A polar bear is standing on a small, white ice floe in the middle of a dark sea. The sea is filled with many other ice floes of various sizes, creating a fragmented and textured surface. The bear is facing left, and its fur appears slightly yellowish due to the environment. The overall scene is a stark, cold, and desolate Arctic landscape.

# An anisotropic, elastic-decohesive constitutive relation for modeling Arctic sea ice

Gunter Leguy and Deborah Sulsky

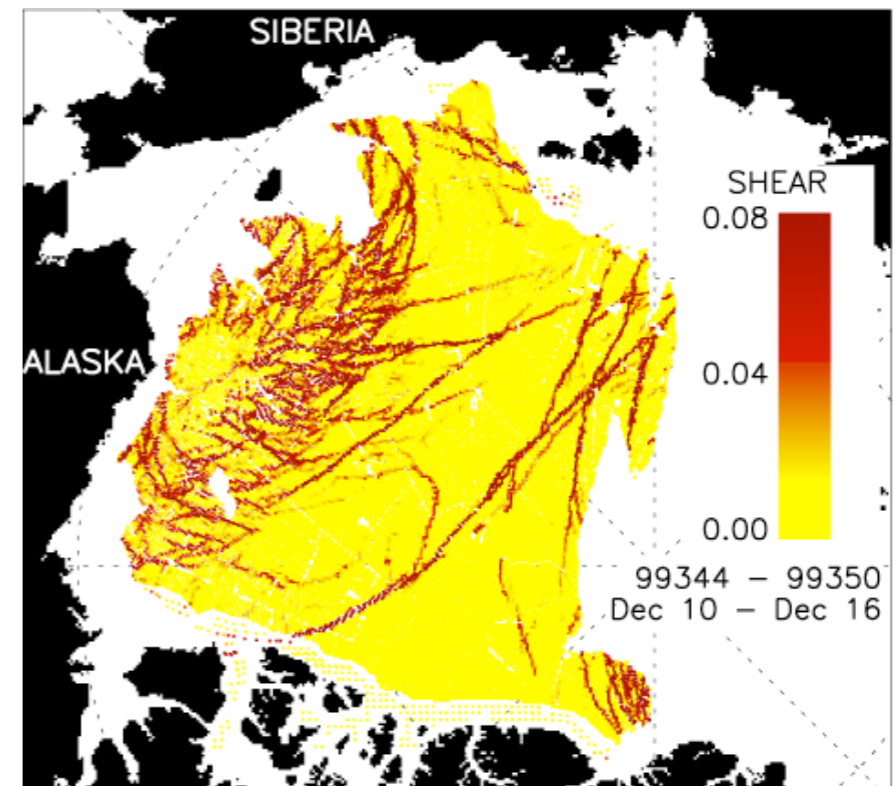
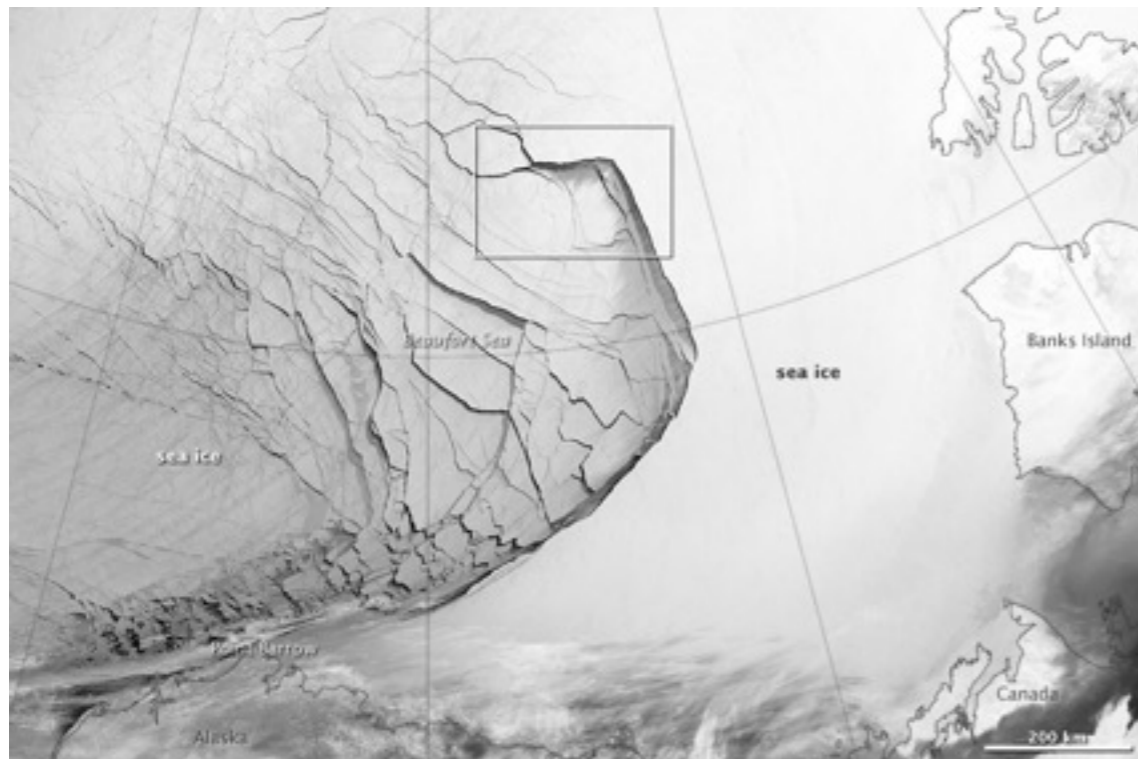
University of New Mexico, Albuquerque, New Mexico

This work is partially supported by grant #NA150AR4310165 to the University of New Mexico from the Climate Variability and Predictability Program, NOAA, US Dept. of Commerce.

# Motivation

## Explicitly represent lead formation

RGPS (Kwok, 1998) analysis of satellite images shows large ice deformation events occurring in long-lasting linear features that appear to correspond to displacement (or velocity) discontinuities in the deformation field due to leads.



