Land Ice Working Group, 19th Annual CESM Workshop

Verification of the two dimensional first order thermo-mechanical flow line land-terminating glacier model

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Importance of glaciers

- Glacier melt water influence over 1 billion people in Asia!

Climate Change Will Affect the Asian Water Towers

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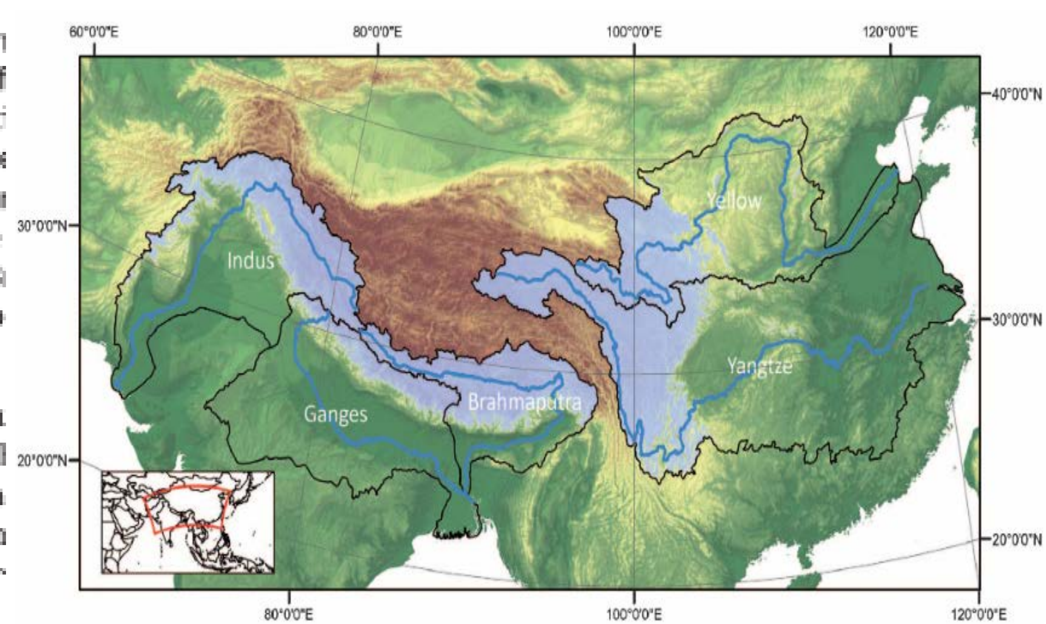
More than 1.4 billion people depend on water from
and Yellow rivers. Upstream snow and ice reserves of
water availability, are likely to be affected substanti
yet unclear. Here, we show that meltwater is extreme
for the Brahmaputra basin, but plays only a modest r
A huge difference also exists between basins in the
to affect water availability and food security. The B
susceptible to reductions of flow, threatening the fo

Mountains are the water towers of the world (1), including for Asia, whose rivers all are fed from the Tibetan plateau and adjacent mountain ranges. Snow and



Numerical ice flow models

The common ice flow models are:

- **1D** depth-integrated shallow ice approximation models;
- **2D** shallow ice/first order (higher order) approximation models (flow line; flow band);
- **3D** shallow ice/first order approximation/full Stokes models;

Difficulty in data acquisition

At 5800 m a.s.l. Mt Everest, 2009

Numerical models

- **3D models** 

- **2D first order flow line model (FLM)**
  - ✓ Finite Difference Method;
  - ✓ Terrain-following coordinate transformation;

- **3D full Stokes models (FSM);**
  - ✓ Finite Element Method;
  - ✓ P2-P1 element for u, P1 element for T ;

Physics basis of ice flow

- Momentum balance equation

$$\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g} = 0,$$

$\mathbf{g}$ equals to $(0, 0, g)$ and $(0, fg)$ for FSM and FLM

- Energy balance equation

$$\rho c \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = k \nabla^2 T + 2\eta \dot{\epsilon} : \dot{\epsilon},$$

- Mass conservation equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathrm{u}) = 0$$

Boundary conditions

- surface

$$T = T_{\mathrm{s}} = T_{\mathrm{t}} + \gamma_{\mathrm{e}}(s - s_{\mathrm{t}}),$$

$$\boldsymbol{\sigma} \cdot \mathbf{n} \simeq 0,$$

- bottom

$$\frac{\partial T}{\partial z} = -\frac{G}{k},$$

FSM

$$\mathbf{u} \cdot \mathbf{n} = 0,$$

$$\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{t} + \beta \mathbf{u} \cdot \mathbf{t} = 0.$$

FLM

$$\sigma_{xz} + \beta u = 0,$$

Numerical experiments

- **ESD: Geometry induced**
  - ✓ Steady-state thermo-mechanically decoupled modeling
  - ✓ Constant A and compute T once after u converges
  - ✓ Haut Glacier d' Arolla and ice slabs, uniform width
- **ESC: Temperature induced**
  - ✓ Steady-state ($\partial T/\partial t = 0$) thermo-mechanically coupled modeling
  - ✓ Update T every time after u with frozen/slip beds
  - ✓ Haut Glacier d' Arolla, uniform width
- **ETC: Time induced**
  - ✓ Transient ($\partial T/\partial t \neq 0$) thermo-mechanically coupled modeling
  - ✓ Update T every time after u with frozen/slip beds
  - ✓ Haut Glacier d' Arolla with different time periods, uniform width

