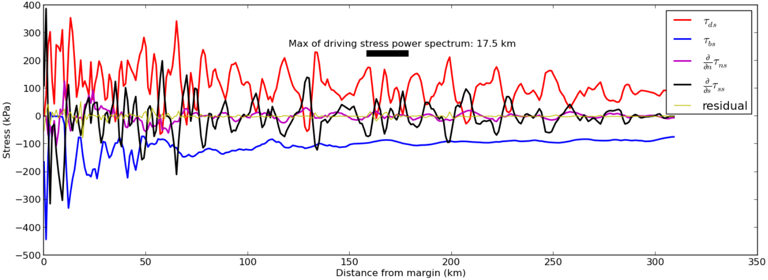
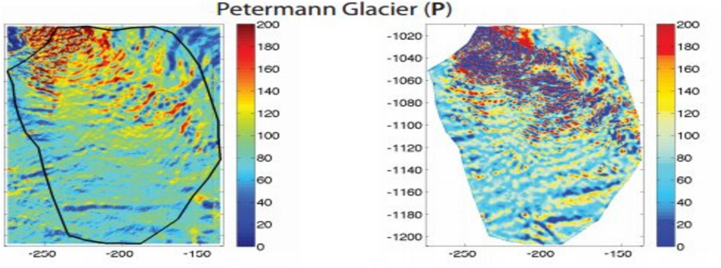


Conclusions

- Longitudinal stress gradients are a large part of the force budget
- These gradients vary very quickly and out of phase with driving stress.
- This is a highly local phenomena
- Speed up in Jakobshavns would depend on integral over coupling length, this is still small.
- Beyond back stress?

What about the rib-like features?

Can they be seen in our basal traction?



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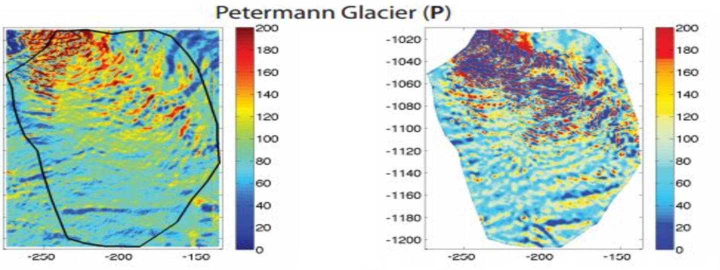

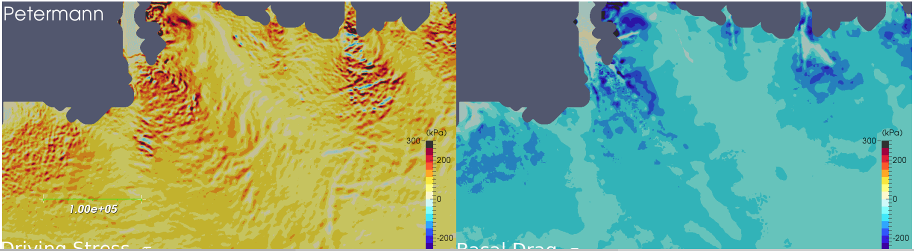

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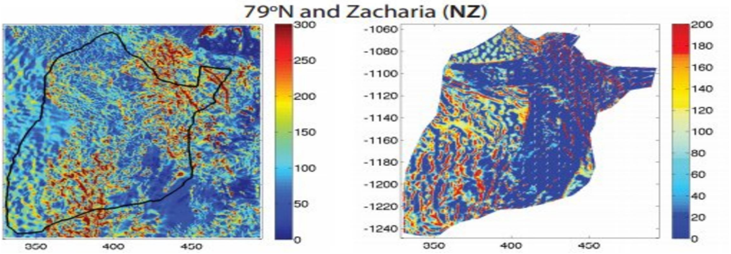