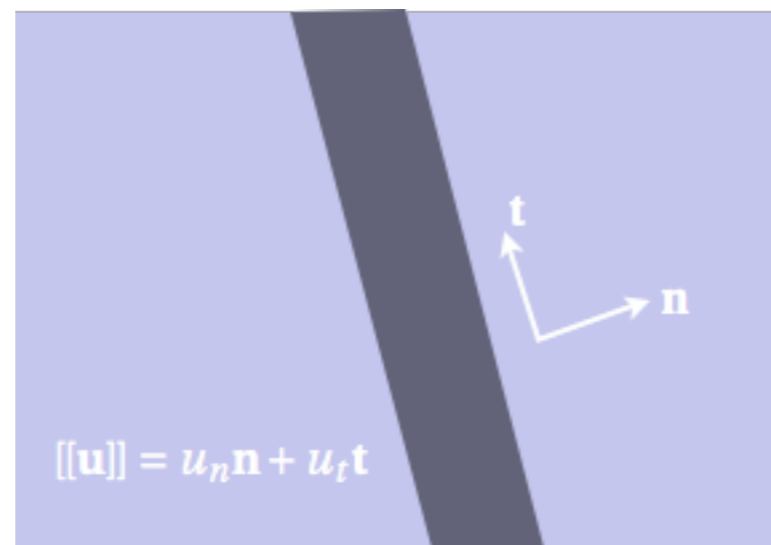
Cracks in the ice (leads) occupy 1-2% of the ice cover in winter but account for half of the ocean-air heat flux. Heat flux through intact ice is 2-5  $\text{Wm}^2$  compared with 300-500  $\text{Wm}^2$  through leads.

# Model

- Ice dynamics (horizontal momentum equation) is solved using the material point method (Peterson and Sulsky, 2012)
- Mass is conserved for each material point (continuity equation)
- Each material point solves column thermodynamics equations and tracks ice thickness distribution
- The sea ice code is coupled to the MITgcm (Marshall et al., 1997) ocean code through fluxes
- Atmospheric forcing is JRA-25 reanalysis data (Onogi et al., 2007)
- Use of an elastic-decohesive constitutive model for the ice

# The Elastic-Decohesive Constitutive Model

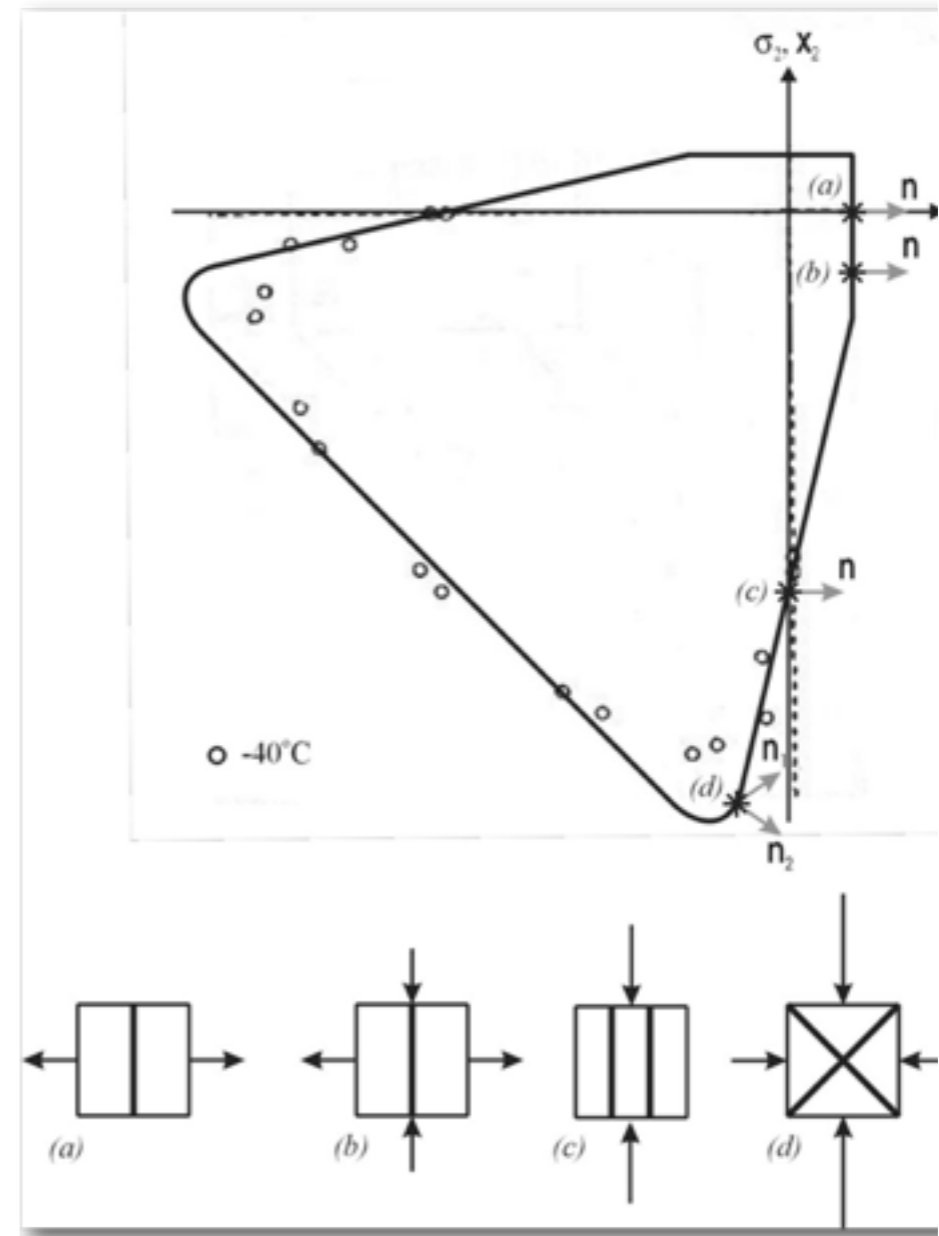
- Intact ice is modeled as elastic
- Leads (cracks) are modeled as discontinuities
- Model predicts initiation, orientation and opening of leads
- Traction is reduced with lead opening until a complete fracture forms



The model introduces a jump in displacement as a crack is initiated in the simulation. Crack initiation is governed by a curve in stress space. What is that curve?

# Laboratory data

Measurement by Schulson (2001) show the stress state when a crack forms and the orientation of the crack. The observed failure envelope in stress space that describes initiation of failure could be described mathematically by a function  $F(\sigma) = 0$ .



What is  $F$ ?

- (a) Loading is purely tensile.
- (b) Biaxial loading - tension and compression.
- (c) Axial loading - pure compression.
- (d) Biaxial compression.

In (a-c) the crack has a normal in the direction of maximum principal stress. (d) transitions to shear failure with two possible crack orientations.