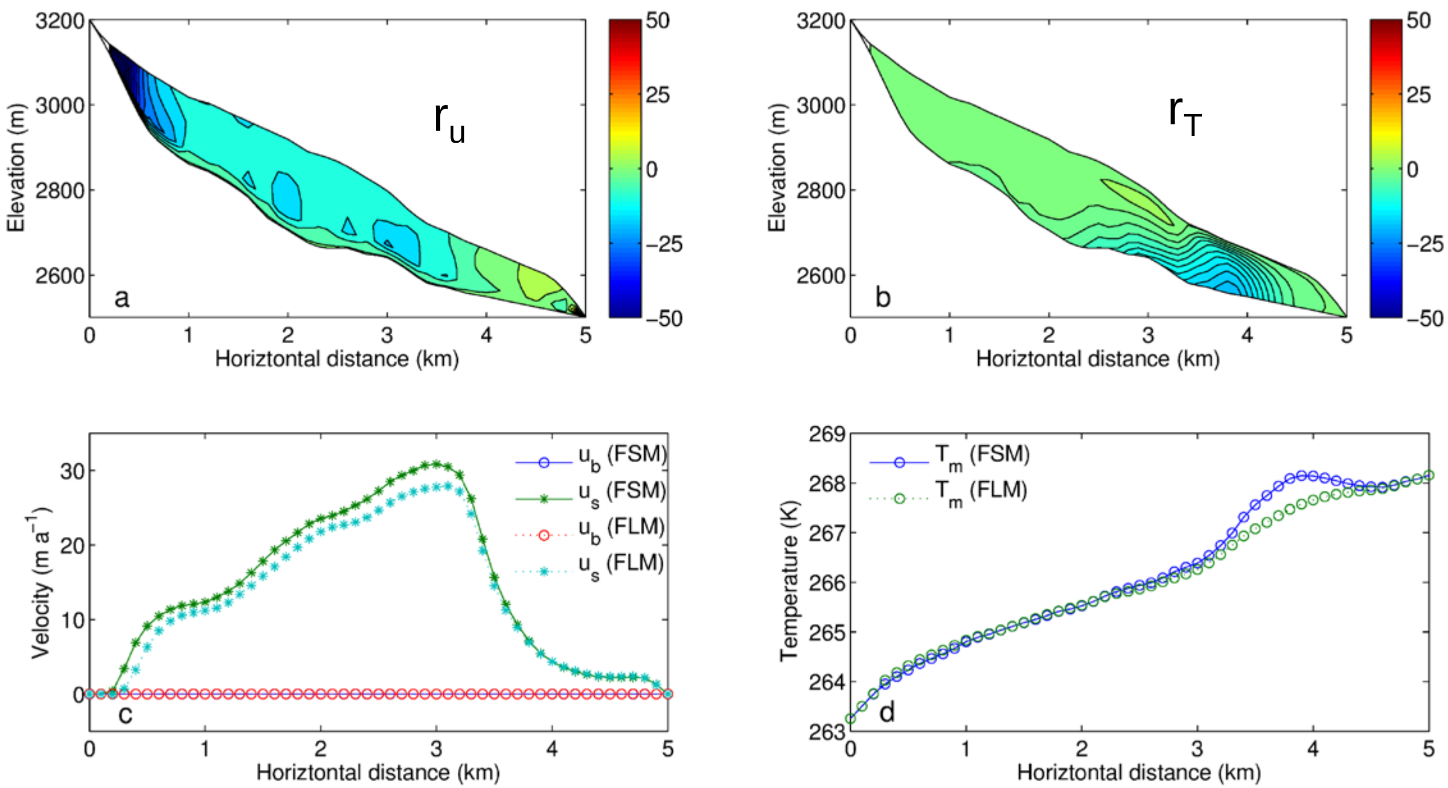
Numerical experiments

$$r_u = \frac{\boldsymbol{u}_{\mathrm{FLM}} - \boldsymbol{u}_{\mathrm{FSM}}}{\boldsymbol{u}_{\mathrm{FSM}}} \times 100, \qquad (\%)$$

$$r_T = \frac{\boldsymbol{T}_{\mathrm{FLM}} - \boldsymbol{T}_{\mathrm{FSM}}}{\boldsymbol{T}_{\mathrm{FSM}}} \times 100, \qquad (\%)$$

Model results: ESD

Haut Glacier d' Arolla



FLM generally underestimates horizontal velocity u

FLM underestimate ice temperature at the downstream basal ice

Why FLM underestimates u field ?

Stress balance:

$$\underbrace{\rho g_z h \alpha_{sx}}_{\tau_d} = \underbrace{R_{xz}(b) - R_{xx}(b)\alpha_{bx} - R_{xy}(b)\alpha_{by}}_{\tau_b}$$
$$- \underbrace{\frac{\partial}{\partial x}\int_b^s R_{xx}\,dz}_{\tau_{lon}} - \underbrace{\frac{\partial}{\partial y}\int_b^s R_{xy}\,dz}_{\tau_{lat}},$$

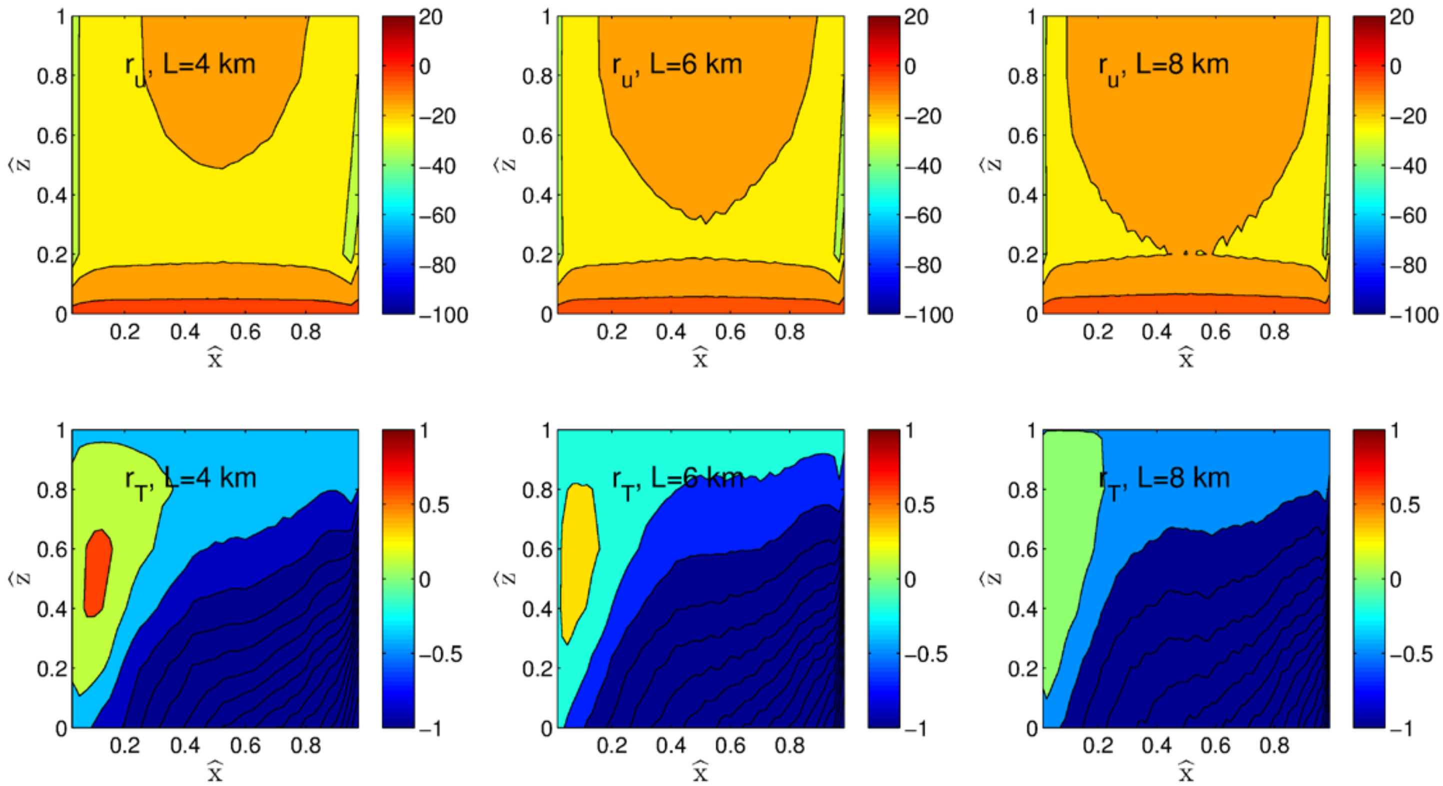
Shape factor:

