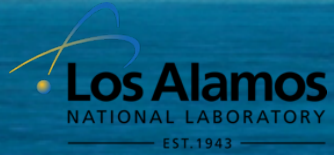


# Greenland subglacial drainage evolution regulated by weakly-connected regions of the bed



Matthew Hoffman  
Stephen Price



Lauren Andrews  
Ginny Catania



Jason Gulley



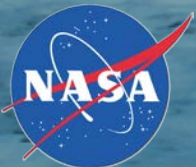
Martin Lüthi



Claudia Ryser

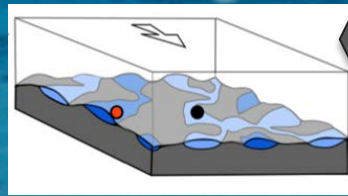


Robert Hawley

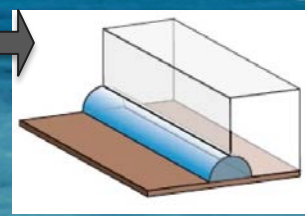
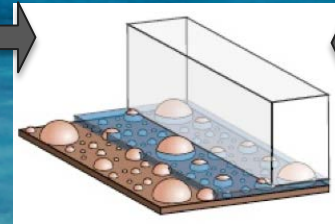


Thomas Neumann

Weakly-connected  
Drainage



Distributed Drainage Channelized Drainage

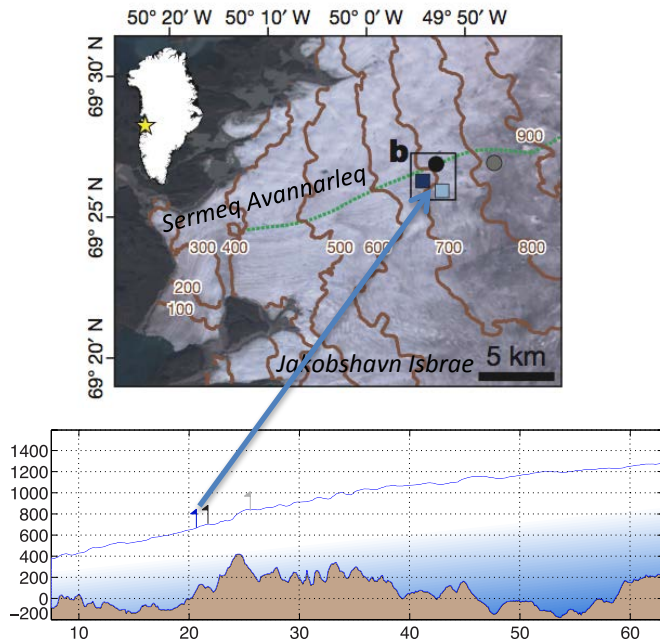


Field campaign and modeling supported by:



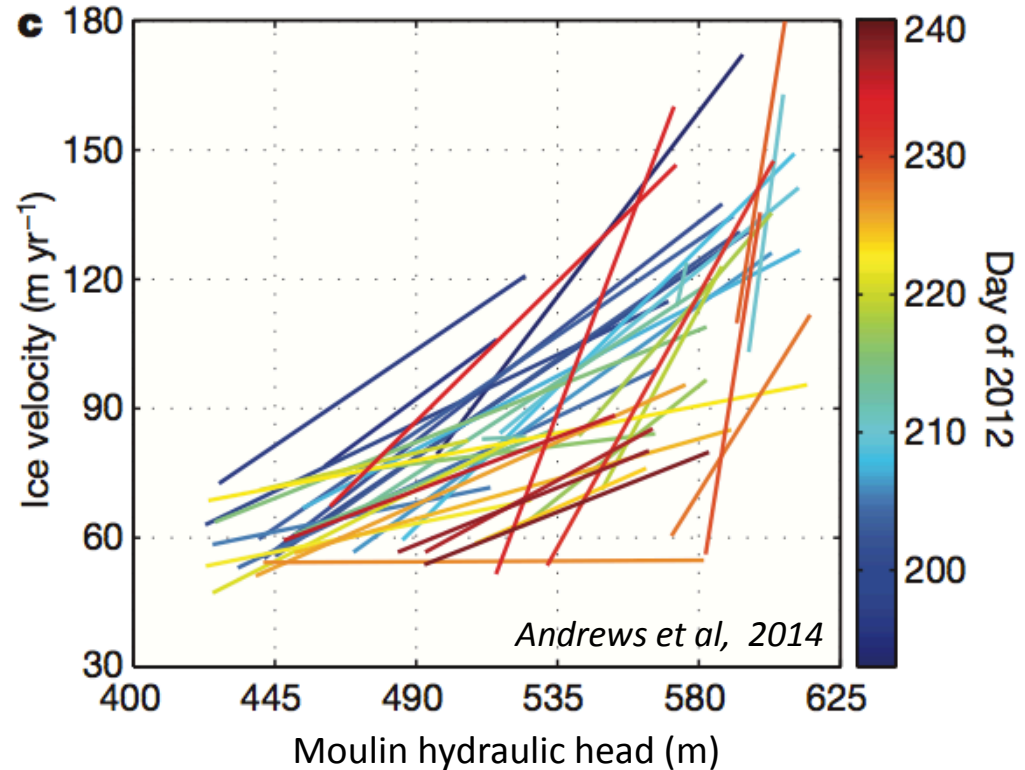
## Field campaign, 2012

- moulin water pressure
- borehole water pressure
- ice velocity



Moulin water pressure indicates equilibrated subglacial **channels**.

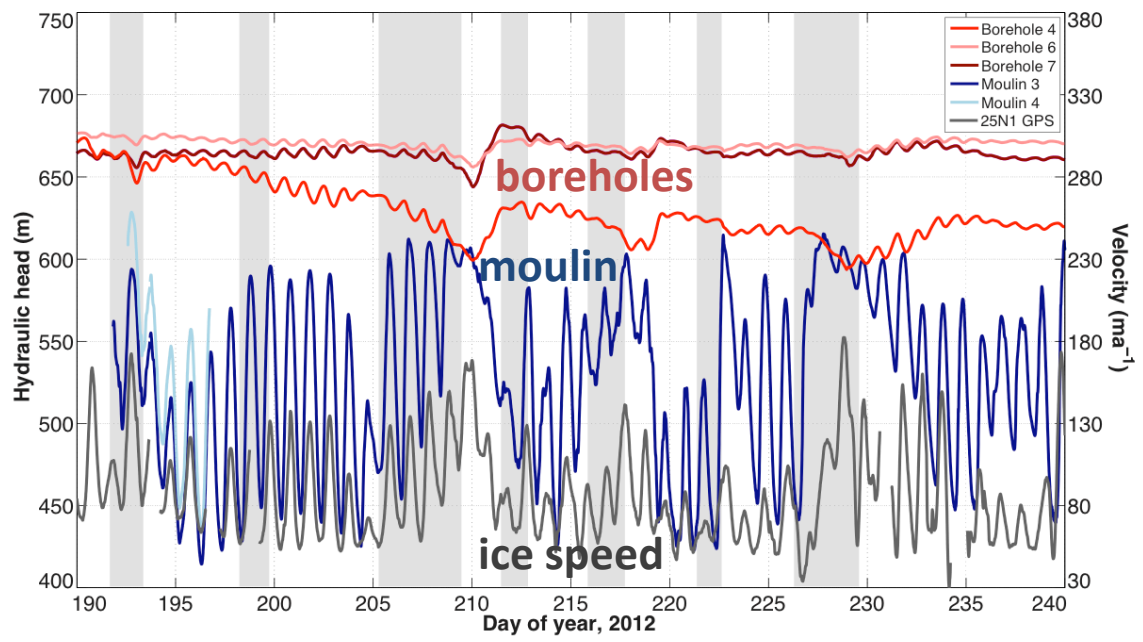
However, ice speed continues to drop.



...continued evolution elsewhere in the drainage system?