# Corresponding Model

F is a function of stress (Schreyer et al. (2005), Sulky et al. (2006)).

$$F = \max_n F_n(\sigma), \quad [\sigma] = \begin{pmatrix} \tau_n & \tau_t \\ \tau_t & \sigma_{tt} \end{pmatrix}$$

$$F_n = \left( \frac{\tau_t}{s_m \tau_{sf}} \right)^2 + e^{\kappa B_n} - 1$$

$$B_n = \frac{\tau_n}{\tau_{nf}} + \frac{\langle -\sigma_{tt} \rangle^2}{f_c'^2} - 1$$

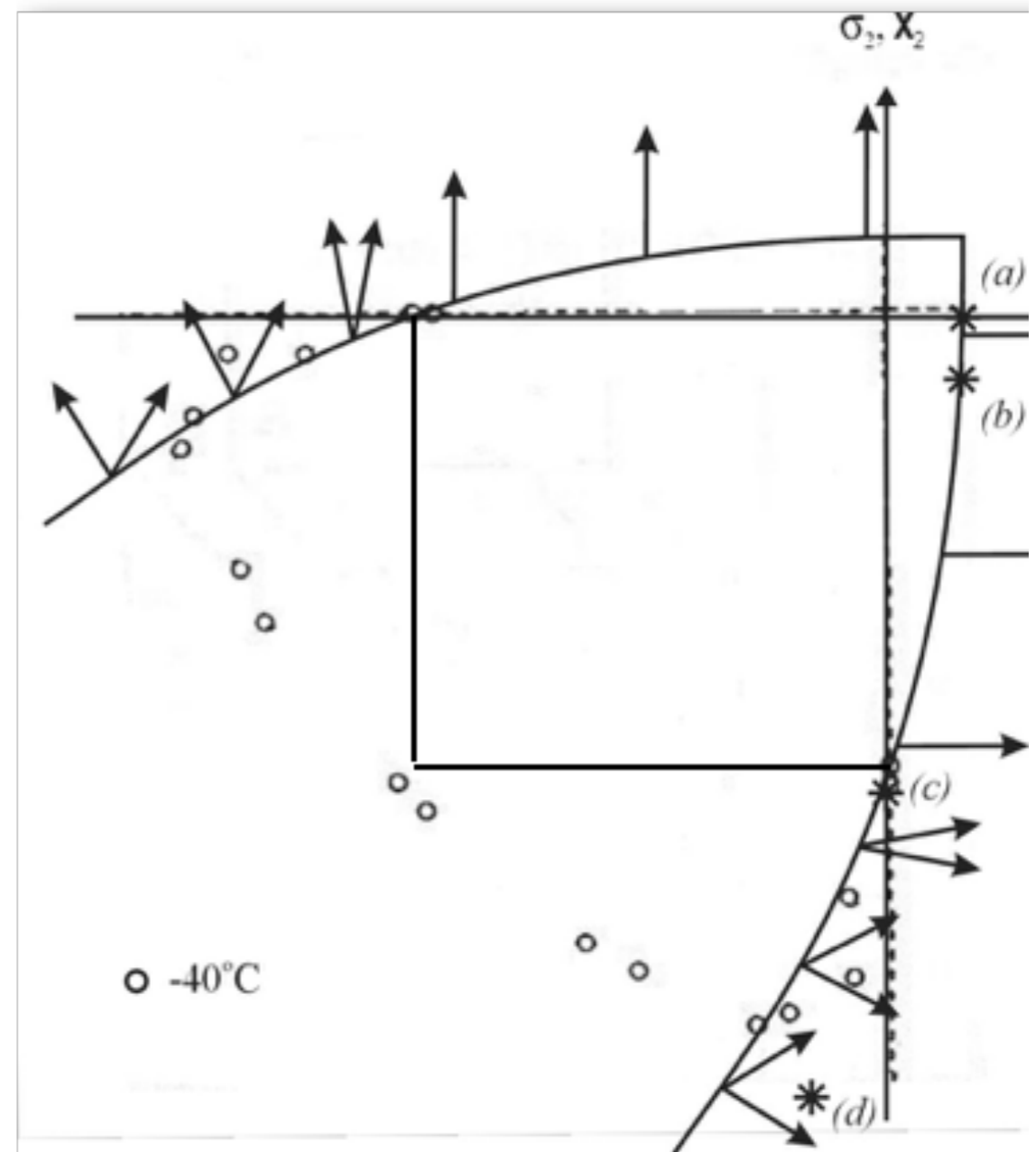
Model parameters:

$\tau_{nf}$  = tensile strength

$\tau_{sf}$  = shear strength

$f_c'$  = compressive strength

$s_m$  = shear magnification



Modeled failure envelope  $F=0$ . Arrows show the predicted direction of the normal to the crack surface. Directions match experiments at (a-d).