$$f_* = \left(\tau_b + \tau_{lon}\right)/\tau_d$$

From (Adhikari and Marshall, 2012)

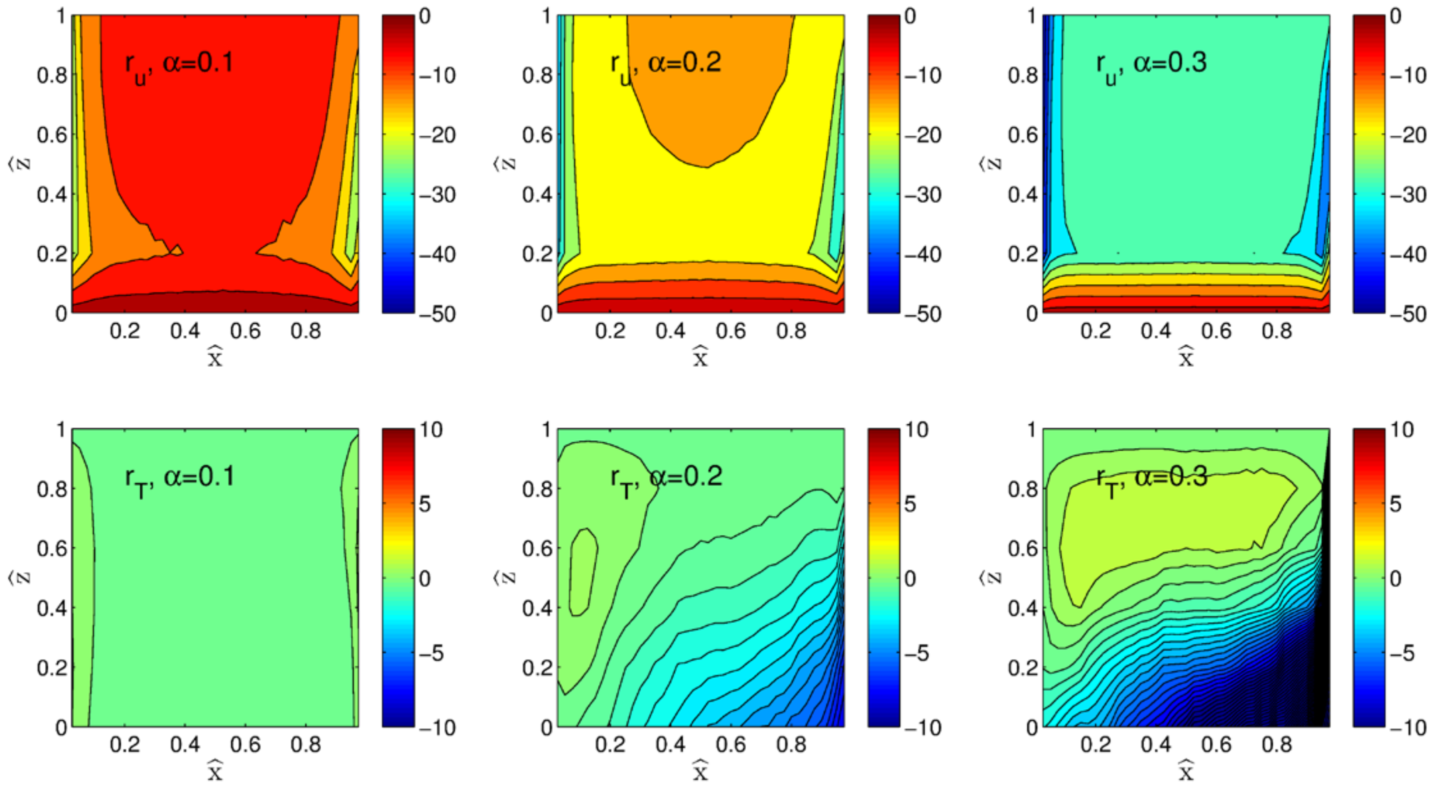
Impacts of longitudinal stress

Ice slabs with varied lengths, 4 km, 6 km, 8 km



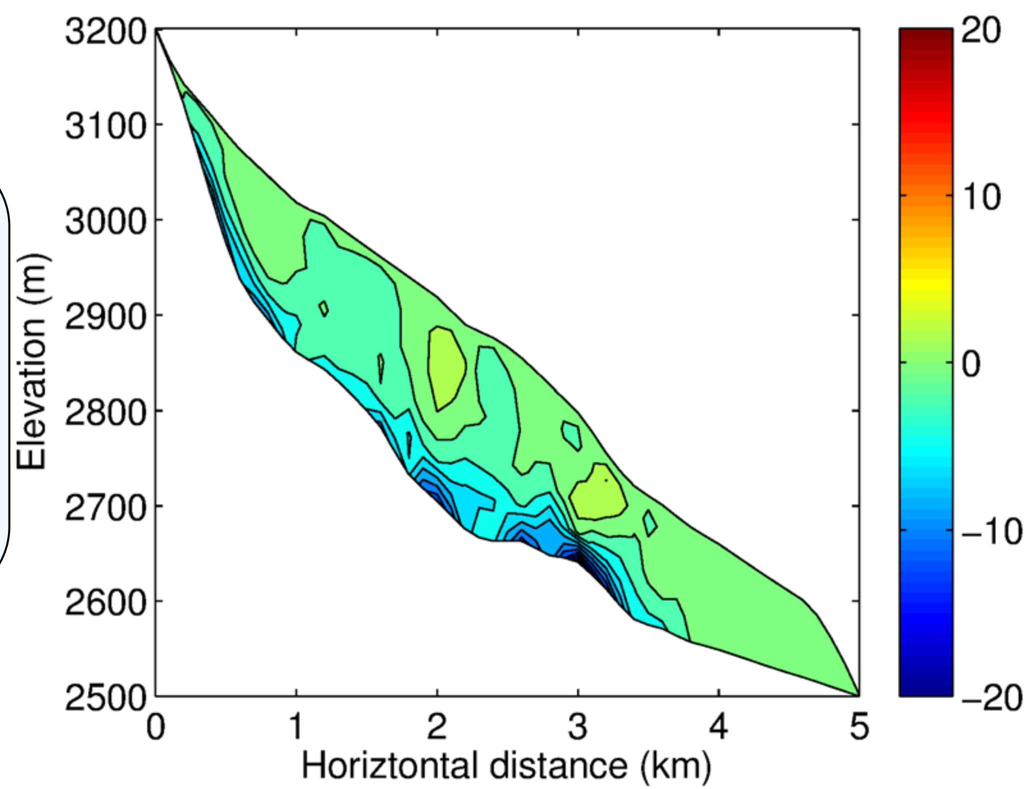