# Subglacial changes outside the channelized regions

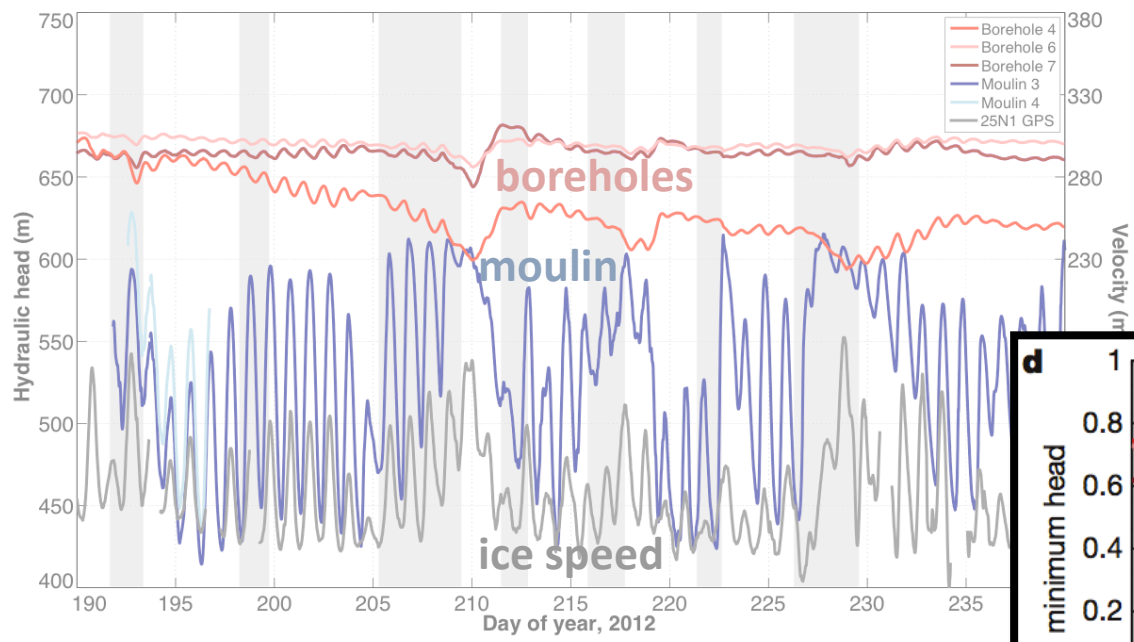
- Low amplitude diurnal changes in boreholes
- Borehole head out of phase with velocity
  - Sampled 'disconnected' or 'isolated' distributed system
- Seasonal trends in some boreholes match seasonal trends in ice velocity.



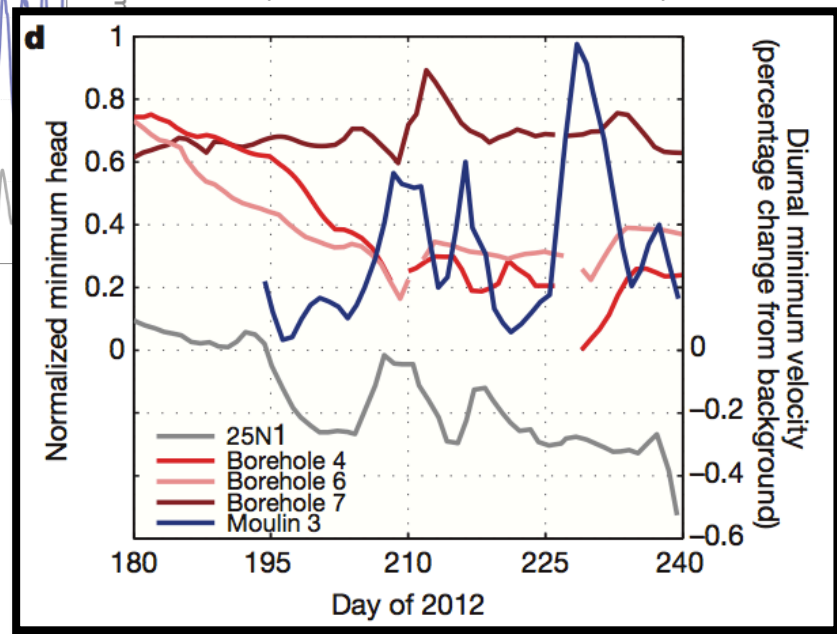


# Subglacial changes outside the channelized regions

- Low amplitude diurnal changes in boreholes
- Borehole head out of phase with velocity
  - Sampled 'disconnected' or 'isolated' distributed system
- Seasonal trends in some boreholes match seasonal trends in ice velocity.



## Seasonal Variation (smoothed, normalized)



## Observational Summary

- Channels control short-term variations in velocity but *not* late-summer evolution.
- Late-summer evolution may be affected by changes in **"isolated regions"** of the bed.



# Importance of Isolated Drainage?

Ice dynamics respond to the integrated basal traction over both **connected** and **disconnected (isolated)** regions (Iken & Truffer 1997).

If water pressure lowers in the disconnected region, that should increase the overall basal traction, causing less sliding.

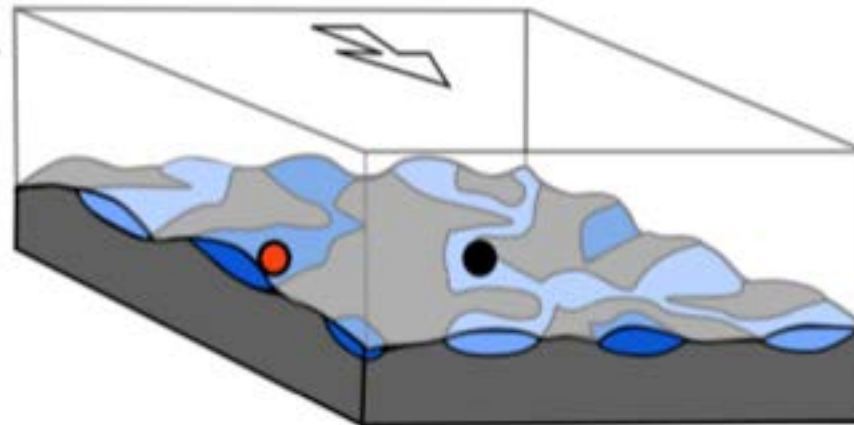
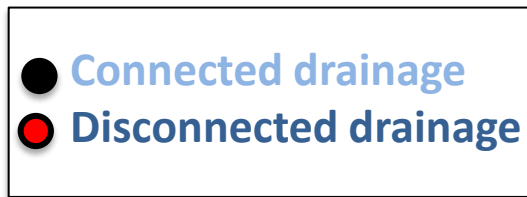


Figure modified from Ian Hewitt

Ample evidence for **extensive** and **dynamic** isolated system from mountain glaciers, e.g.:

- Hodge (1979): 22/24 boreholes drilled in South Cascade Glacier intercept 'inactive' regions:  
*"Most of the bed, **possibly as much as 90%**, appears to be hydraulically inactive and isolated from a few active subglacial conduits"*
- Murray & Clarke (1995), Gordon et al. (1998): Inactive regions can **change in pressure** or switch to active as water pressure in the active system rises.
- Iken & Truffer (1997): isolated cavities moderate active drainage regions

# Subglacial Hydrology Model

(Modified version of Hoffman & Price, 2014, JGR)

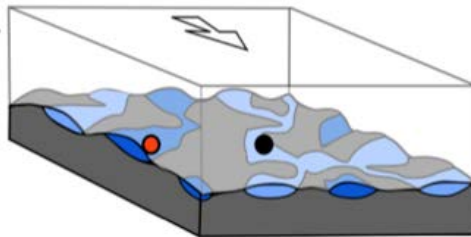


Image: Ian Hewitt

## Weakly-connected Drainage

*new component*

*(0-d subgrid representation)*

Cavities open by sliding,  
close by creep.

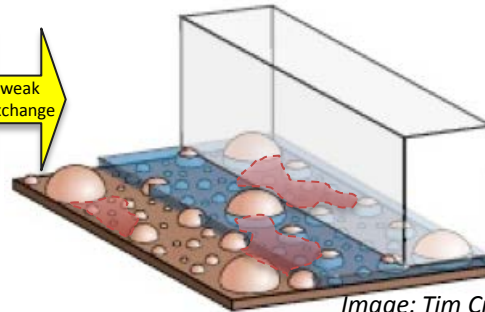


Image: Tim Creyts

## Distributed Drainage

*e.g. Hewitt 2011, J.Glac.*

*(Macroporous sheet)*

Cavities open by sliding,  
close by creep.

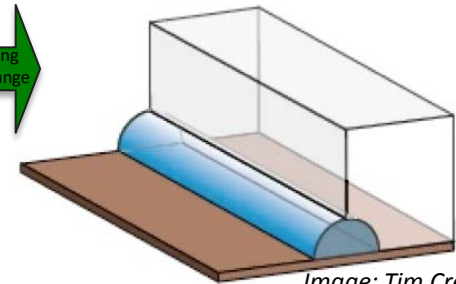


Image: Tim Creyts

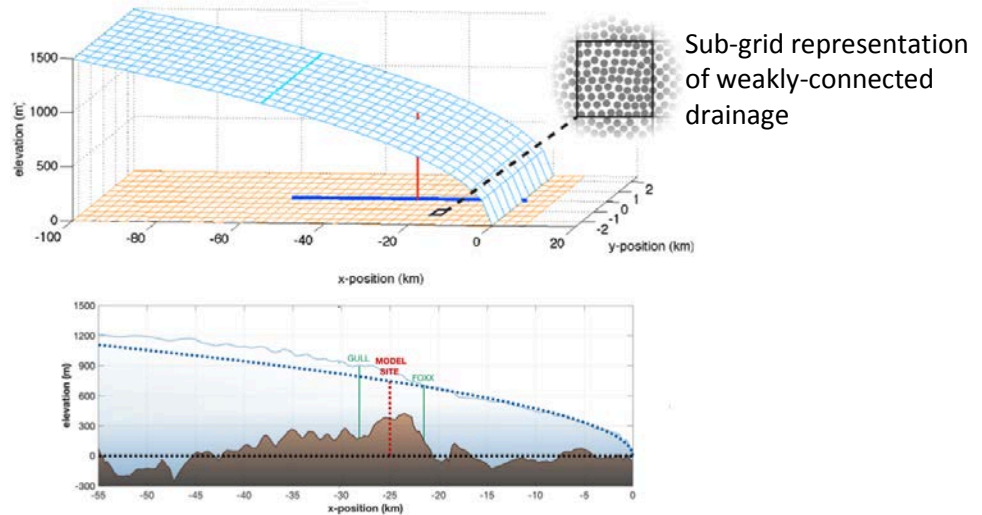
## Channelized Drainage

*e.g. Hewitt 2011, J.Glac.*

Channels open by melting,  
close by creep.