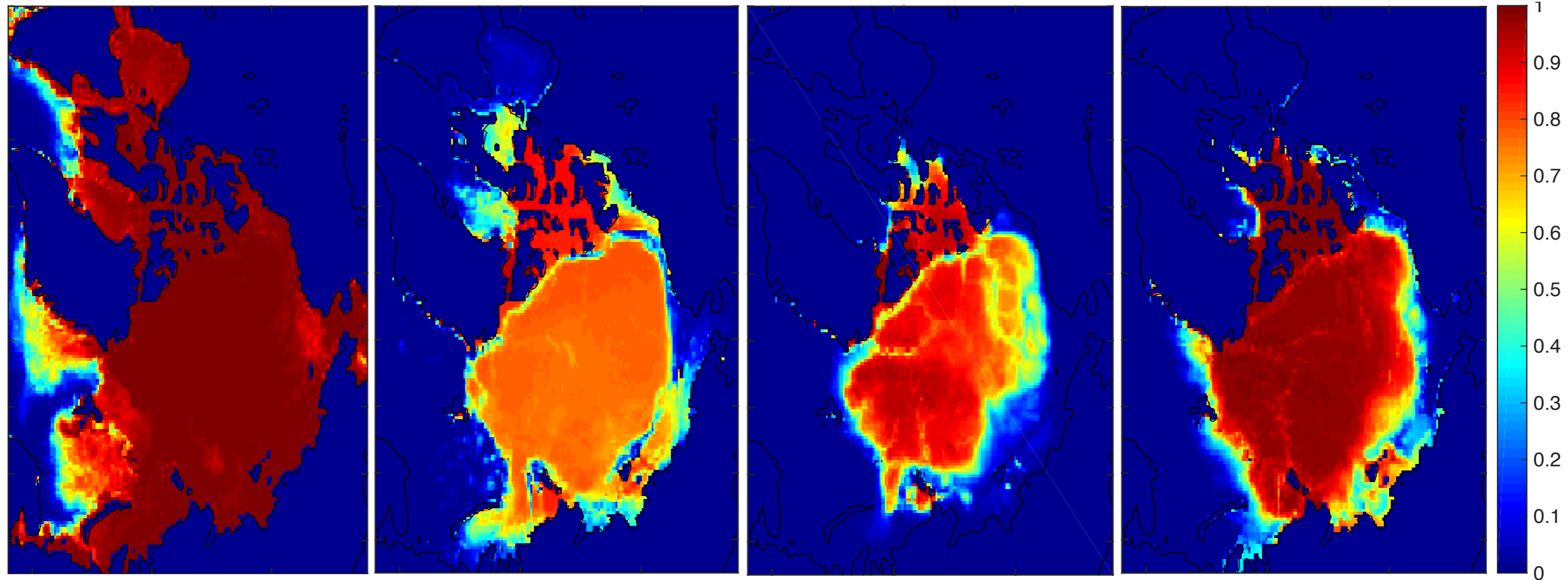
# Sea ice concentration 2003

March

July

September

October



Grid resolution = 36km

# Metrics



# Multi-category contingency table

		Observed category				
		1	2	...	K	Total
Forecast category	ij					
	1	$n(F_1, O_1)$	$n(F_1, O_2)$	...	$n(F_1, O_K)$	$N(F_1)$
	2	$n(F_2, O_1)$	$n(F_2, O_2)$	...	$n(F_2, O_K)$	$N(F_2)$
	...	...	...	...	...	...
	K	$n(F_K, O_1)$	$n(F_K, O_2)$	...	$n(F_K, O_K)$	$N(F_K)$
Total	$N(O_1)$	$N(O_2)$	...	$N(O_K)$	N	

$hits [i] = n(F_i, O_i)$ , event forecast to occur and did occur

$false\ alarm [i] = \sum_{j \neq i} n(F_i, O_j)$ , event forecast to occur, but did not occur

$misses [i] = \sum_{j \neq i} n(F_j, O_i)$ . event forecast not to occur, but did occur

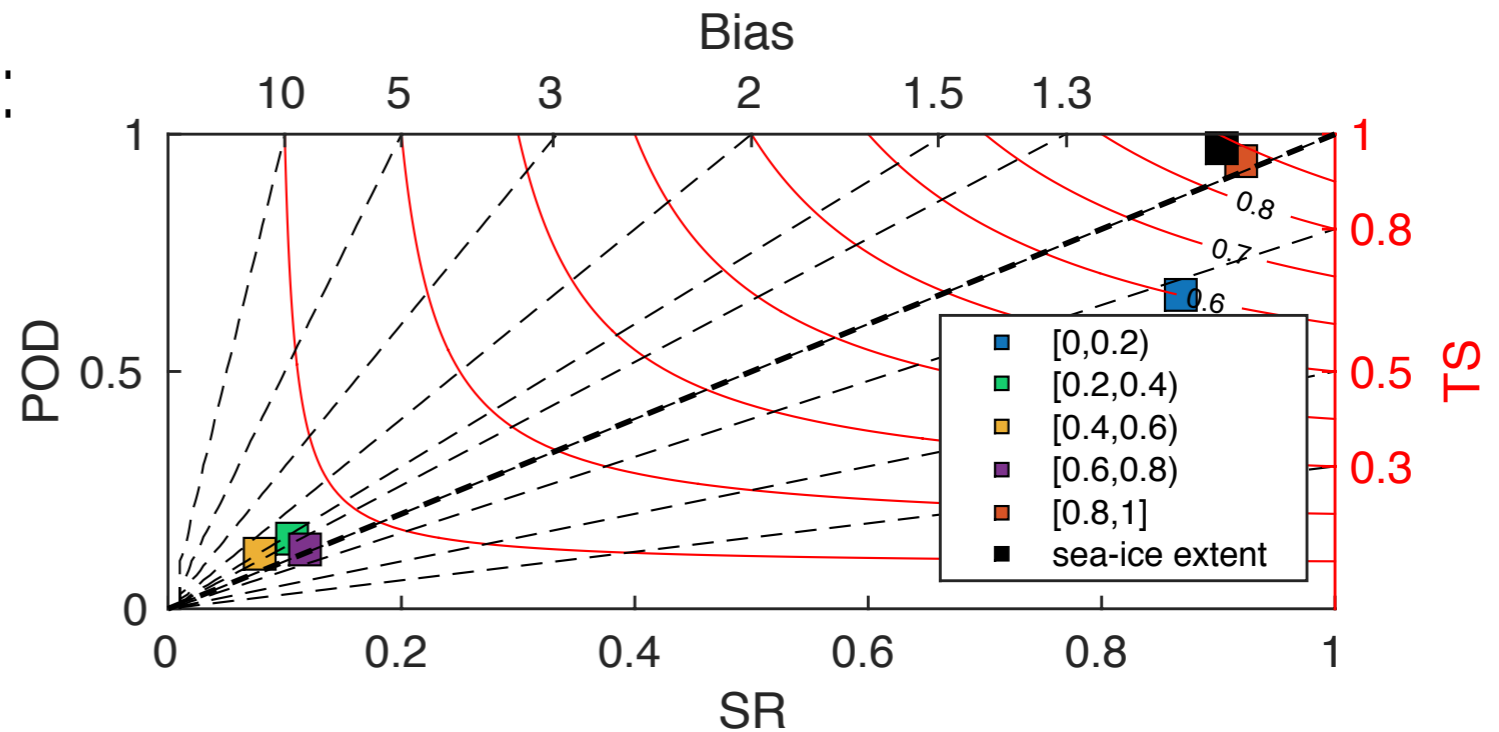
Name	Perfect	Definition	Interpretation
Bias	1	$\frac{h + fa}{h + m}$	How did the forecast frequency of 'yes' events compare to the observed frequency of 'yes' events?
POD	1	$\frac{h}{h + m}$	What fraction of the observed 'yes' events were correctly forecast?
SR	1	$\frac{h}{h + fa}$	What fraction of the forecast 'yes' events were correctly observed?
TS	1	$\frac{h}{h + m + fa}$	How well did the forecast 'yes' events correspond to the observed 'yes' events?