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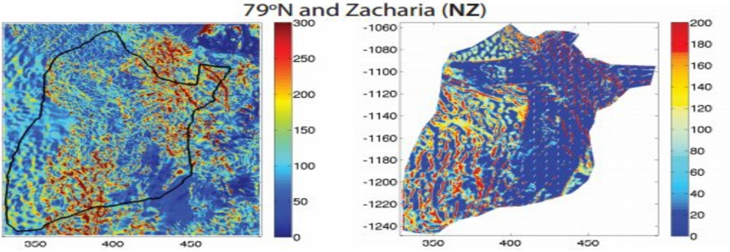
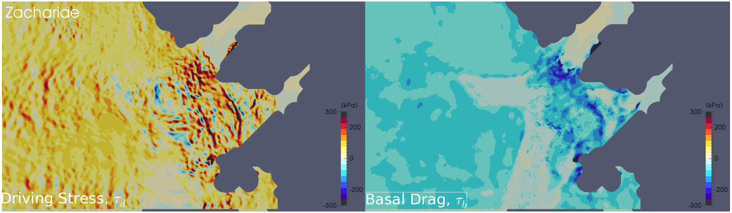
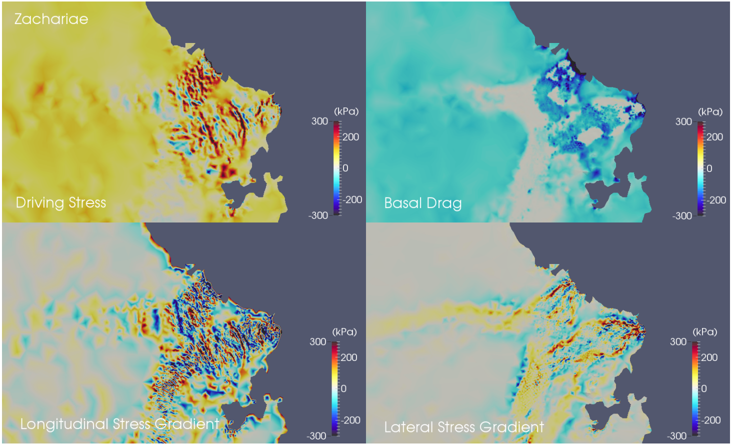


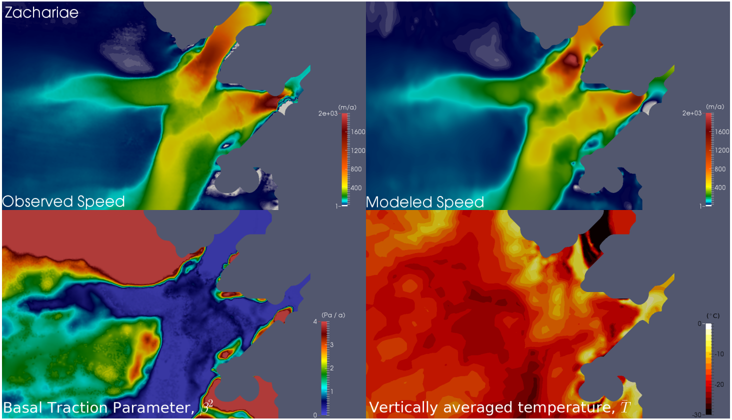


What about the rib-like features?

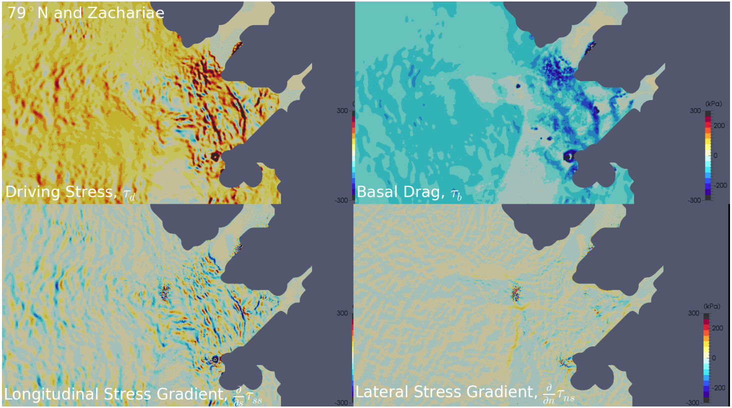
Is it the full-Stoke's treatment?