# Idealized "ROGUE" Experiment

- 100km long domain
- "Plastic" glacier shape (constant  $\tau_d = 10^5$  Pa)
- 5 km wide "catchment-scale" domain with laterally periodic boundaries & potential channel along centerline
- Study site:  
25km inland,  $H \sim 750$ m,  $ds/dx \sim 0.01$



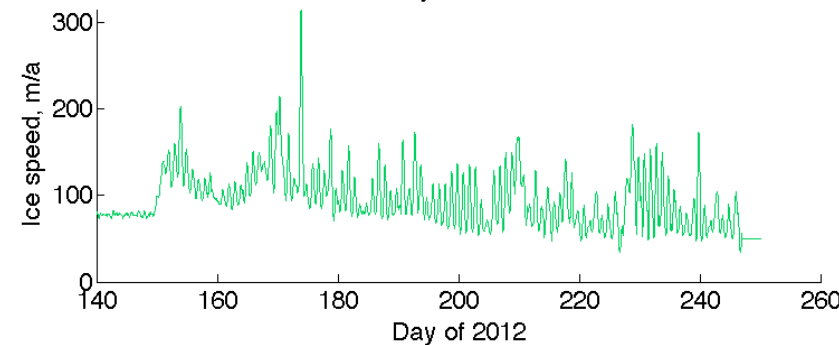
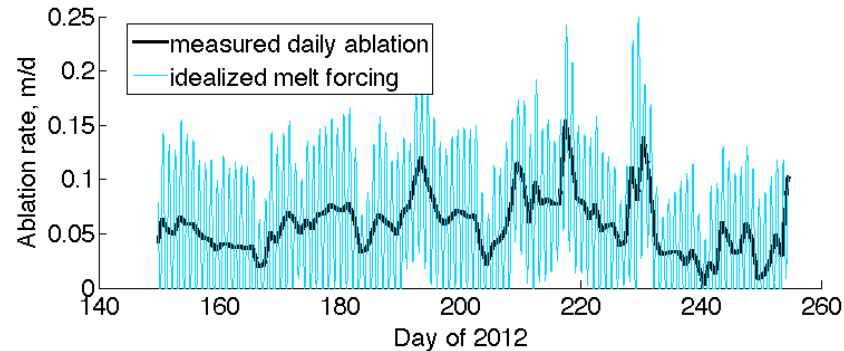
## 1. Springtime initial condition:

spinup 3-component hydrology to steady state

## 2. Summer forcing experiment:

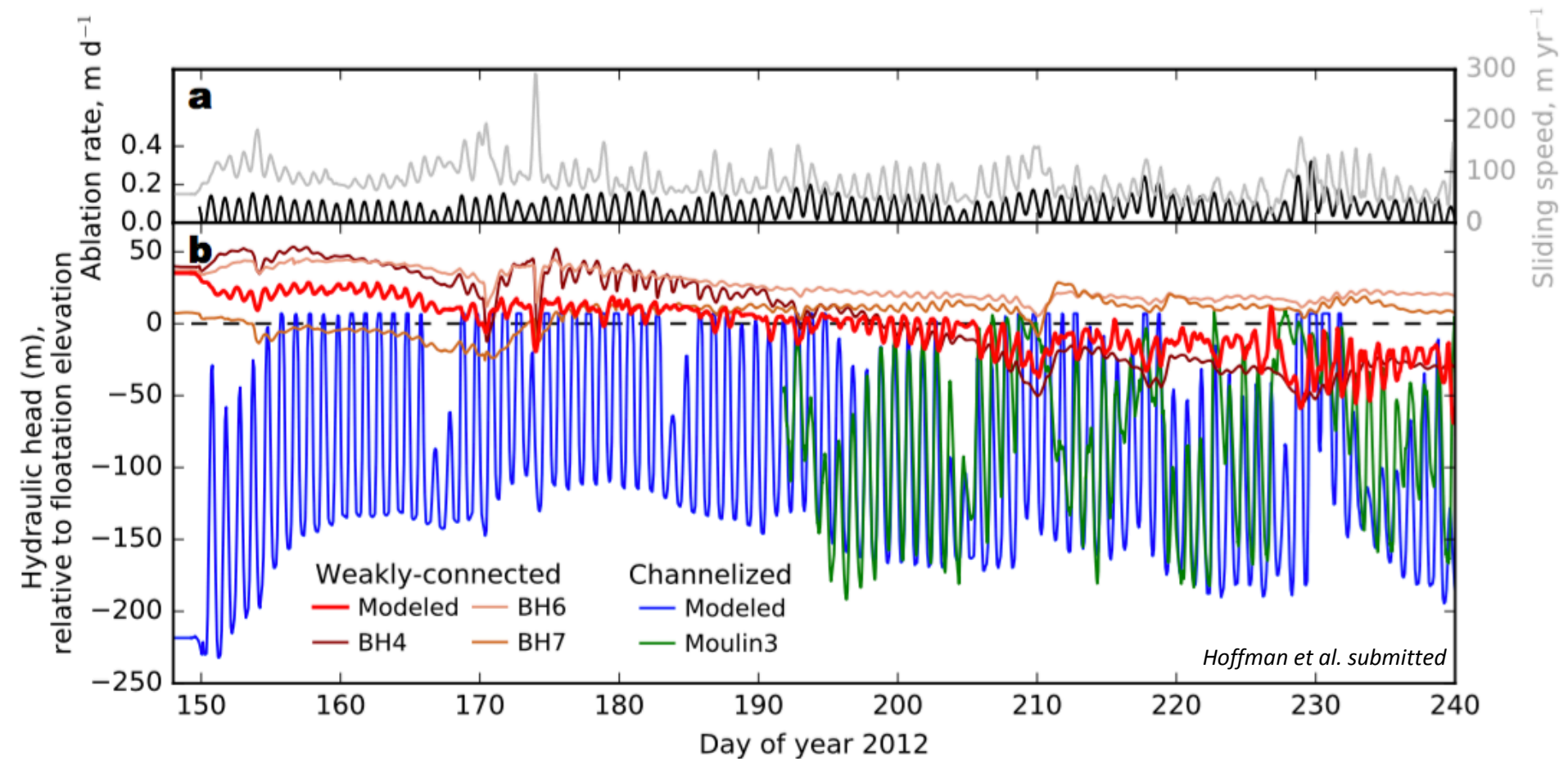
- **Supraglacial meltwater input** along centerline
  - Based on measured daily ablation rates
  - Diurnally-varying sinusoidal shape added
  - Lapse rate extends forcing from ELA to terminus
- **Diurnally-varying sliding** based on GPS ice velocity observations