# Performance diagram

- Roebber (2009)
- Use geometric relationship of 4 metrics:

$$\text{bias} = \frac{\text{POD}}{\text{SR}},$$

$$\text{TS} = \frac{1}{\frac{1}{\text{SR}} + \frac{1}{\text{POD}} - 1}.$$



- Easy to read and display

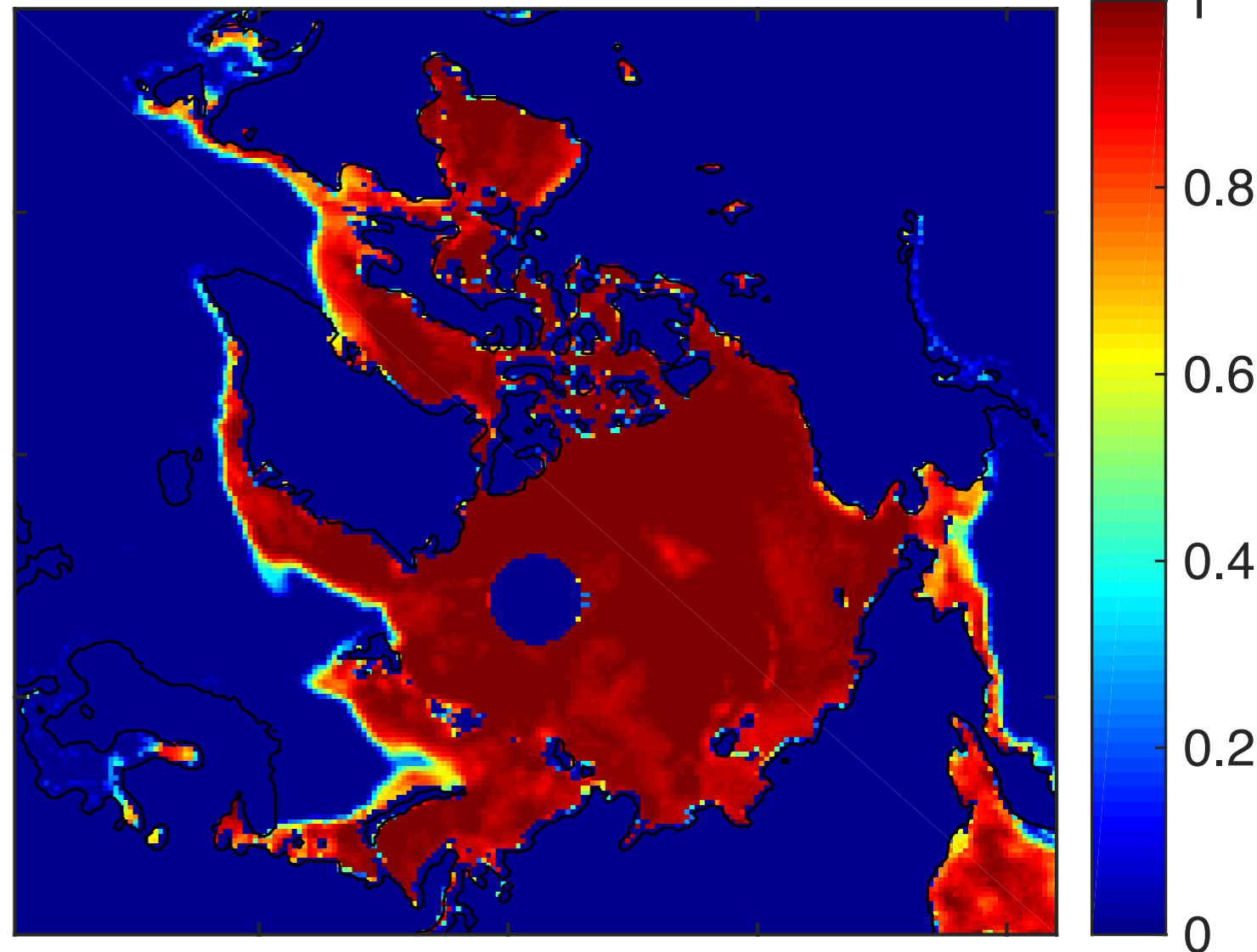
# Model comparison

# Sea ice concentration

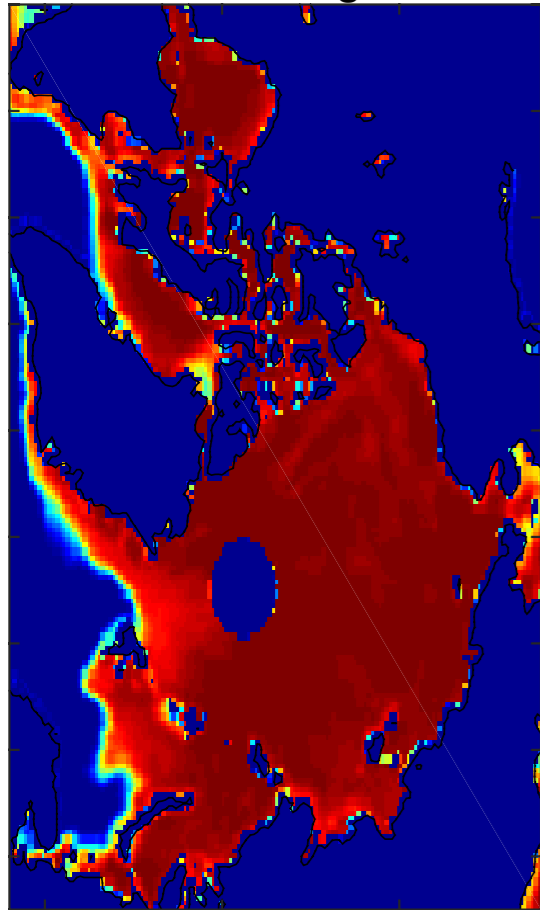
## Observations

- Nimbus-7 passive microwave data (Cavalieri et. al, 1996)
- Gridded resolution: 25 km \* 25 km
- Sensitivity:  $\pm 5\%$  in winter and  $\pm 15\%$  in summer