Impacts of longitudinal stress

Ice slabs with varied slopes, 0.1, 0.2, 0.3



Why FLM has biased T field ?

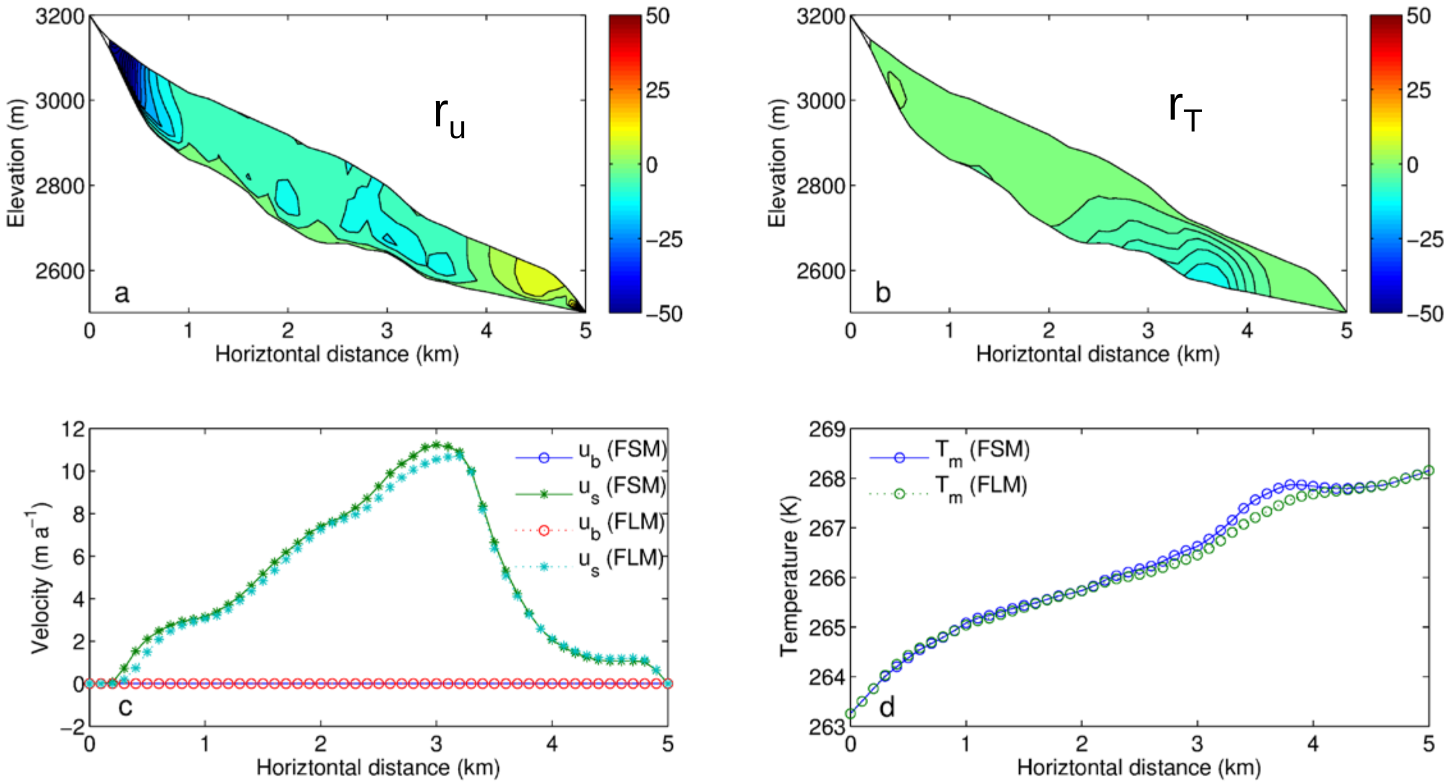
1. Velocity bias
2. Model simplification
3. Constant shape factor