**Observe seasonal evolution of each component of drainage system at study location.**



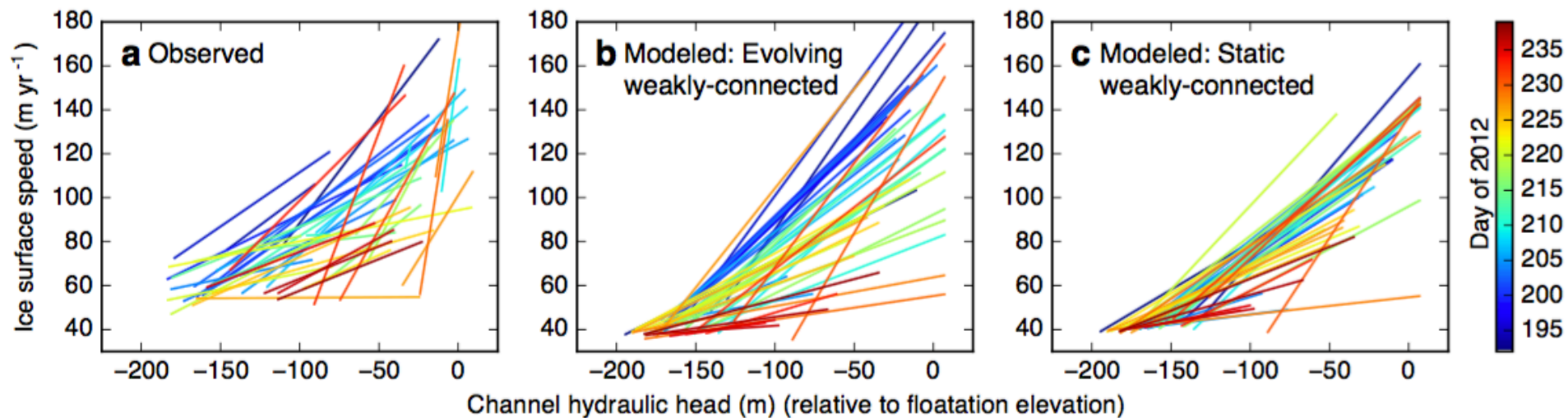


# Model results: water pressure

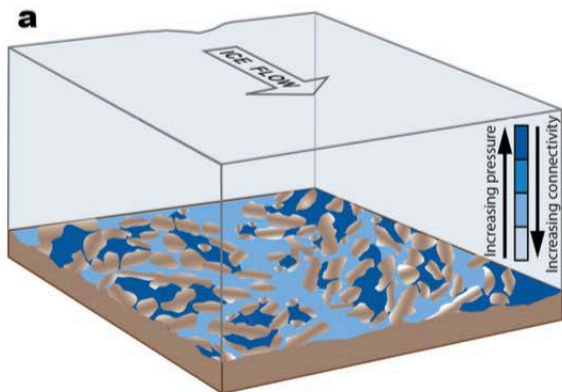


# Model results: ice speed

Solve ice surface speed using CISM 2.1 with Coulomb basal friction law

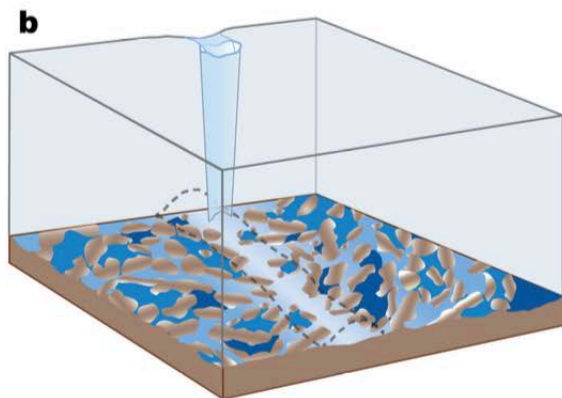


# Proposed conceptual model



## Onset of the melt season

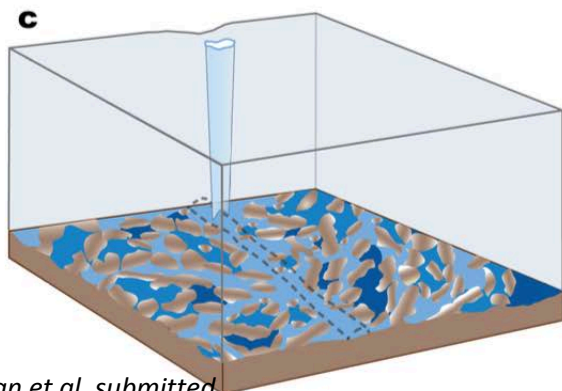
Large fraction of the bed is composed of weakly-connected cavities at a higher water pressure than the surrounding distributed system.



## Middle of the melt season

Meltwater draining through moulin is largely accommodated by the formation of efficient channels.

Concurrently, some of the weakly-connected cavities have leaked water, lowering their water pressure.

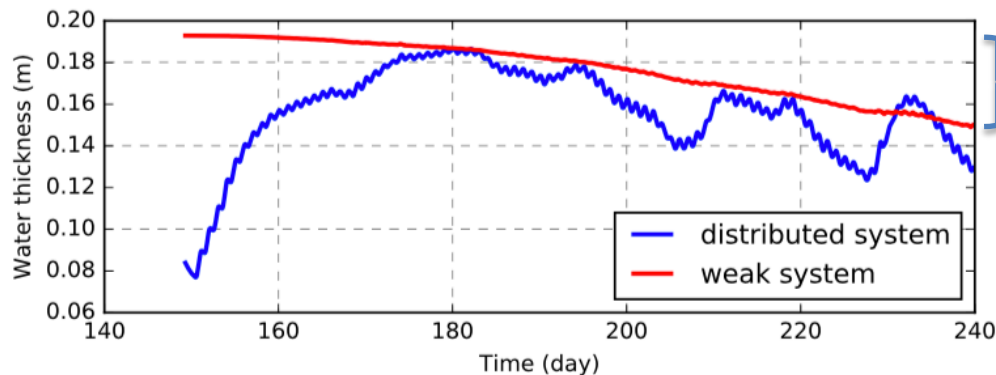
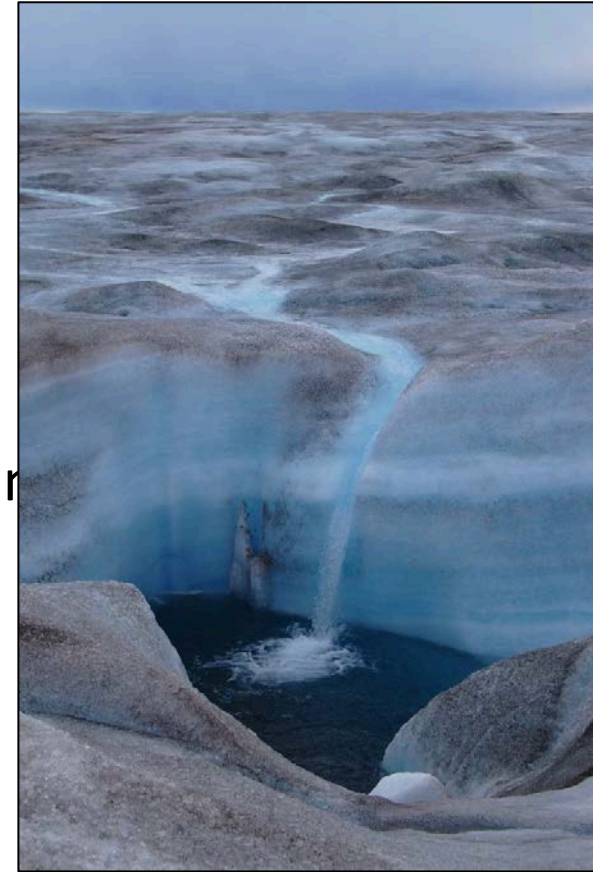


## End of the melt season

Channels collapse within days after melt inputs cease, but the partially drained weakly-connected cavities take months to recharge by basal melting, leaving higher integrated basal traction than before summer began.

# Conclusions

- Observations and modeling suggest a 3-component conceptual model for drainage
  - Distributed
  - Channelized
  - "Isolated" or weakly connected
- Small changes in isolated drainage could play an important role in seasonal evolution due to covering a large area fraction of the bed.
- Isolated system may take longer to recover in fall/winter → winter slowdown mechanism?