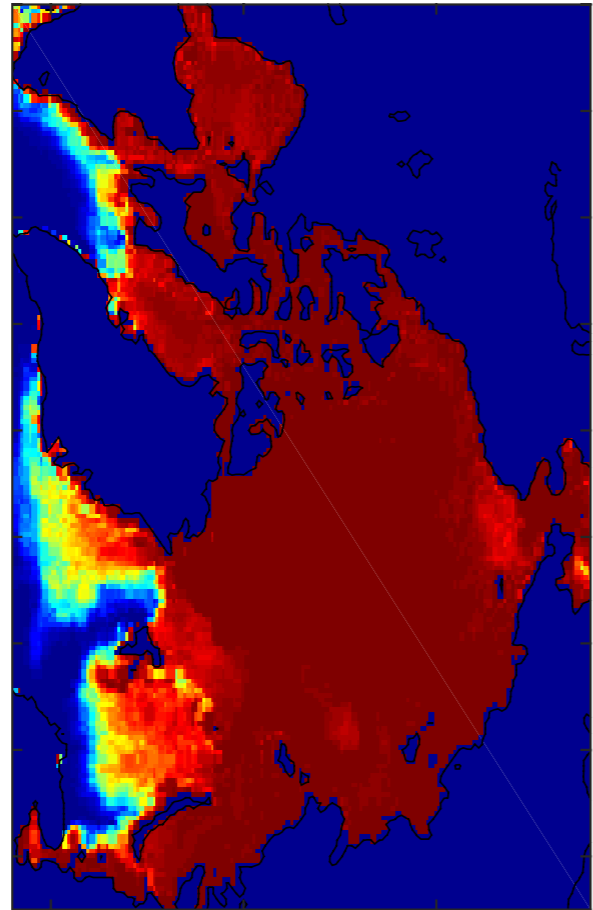
## Ice compactness on Mar-15-2001



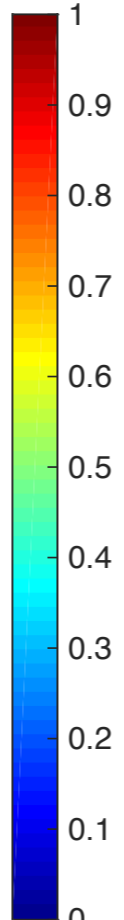
**Averaged ice compactness in Mar**



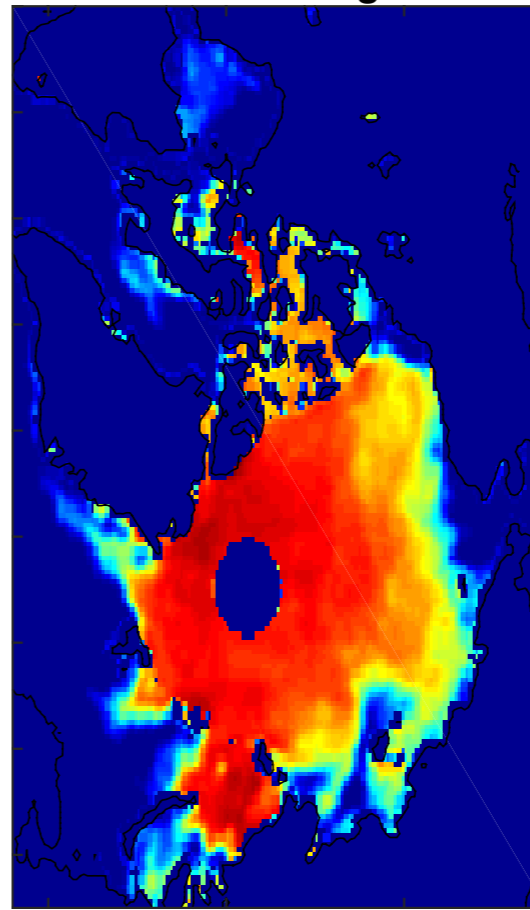
Observation



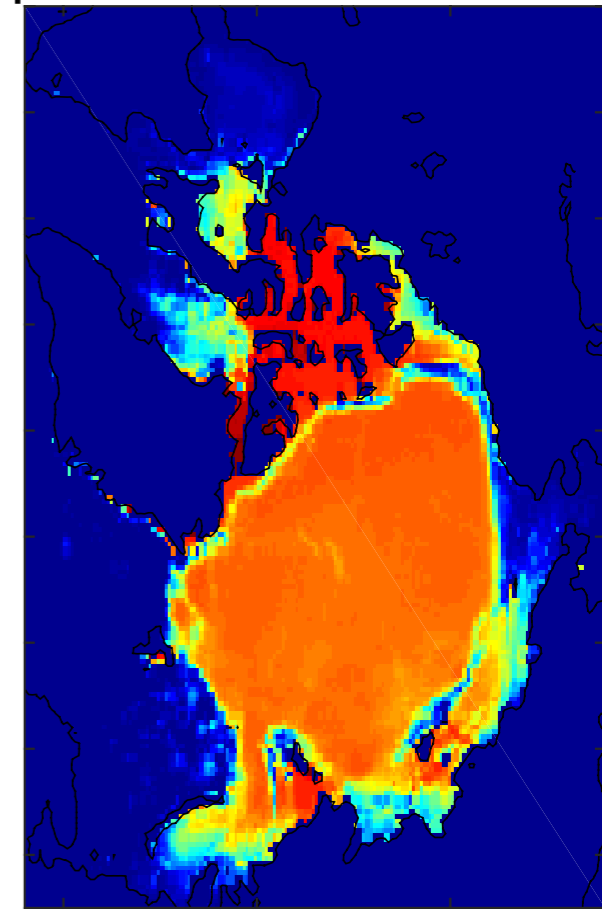
Forecast



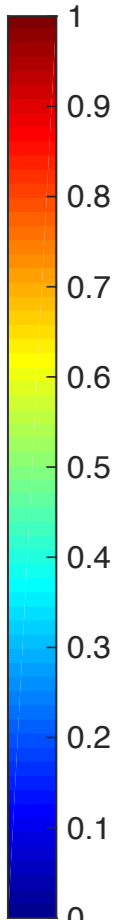