∂u/∂z difference between FSM and FLM

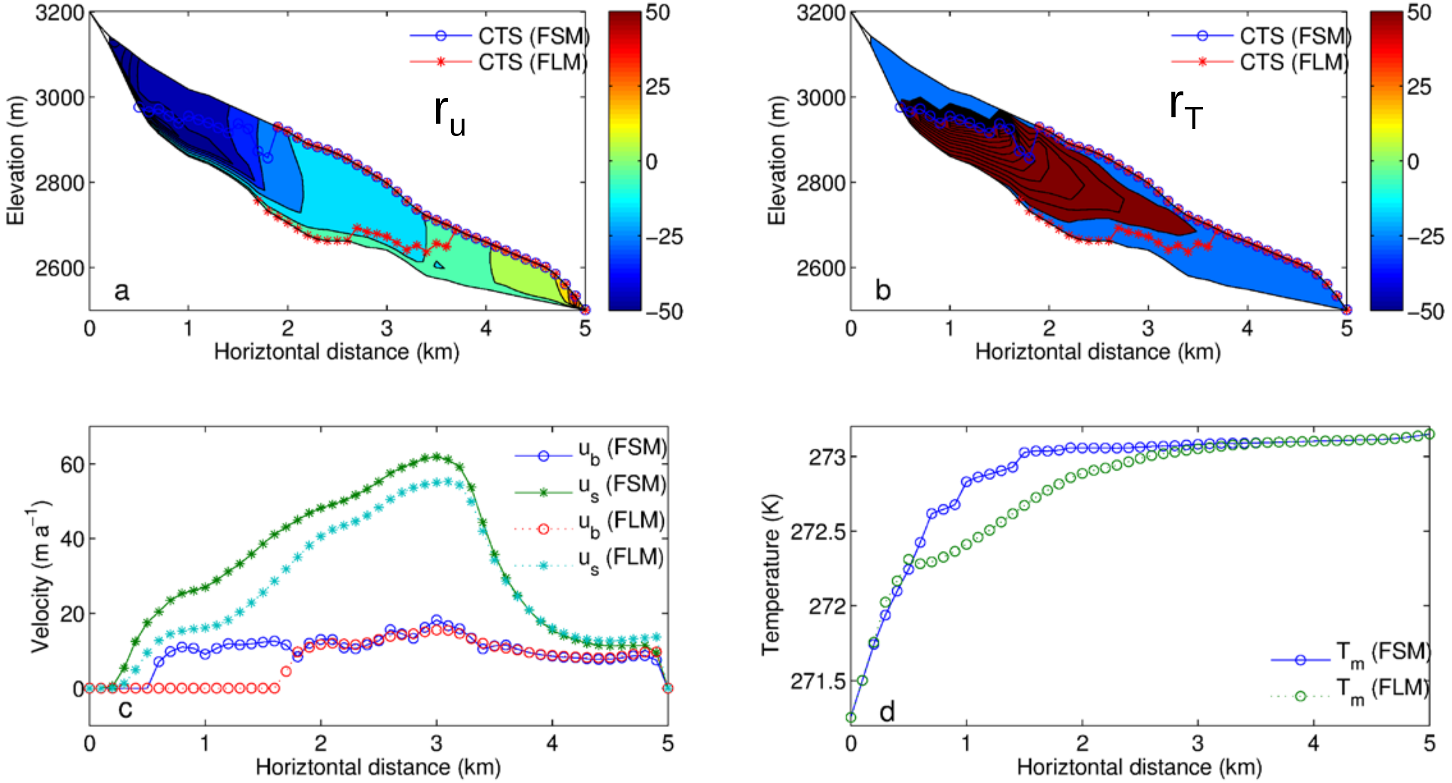
Model results: ESC

Temperature coupling could make some model improvements



Model results: ESC

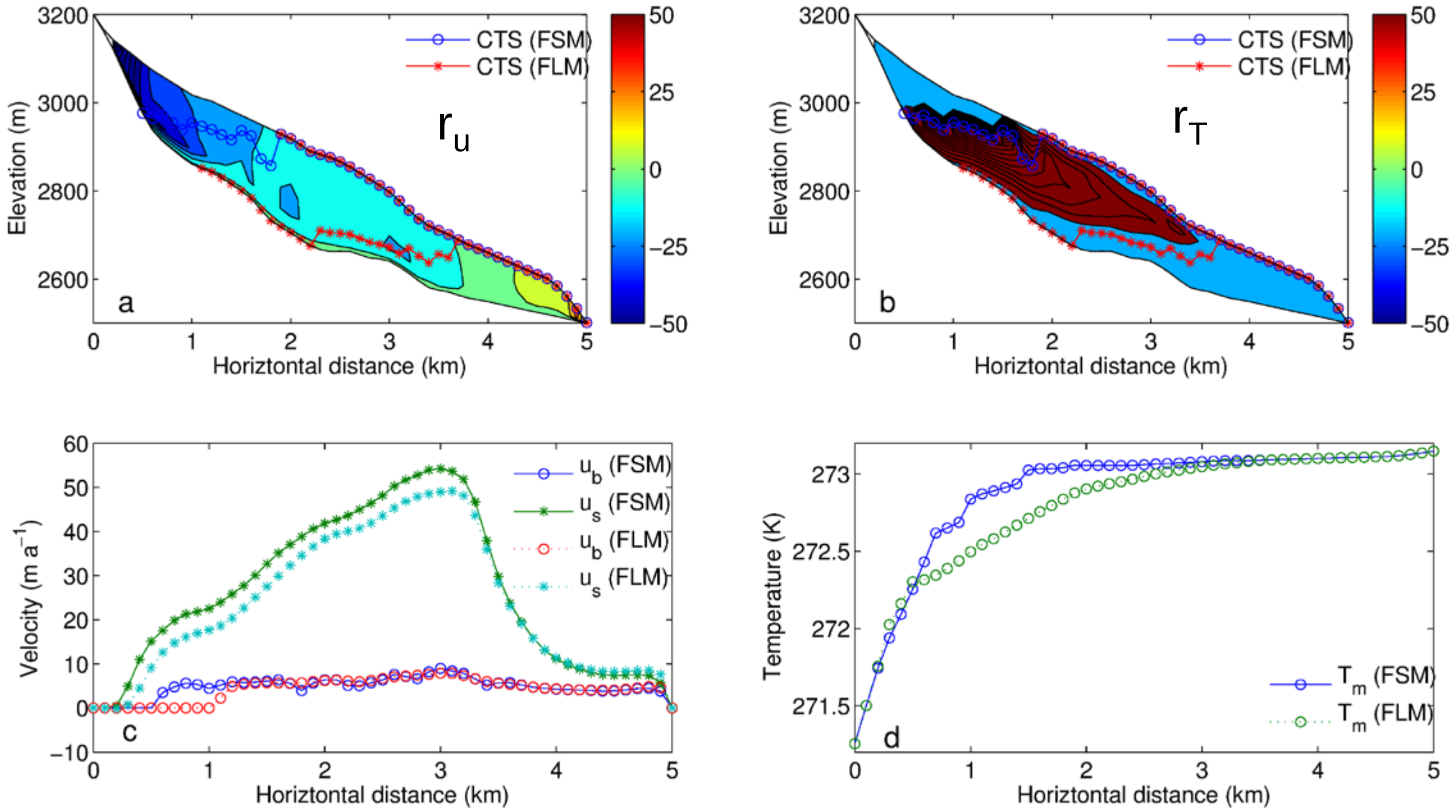
Sliding parameter β = 10^4 Pa a m^{-1}



Basal sliding could enhance the model discrepancies

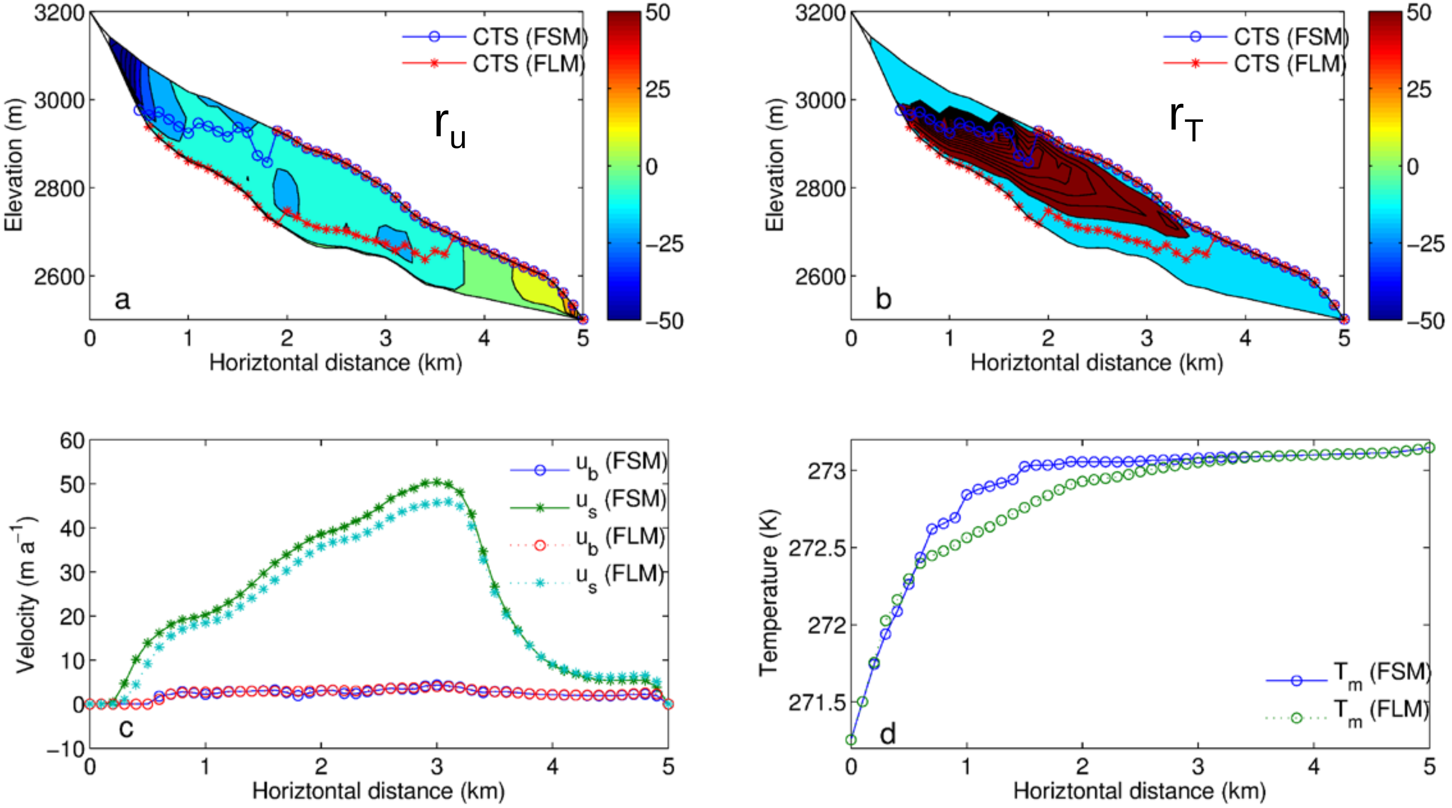