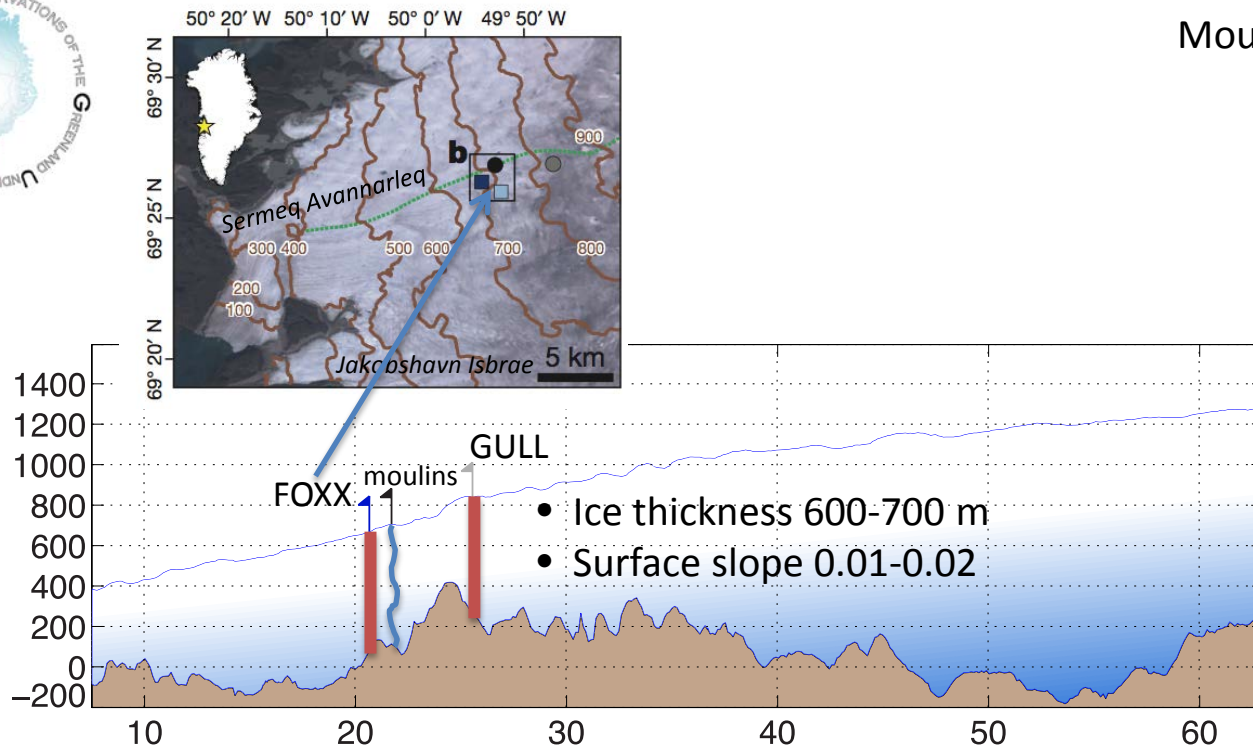
~2-5 years recharge time







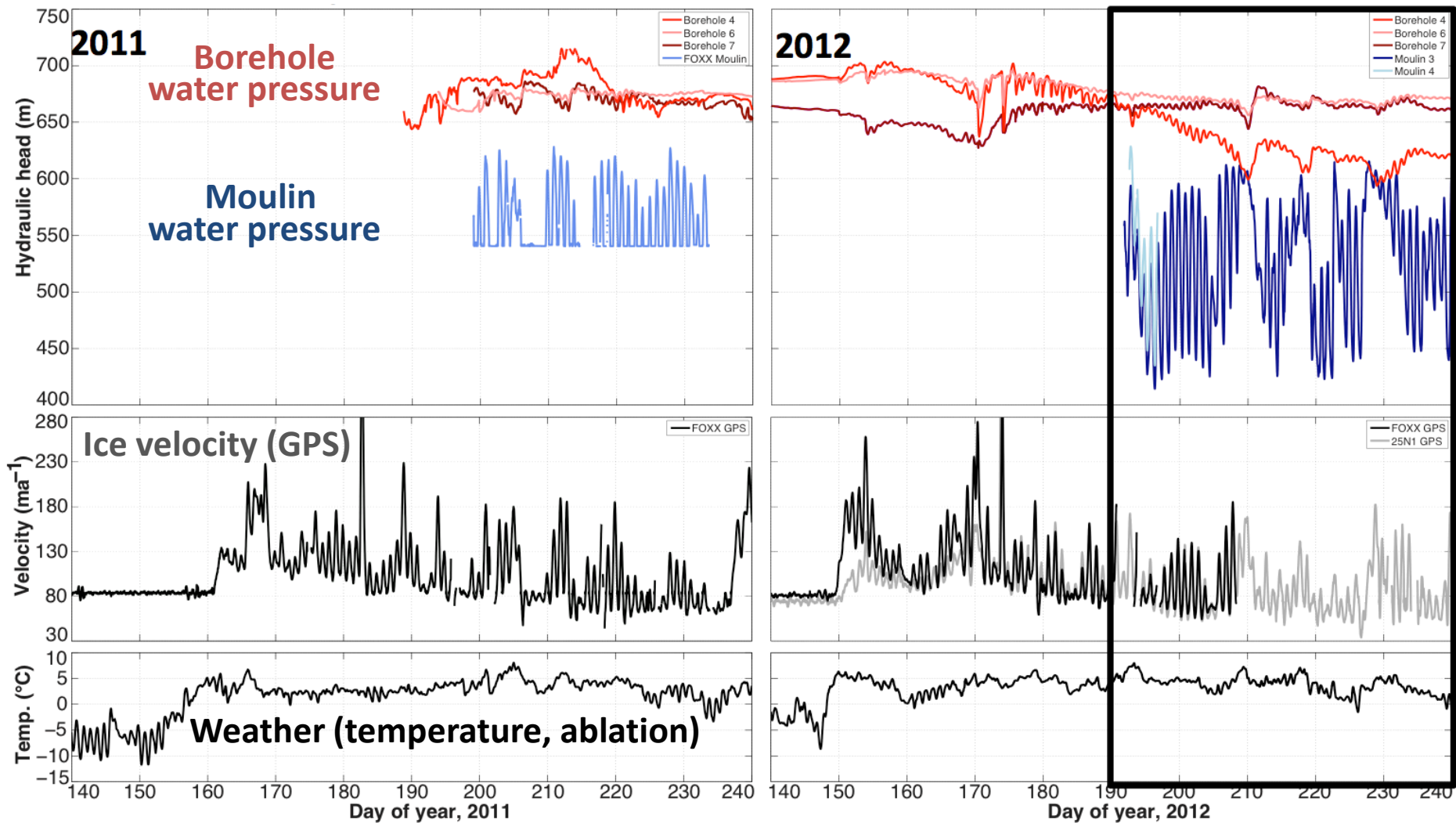
## Moulin pressure sensor installation: Summers 2011 & 2012



## Hot Water Drilling & Borehole sensor installation: Summer 2011



# Measurements during two melt seasons – site FOXX



# Subglacial Hydrology Model

(Modified version of Hoffman & Price, 2014, JGR)

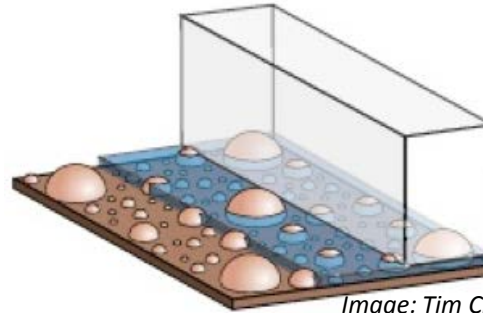


Image: Tim Creyts

## Distributed Drainage

e.g. Hewitt 2011, J.Glac.

(Macroporous sheet)

Mass Conservation  
of Water

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = \frac{m}{\rho_w} + \omega$$

Flux  
divergence
Basal  
melt
Water input from  
surface & other  
drainage elements

Evolution of  
Drainage Element  
Volume

$$\frac{\partial h}{\partial t} = V_O - V_C = \left( \frac{m}{\rho_i} + |\mathbf{u}_b| \frac{h_r - h}{l_r} \right) - \left( \frac{hN}{\eta_i} \right)$$

Melt  
opening
Sliding  
over  
bumps
Creep closure  
of ice

Flow Law

Darcy style:

$$\mathbf{q} = -\frac{k_0 h^3}{\eta_w} \nabla \phi$$

Energy Balance

$$mL = G + \mathbf{u}_b \cdot \boldsymbol{\tau}_b$$

Geothermal
Basal friction



# Subglacial Hydrology Model

(Modified version of Hoffman & Price, 2014, JGR)

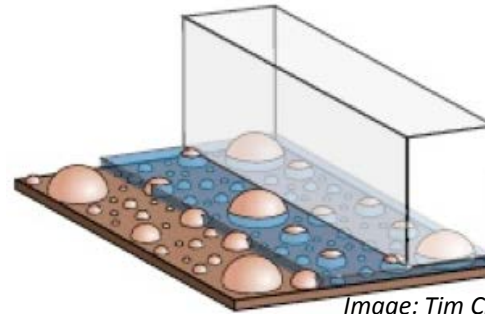


Image: Tim Creyts

**Distributed Drainage**  
e.g. Hewitt 2011, J.Glac.  
(Macroporous sheet)

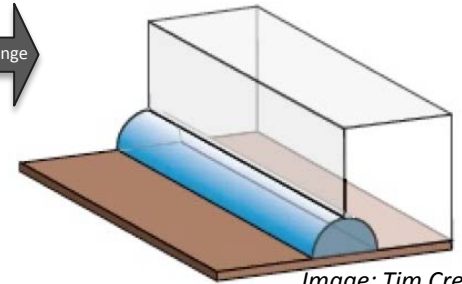


Image: Tim Creyts

**Channelized Drainage**  
e.g. Hewitt 2011, J.Glac.

**Mass Conservation of Water**

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = \frac{m}{\rho_w} + \omega$$

Flux divergence
Basal melt
Water input from surface & other drainage elements

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = \frac{M}{\rho_w} + \Omega_c$$

flow divergence
melt from flow
water input from sheet

**Evolution of Drainage Element Volume**

$$\frac{\partial h}{\partial t} = V_O - V_C = \left( \frac{m}{\rho_i} + |\mathbf{u}_b| \frac{h_r - h}{l_r} \right) - \left( \frac{hN}{\eta_i} \right)$$

Melt opening
Sliding over bumps
Creep closure of ice

$$\frac{\partial S}{\partial t} = \frac{M}{\rho_i} - \frac{SN_c}{\eta_i}$$

melt from flow
creep closure of ice

**Flow Law**

Darcy style:

$$\mathbf{q} = -\frac{k_0 h^3}{\eta_w} \nabla \phi$$

Turbulent flow:

$$FQ^2 = S^{8/3} \left( \Psi + \frac{\partial N_c}{\partial x} \right),$$

**Energy Balance**

$$mL = G + \mathbf{u}_b \cdot \boldsymbol{\tau}_b$$

Geothermal
Basal friction

$$ML = Q \left( \Psi + \frac{\partial N_c}{\partial x} \right)$$

Dissipation

# Subglacial Hydrology Model

(Modified version of Hoffman & Price, 2014, JGR)