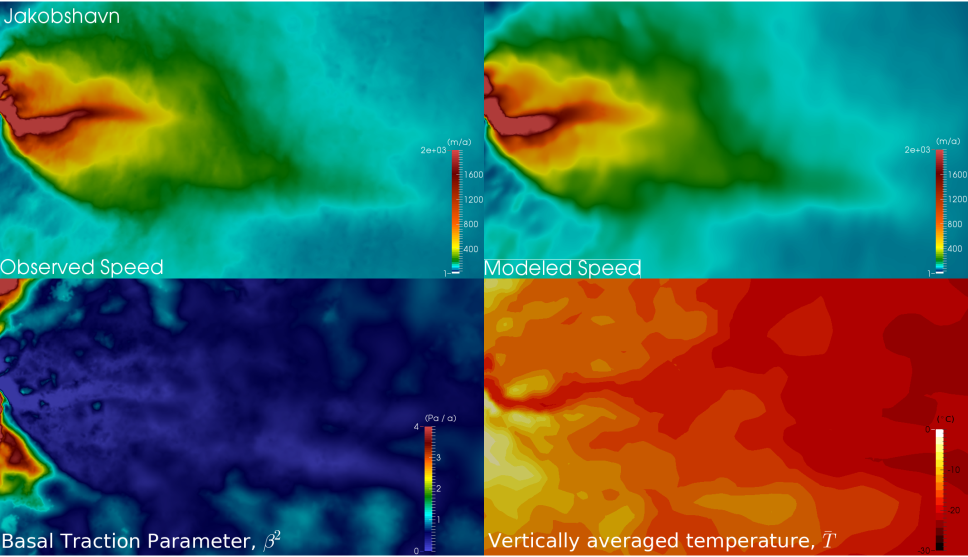


What about the rib-like features?
Rate factor?



What about the rib-like features?
With temperatures -2°C everywhere

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(°C)
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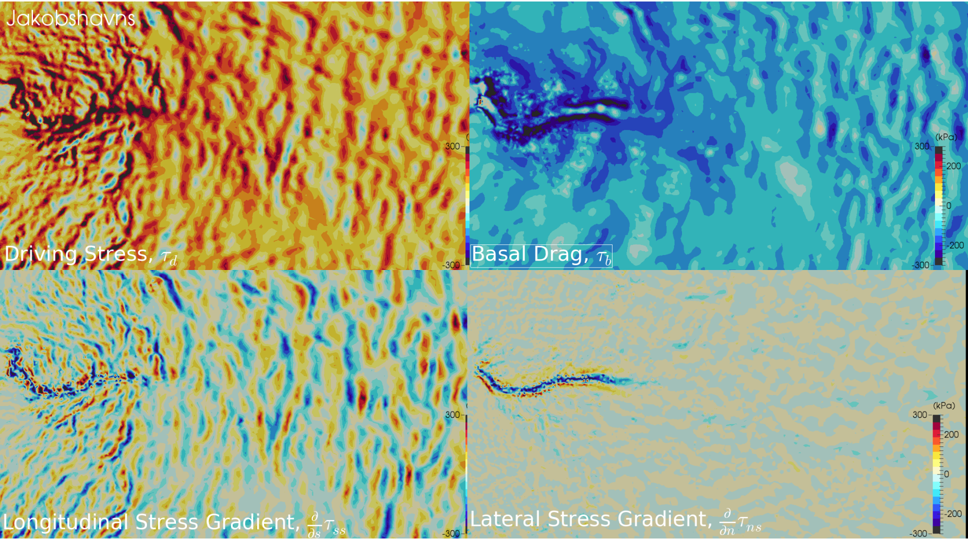
-30