**Averaged ice compactness in Jul**



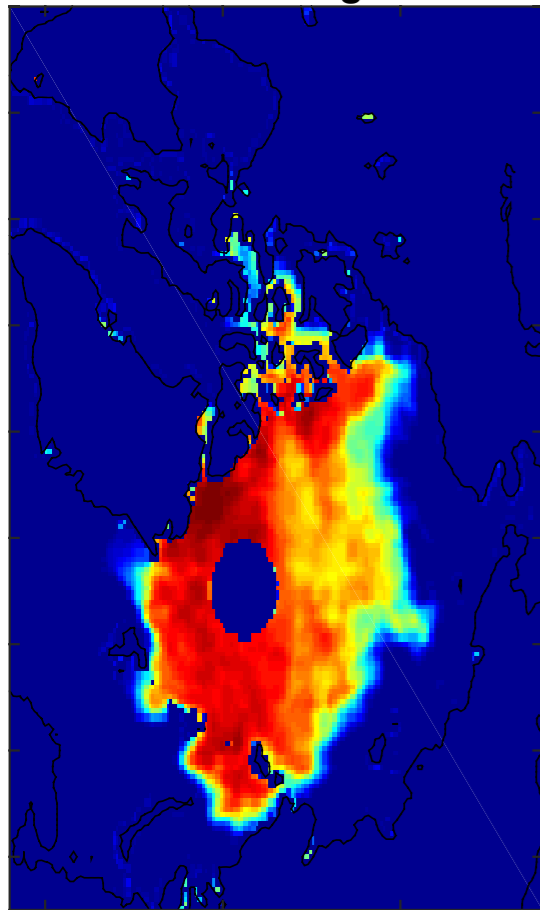
Observation



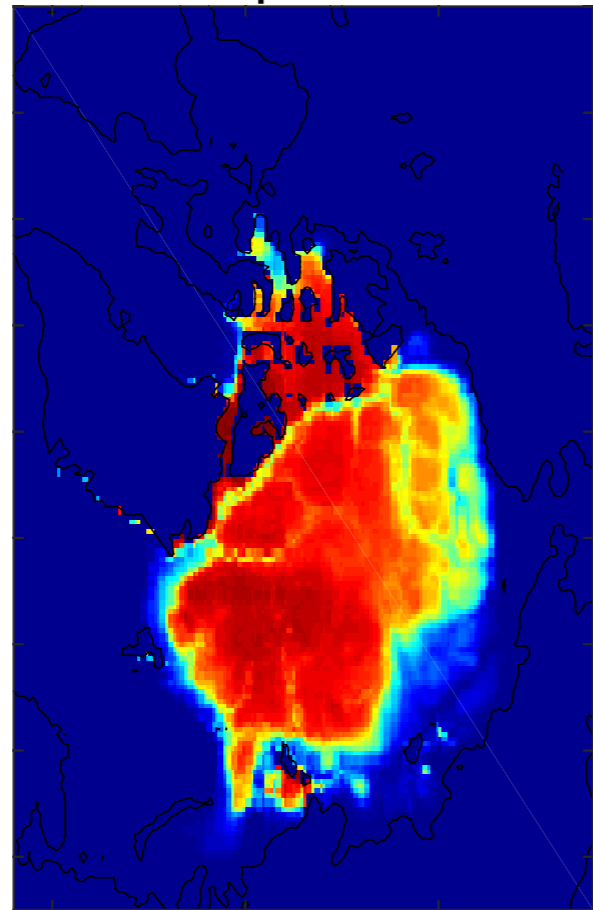
Forecast



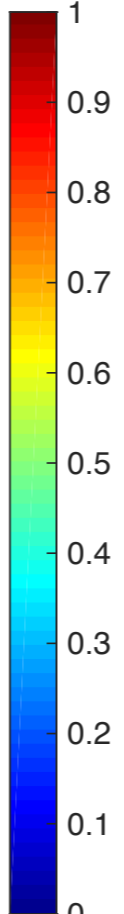
**Averaged ice compactness in Sep**



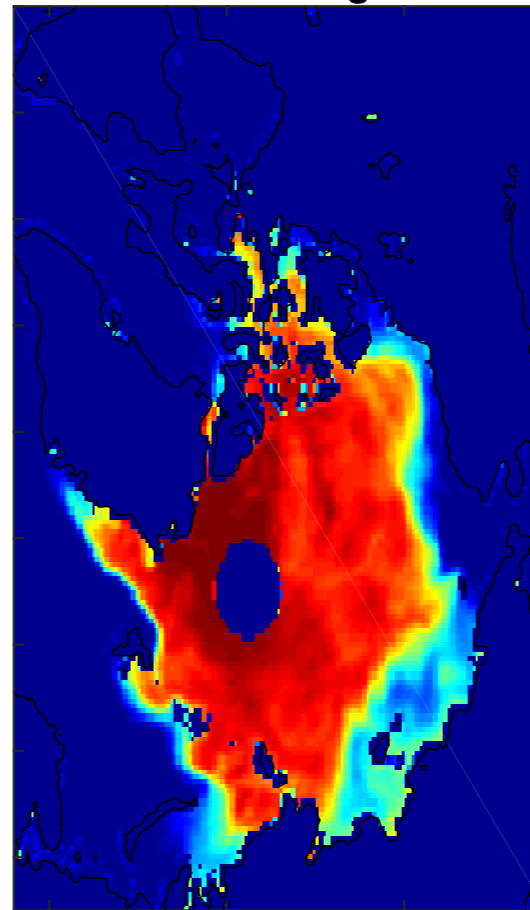
Observation