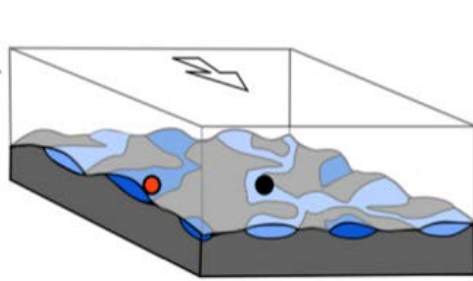


Image: Ian Hewitt

**Isolated Drainage**  
new component  
(0-d subgrid representation)

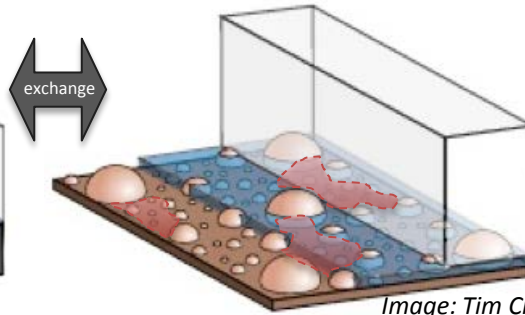


Image: Tim Creyts

**Distributed Drainage**  
e.g. Hewitt 2011, J.Glac.  
(Macroporous sheet)

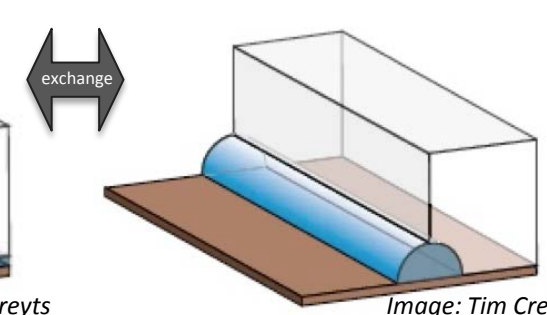


Image: Tim Creyts

**Channelized Drainage**  
e.g. Hewitt 2011, J.Glac.

**Mass Conservation of Water**

$$\frac{\partial h}{\partial t} = \frac{m}{\rho_w} + \gamma$$

Basal melt     Water input from sheet

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = \frac{m}{\rho_w} + \omega$$

Flux divergence     Basal melt     Water input from surface & other drainage elements

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = \frac{M}{\rho_w} + \Omega,$$

divergence     melt from flow     water input from sheet

**Evolution of Drainage Element Volume**

$$\frac{\partial h}{\partial t} = V_O - V_C = \left( \frac{m}{\rho_i} + |\mathbf{u}_b| \frac{h_r - h}{l_r} \right) - \left( \frac{hN}{\eta_i} \right)$$

Melt opening     Sliding over bumps     Creep closure of ice

$$\frac{\partial h}{\partial t} = V_O - V_C = \left( \frac{m}{\rho_i} + |\mathbf{u}_b| \frac{h_r - h}{l_r} \right) - \left( \frac{hN}{\eta_i} \right)$$

Melt opening     Sliding over bumps     Creep closure of ice

$$\frac{\partial S}{\partial t} = \frac{M}{\rho_i} - \frac{SN_c}{\eta_i},$$

melt from flow     creep closure of ice

Darcy style exchange term:

$$\gamma = - \frac{k_i h^3 \phi_i - \phi_s}{\eta_w ds}$$

Darcy style:

$$\mathbf{q} = - \frac{k_0 h^3}{\eta_w} \nabla \phi$$

Turbulent flow:

$$FQ^2 = S^{8/3} \left( \Psi + \frac{\partial N_c}{\partial x} \right),$$

**Flow Law**

**Energy Balance**

$$mL = G + \mathbf{u}_b \cdot \boldsymbol{\tau}_b$$

Geothermal     Basal friction

$$mL = G + \mathbf{u}_b \cdot \boldsymbol{\tau}_b$$

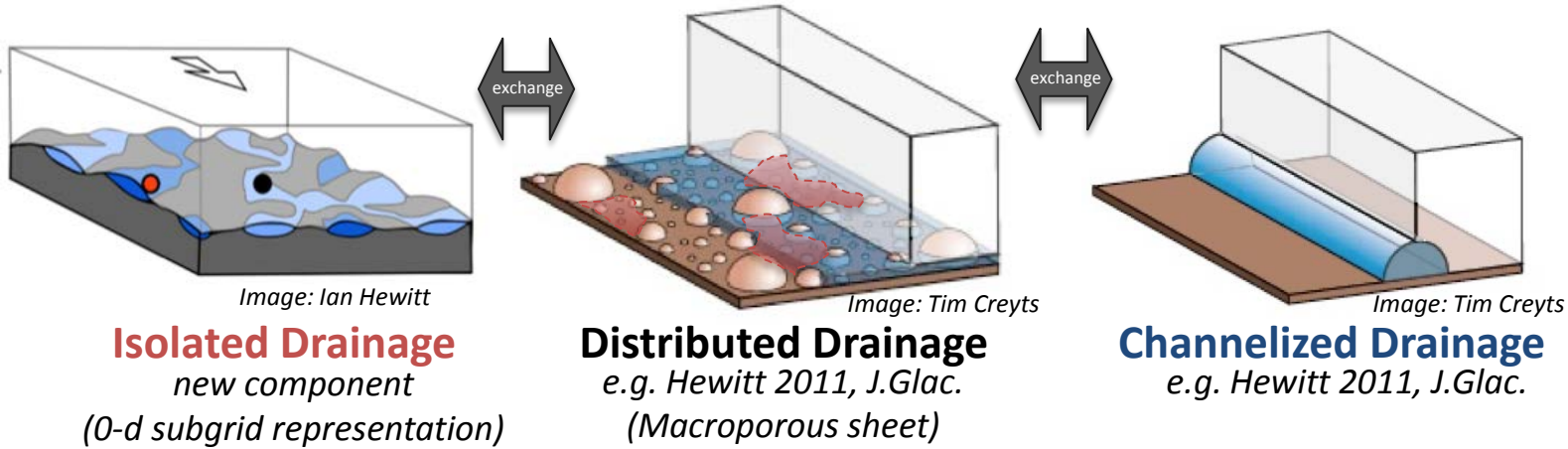
Geothermal     Basal friction

$$ML = Q \left( \Psi + \frac{\partial N_c}{\partial x} \right).$$

Dissipation

# Subglacial Hydrology Model

(Modified version of Hoffman & Price, 2014, JGR)



## Mass Conservation of Water

$$\frac{\partial h}{\partial t} = \frac{m}{\rho_w} + \gamma$$

Basal melt     Water input from sheet

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = \frac{m}{\rho_w} + \omega$$

Flux divergence     Basal melt     Water input from surface & other drainage elements

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = \frac{M}{\rho_w} + \Omega,$$

divergence flow     melt from flow     water input from sheet

## Evolution of Drainage Element Volume

$$\frac{\partial h}{\partial t} = V_O - V_C = \left( \frac{m}{\rho_i} + |\mathbf{u}_b| \frac{h_r - h}{l_r} \right) - \left( \frac{hN}{\eta_i} \right)$$

Melt opening     Sliding over bumps     Creep closure of ice

$$\frac{\partial h}{\partial t} = V_O - V_C = \left( \frac{m}{\rho_i} + |\mathbf{u}_b| \frac{h_r - h}{l_r} \right) - \left( \frac{hN}{\eta_i} \right)$$

Melt opening     Sliding over bumps     Creep closure of ice

$$\frac{\partial S}{\partial t} = \frac{M}{\rho_i} - \frac{SN_c}{\eta_i},$$

melt from flow     creep closure of ice

## Flow Law

Darcy style exchange term:  $\gamma = - \frac{k_i h^3 \phi_i - \phi_s}{\eta_w ds}$

Conductivity function of effective pressure:  $k_i = \frac{C}{N + \eta}$

(inspired by Murray & Clarke (1995), Gordon et al. (1998))