Model results: ESC

Sliding parameter $\beta = 2 \times 10^4$ Pa a m^{-1}



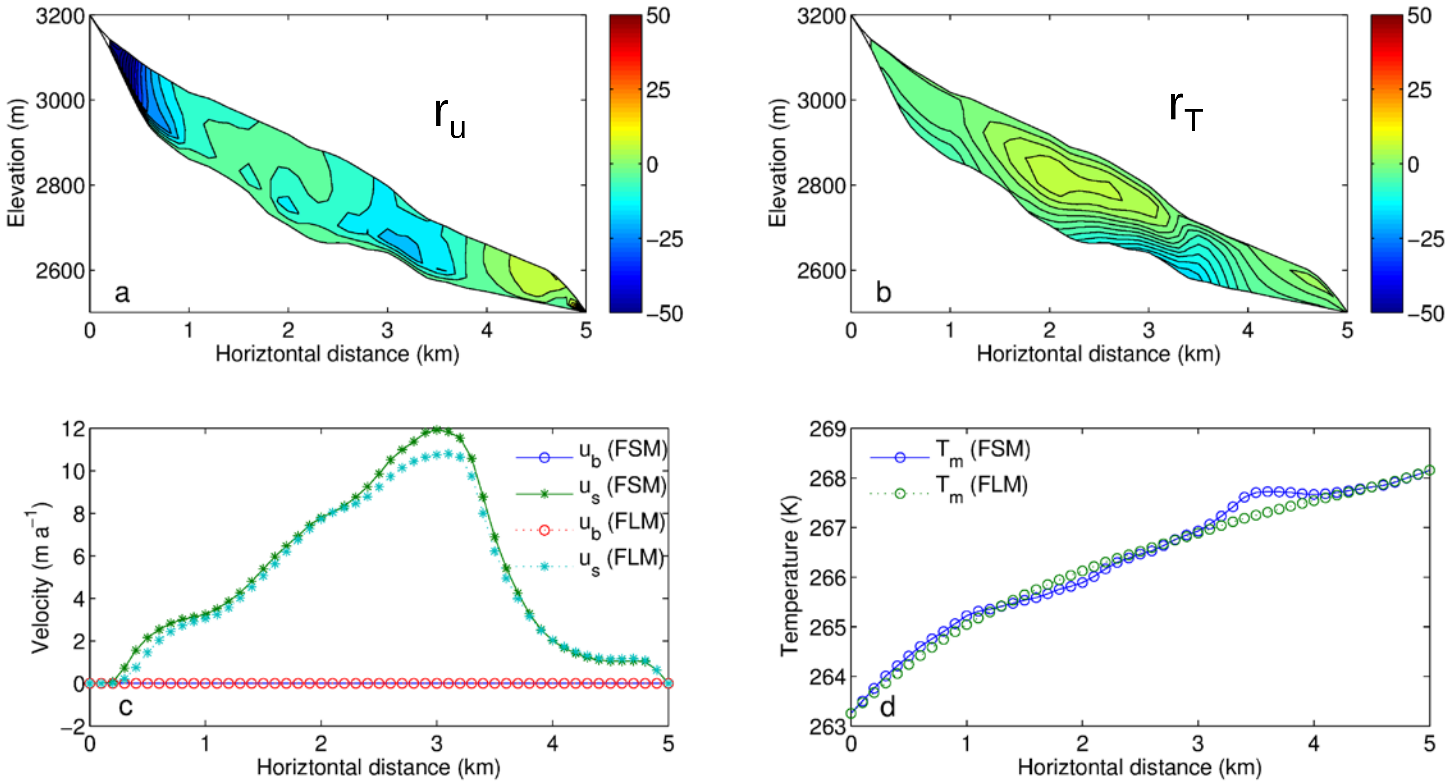
Model results: ESC

Sliding parameter $\beta = 4 \times 10^4$ Pa a m^{-1}



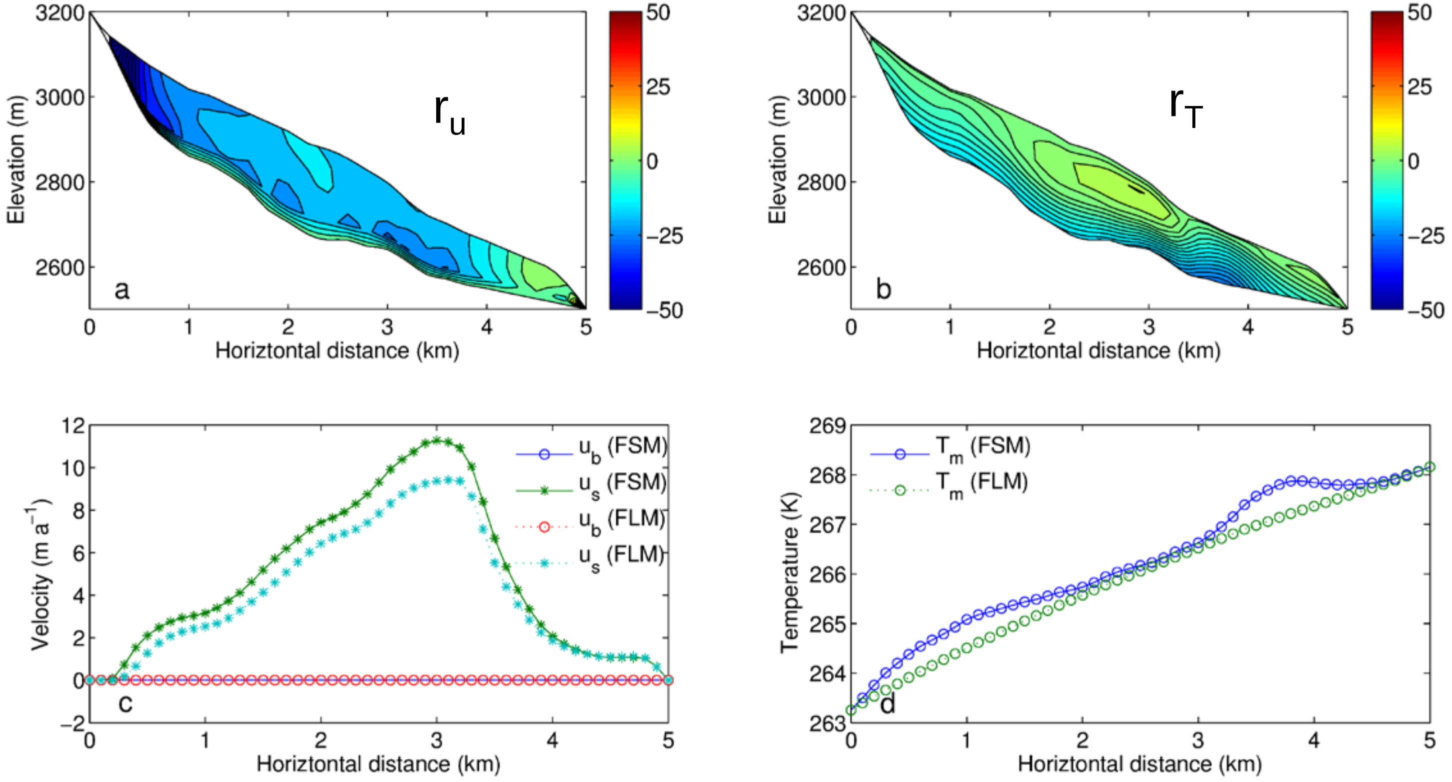
Model results: ETC

Frozen bed, 100 years



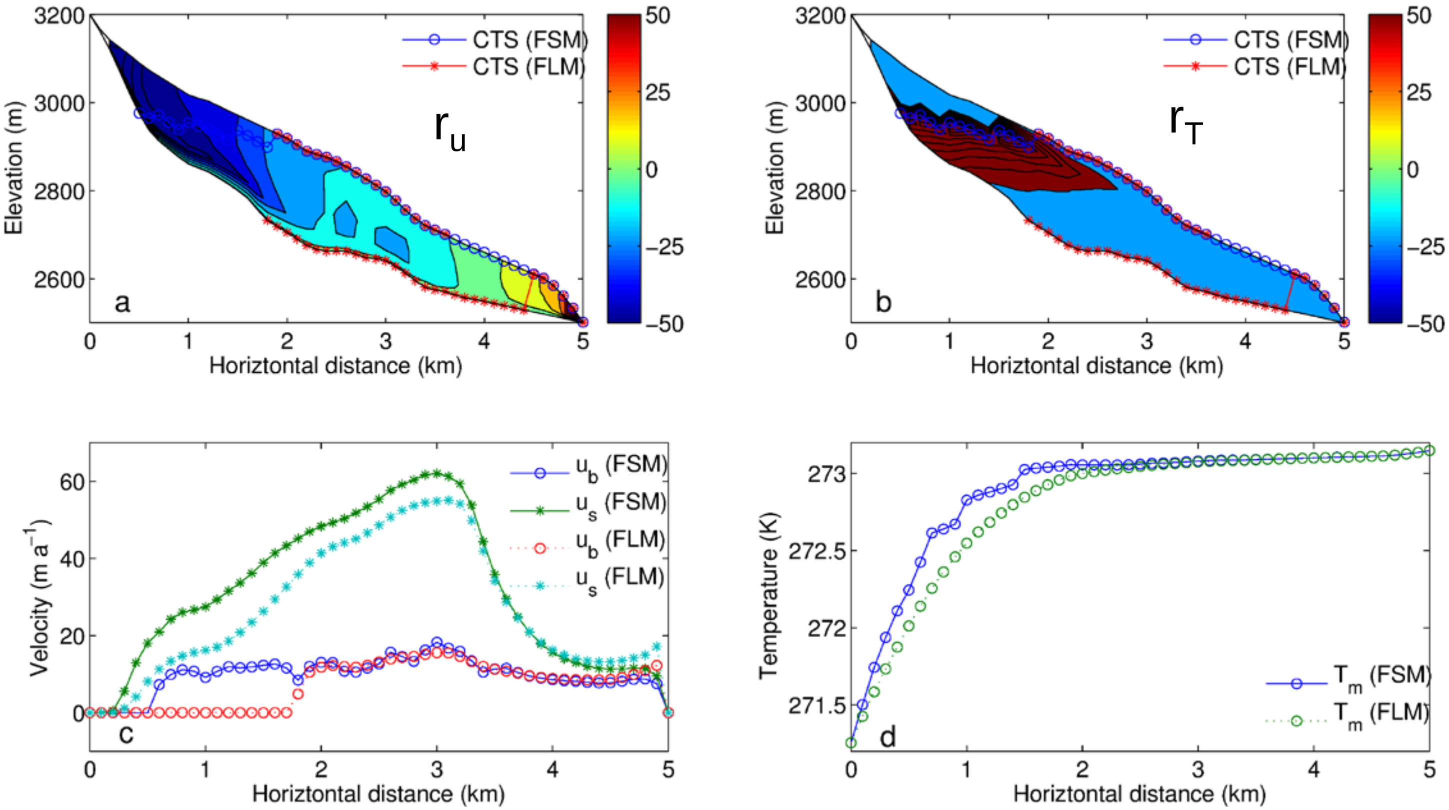
Model results: ETC