Jakobshavns
Driving Stress, τ_d
Basal Drag, τ_b
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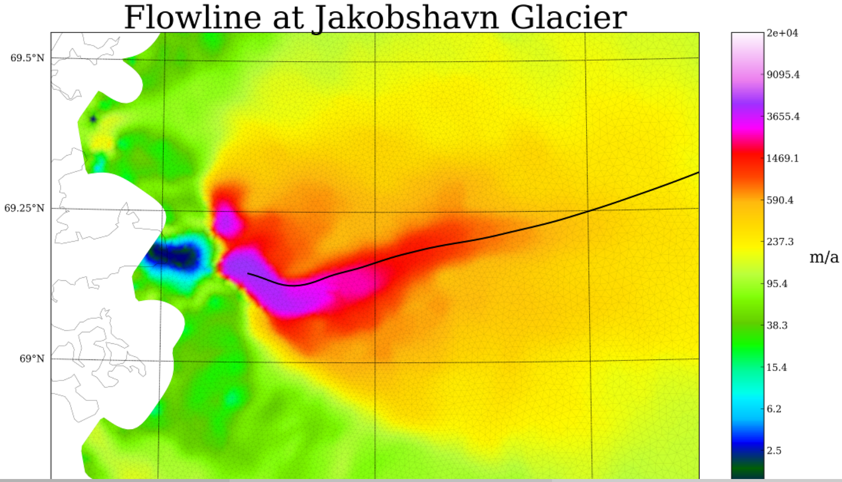
Simplify interpretation



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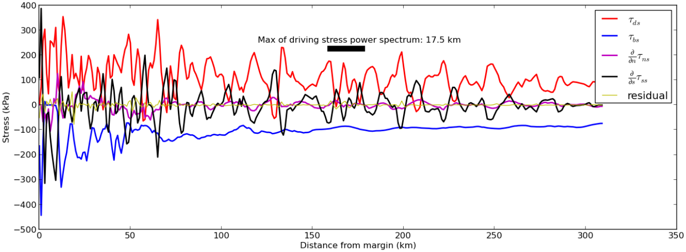

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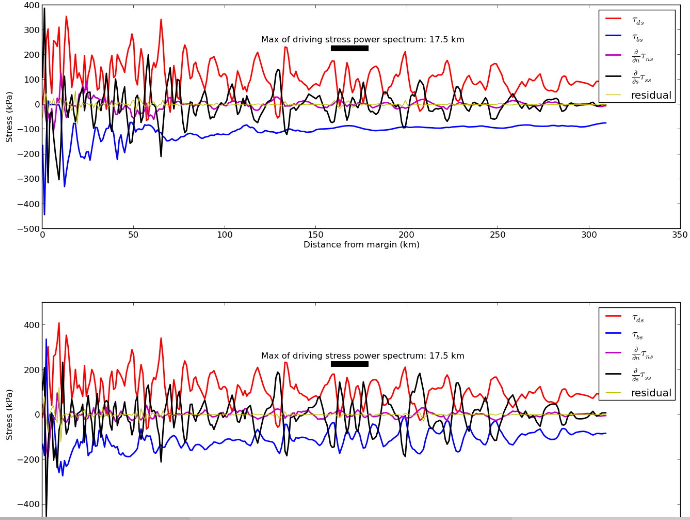
Simplify interpretation



Summary

- Rib like patterns in basal drag are less common in assimilations using full temperature balance.
- Our model has lower resolution (one ice thickness).
- Our model has lower order (L1L2, or SSA hybrid) momentum balance. Alignment of features.
- Our model may be too cold.
- Driving stress anomolies *could* be supported by longitudinal stress gradients.

Along flow force balance

$$\underbrace{\frac{\partial}{\partial s}\left(2\nu H\left(2\frac{\partial \tilde{u}}{\partial s}+\frac{\partial \tilde{v}}{\partial n}\right)\right)}_{\text{Longitudinal stress gradient}}+\underbrace{\frac{\partial}{\partial n}\left(\nu H\left(\frac{\partial \tilde{u}}{\partial n}+\frac{\partial \tilde{v}}{\partial s}\right)\right)}_{\text{Lateral stress gradient}}=\underbrace{\rho g H\frac{\partial z_s}{\partial s}}_{\text{Driving stress}}-\underbrace{\beta^2\tilde{u}}_{\text{Basal drag}}$$