Darcy style:

$$\mathbf{q} = - \frac{k_0 h^3}{\eta_w} \nabla \phi$$

Turbulent flow:

$$FQ^2 = S^{8/3} \left( \Psi + \frac{\partial N_c}{\partial x} \right),$$

## Energy Balance

$$mL = G + \mathbf{u}_b \cdot \boldsymbol{\tau}_b$$

Geothermal     Basal friction

$$mL = G + \mathbf{u}_b \cdot \boldsymbol{\tau}_b$$

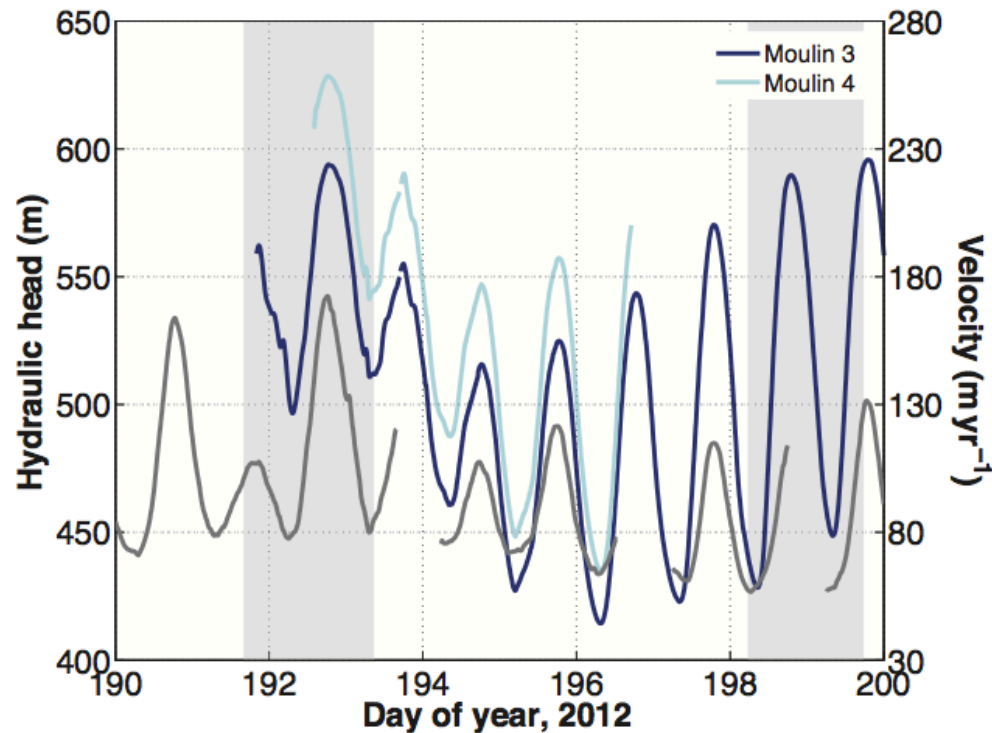
Geothermal     Basal friction

$$ML = Q \left( \Psi + \frac{\partial N_c}{\partial x} \right).$$

Dissipation

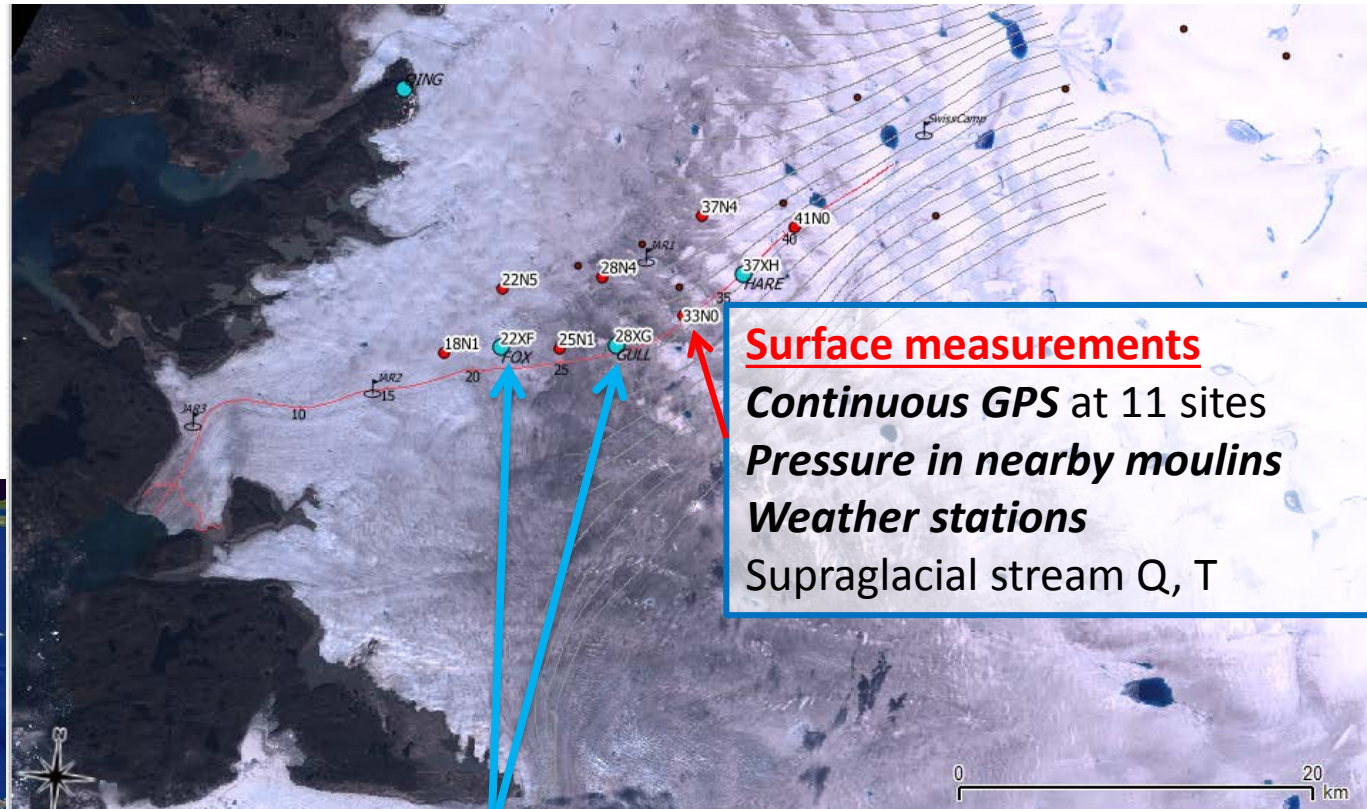
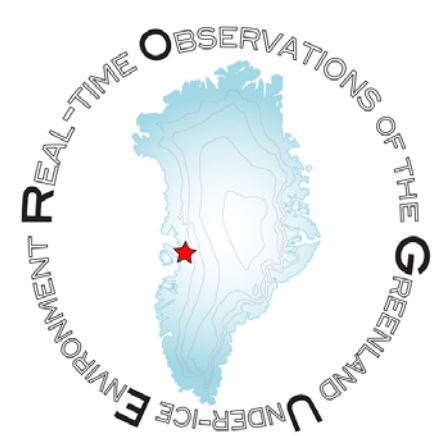
# Moulin water pressure indicates subglacial channels

- Large diurnal variability, specifically, low diurnal minima
- In phase with ice velocities
- Neighboring moulins highly correlated
- Channel model used to confirm quasi-steady state behavior during second half of summer

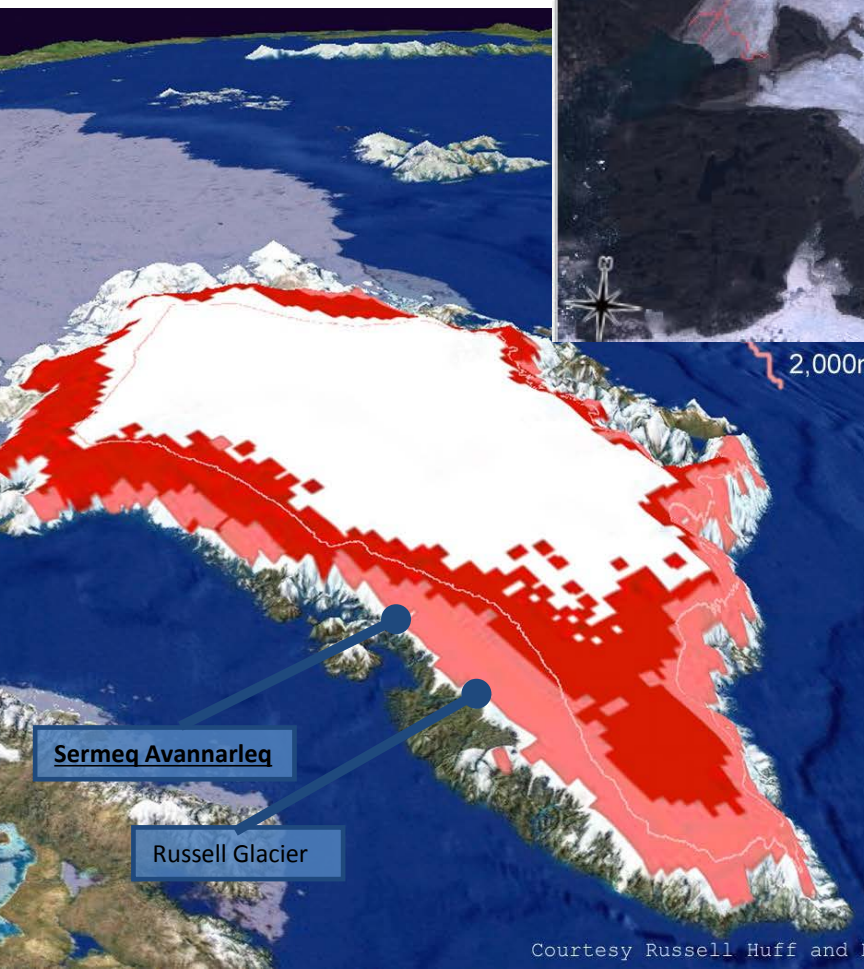


Moulin pressure controls short-term variations in ice speed  
(diurnal and melt-event-scale)





**Surface measurements**  
**Continuous GPS at 11 sites**  
**Pressure in nearby moulins**  
**Weather stations**  
 Supraglacial stream Q, T