Frozen bed, 1000 years



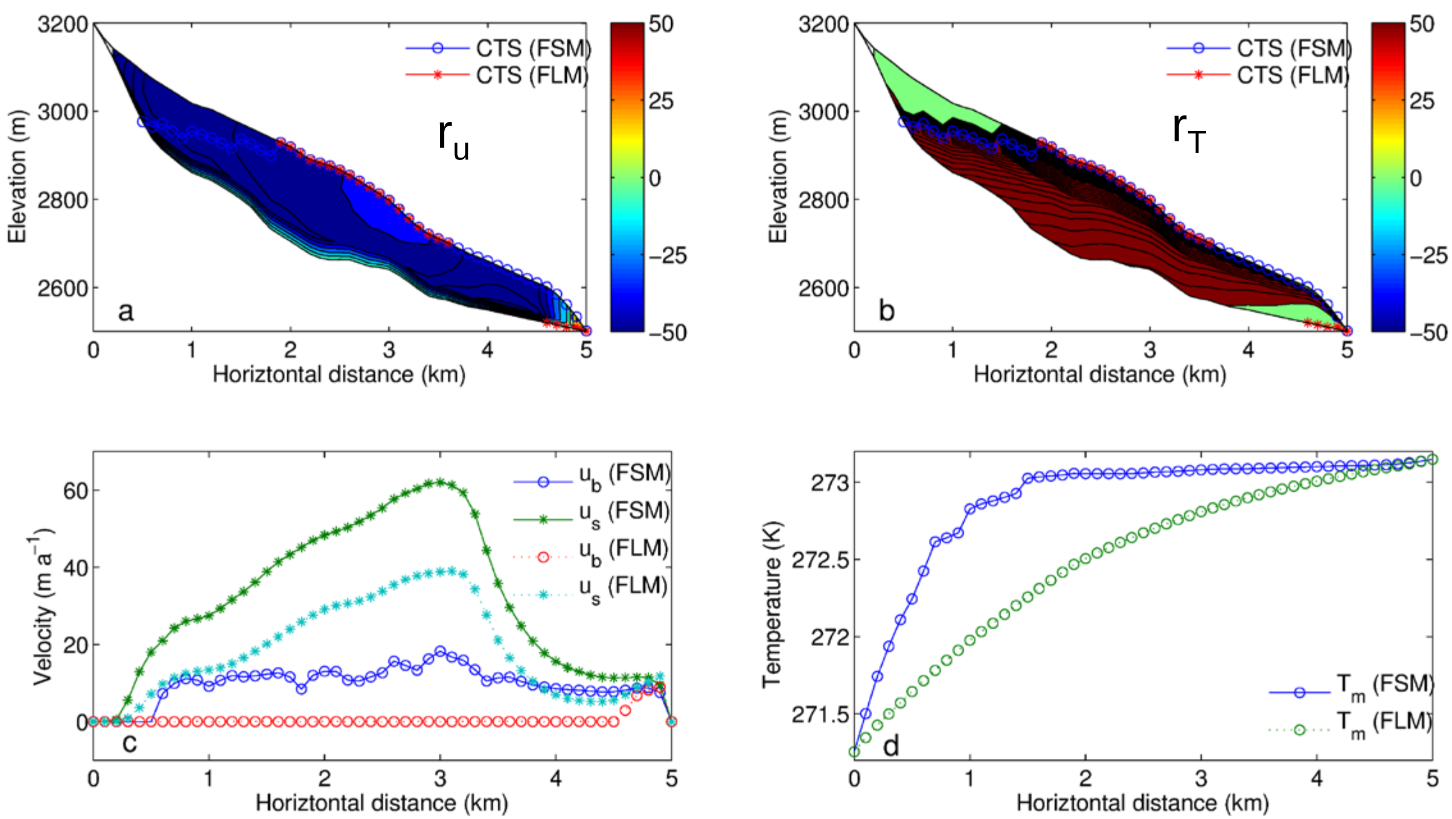
Model results: ETC

Slip bed, 100 years



Model results: ETC

Slip bed, 1000 years



Discussion

- **Geometry:**

  FLM produces smaller u in general, probably due to the shape factor underestimations

- **Temperature:**

  FLM may become unreliable when glaciers become warm and temperate ice zones appear

- **Time:**

  Model time further increase the discrepancies between the FLM/FSM model results

Conclusion

- **We should use FLM with cautions**
- **It is more suitable when**
  - ✓ **Glaciers are long**
  - ✓ **Glaciers are flat**
  - ✓ **Glaciers are cold**
  - ✓ **Model time is short**

Thanks!

Questions?

&

Suggestions?