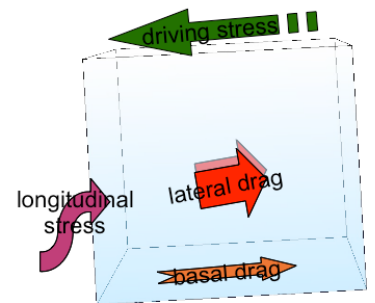


**Borehole measurements**

<u>Englacial</u>	<u>Subglacial</u>
Temperature	<b>Water Pressure</b>
Pressure	Sliding (tethered stake)
Inclination	Water EC
Vertical strain	[Water, bed samples]
Optical Televiewer	

## Ice dynamics:

- Community Ice Sheet Model (CISM)
- Higher-order stress balance



## Sliding law: Couples sheet hydrology (N) to dynamics ( $\tau_b, u_b$ )

- Coulomb friction sliding law (*Schoof 2005, Proc. R. Soc. A*)
- bounded basal drag, cavitation

$$\tau_b = C \left( \frac{u_b}{u_b + N^n \Lambda} \right)^{1/n} N,$$

$$\Lambda = \frac{\lambda_{\max} A}{m_{\max}}$$

Bedrock bump wavelength (points to  $\lambda_{\max}$ )

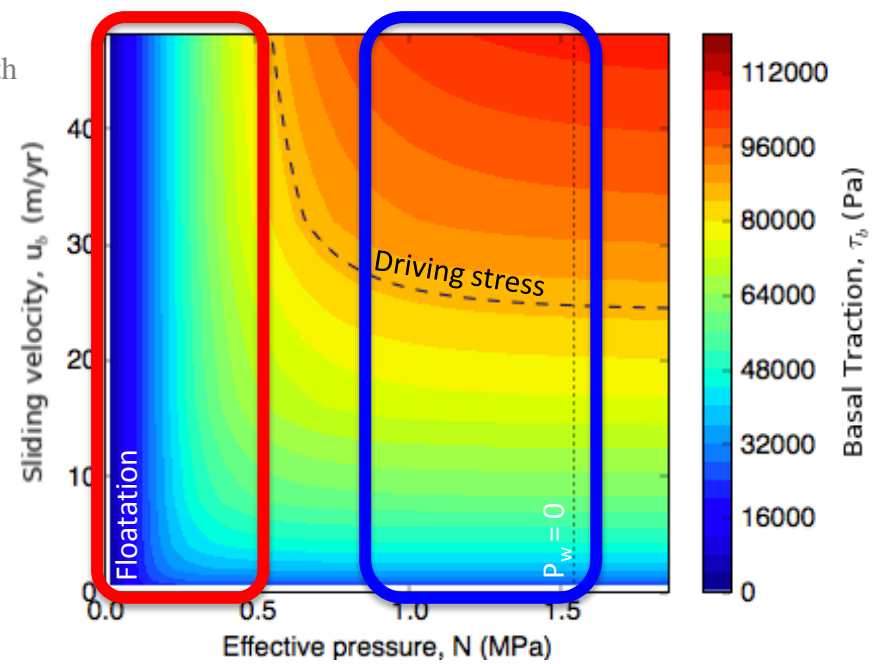
Bedrock bump slope (points to  $m_{\max}$ )

At high N, basal traction is independent of N.

$$\tau_b \propto u_b^{1/n} \quad (\text{nonlinear viscous – the faster the sliding, the more drag from bumps})$$

At low N (near floatation), basal traction is independent of  $u_b$  (Coulomb friction).

$$\tau_b \propto N \quad (\text{near floatation, bumps are drowned})$$



# Distributed Flow Model

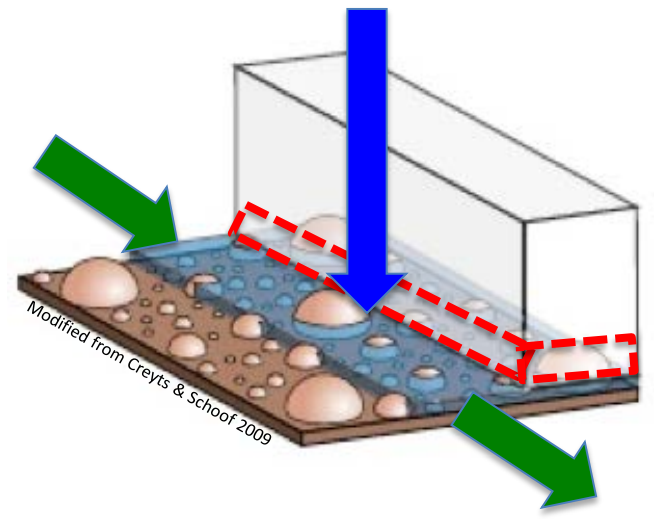
sheet flow, e.g. linked cavities

(e.g. Hewitt 2011, J.Glac.)

## 1) Mass conservation of water

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = \frac{m}{\rho_w} + \omega$$

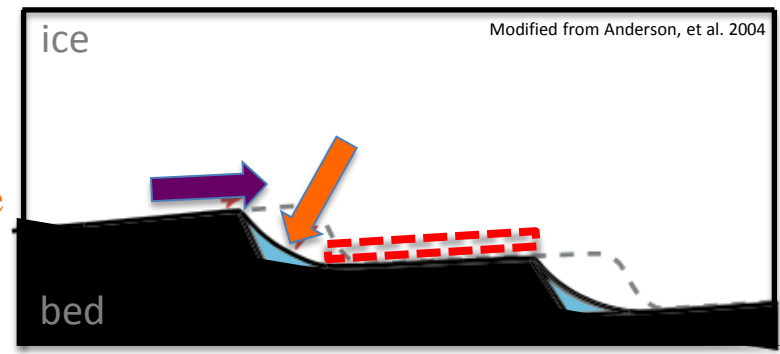
$\frac{\partial h}{\partial t}$ : Flux divergence  
 $\nabla \cdot \mathbf{q}$ : Flux divergence  
 $\frac{m}{\rho_w}$ : Basal melt  
 $\omega$ : Water input from surface



## 2) Evolution of subglacial cavities

$$\frac{\partial h}{\partial t} = V_O - V_C = \left( \frac{m}{\rho_i} + |\mathbf{u}_b| \frac{h_r - h}{l_r} \right) - \left( \frac{hN}{\eta_i} \right)$$

$\frac{m}{\rho_i}$ : Melt opening  
 $|\mathbf{u}_b| \frac{h_r - h}{l_r}$ : Sliding over bumps  
 $\frac{hN}{\eta_i}$ : Creep closure of ice



A Darcy style flow law

$$\mathbf{q} = -\frac{k_0 h^3}{\eta_w} \nabla \phi$$

Heat from passive sources  
(heat from water flow neglected)

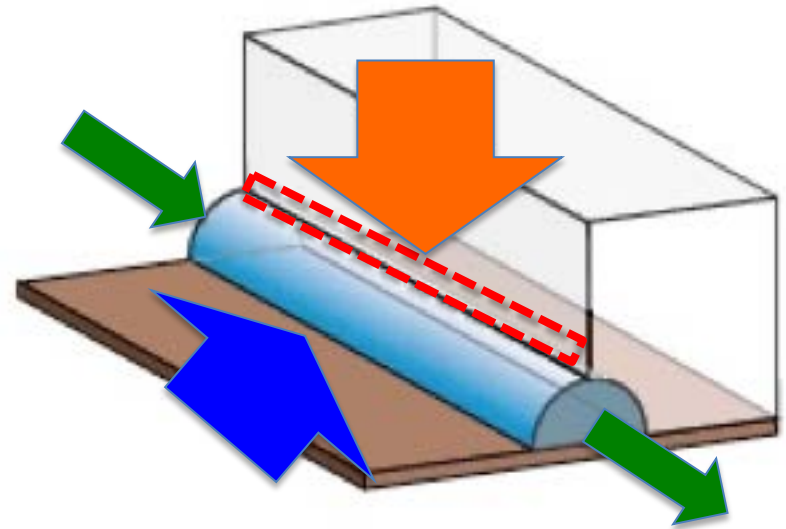
$$mL = G + \mathbf{u}_b \cdot \boldsymbol{\tau}_b - \mathbf{q} \cdot \nabla \phi$$

$G$ : Geothermal  
 $\mathbf{u}_b \cdot \boldsymbol{\tau}_b$ : Basal friction

## 1) Mass conservation of water

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = \frac{M}{\rho_w} + \Omega,$$

flow melt from  
divergence flow water input  
from sheet



## 2) Evolution of subglacial cavities

$$\frac{\partial S}{\partial t} = \frac{M}{\rho_i} - \frac{SN_c}{\eta_i},$$

melt from  
flow creep closure  
of ice

A turbulent flow law

$$FQ^2 = S^{8/3} \left( \Psi + \frac{\partial N_c}{\partial x} \right),$$

melt from flow

$$ML = Q \left( \Psi + \frac{\partial N_c}{\partial x} \right).$$

Coupled to the surrounding sheet via the exchange term  $\Omega$