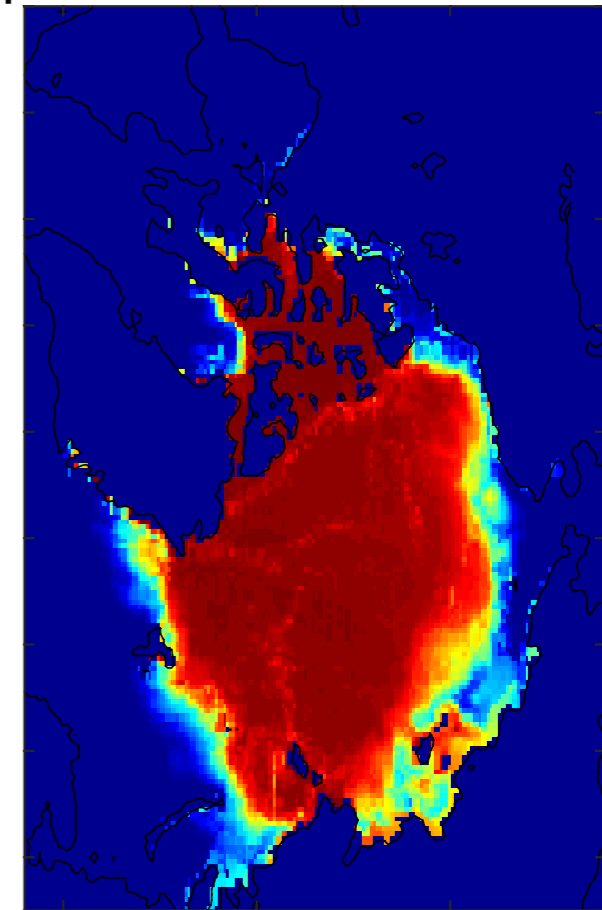
Forecast



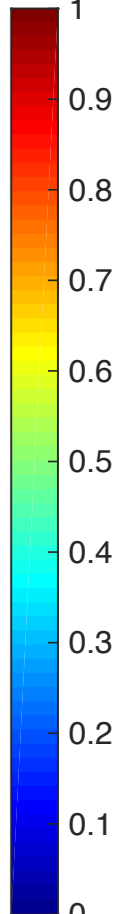
**Averaged ice compactness in Oct**



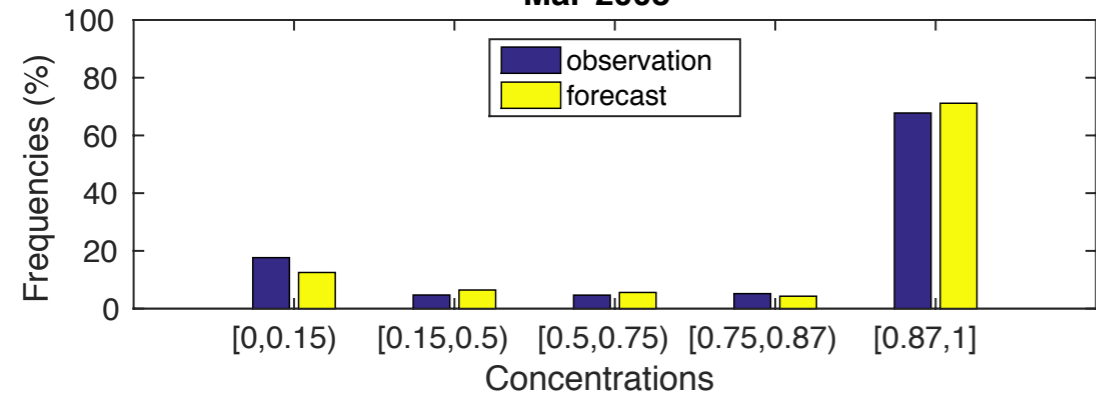
Observation



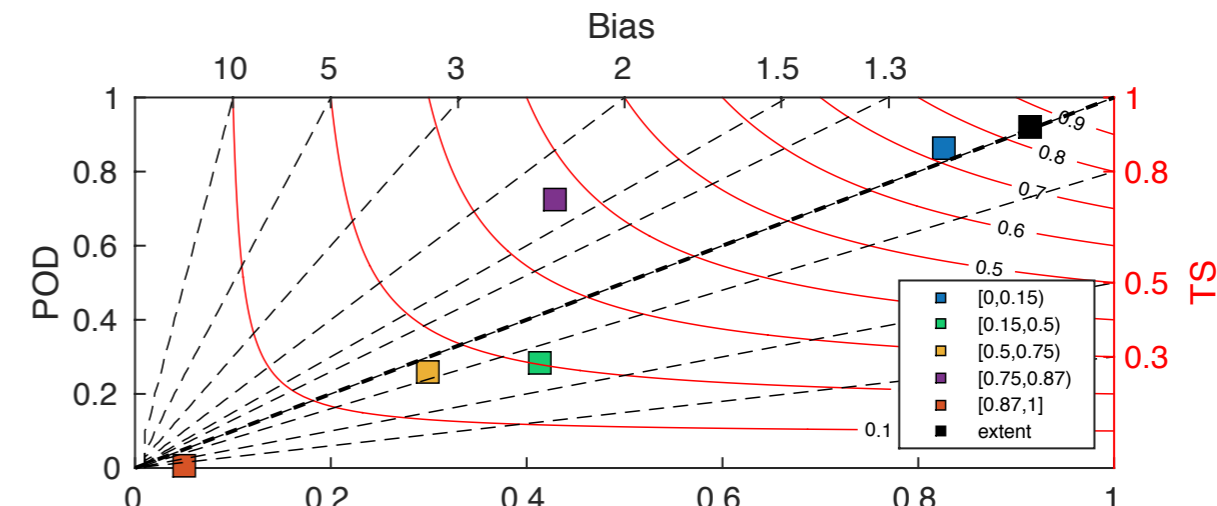
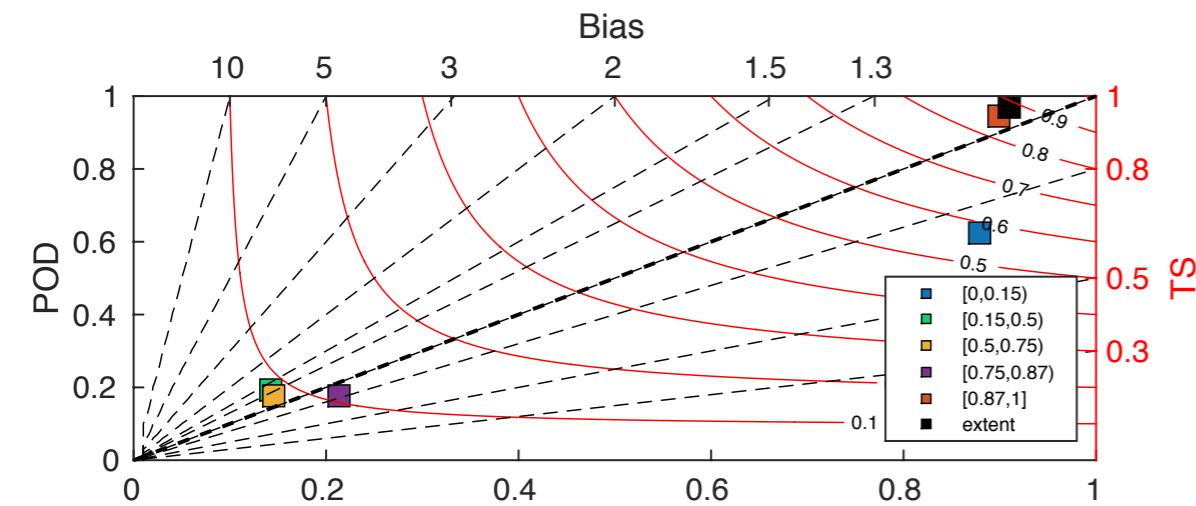
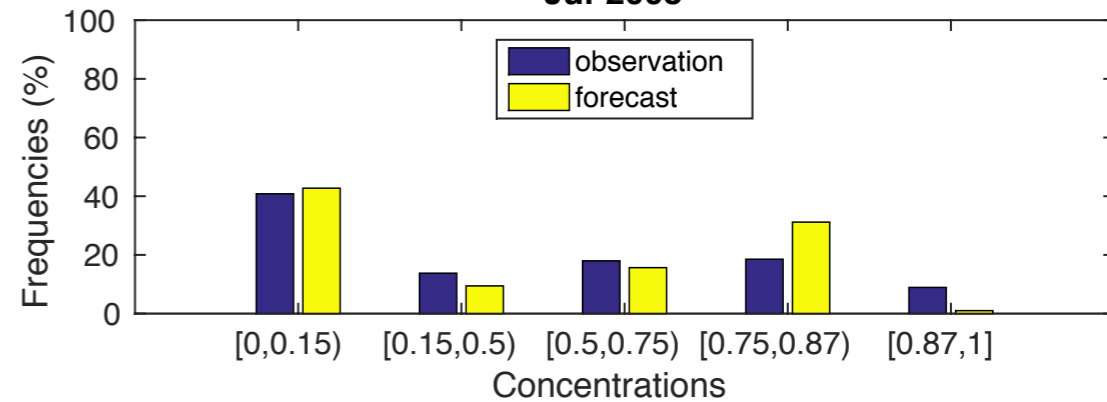
Forecast



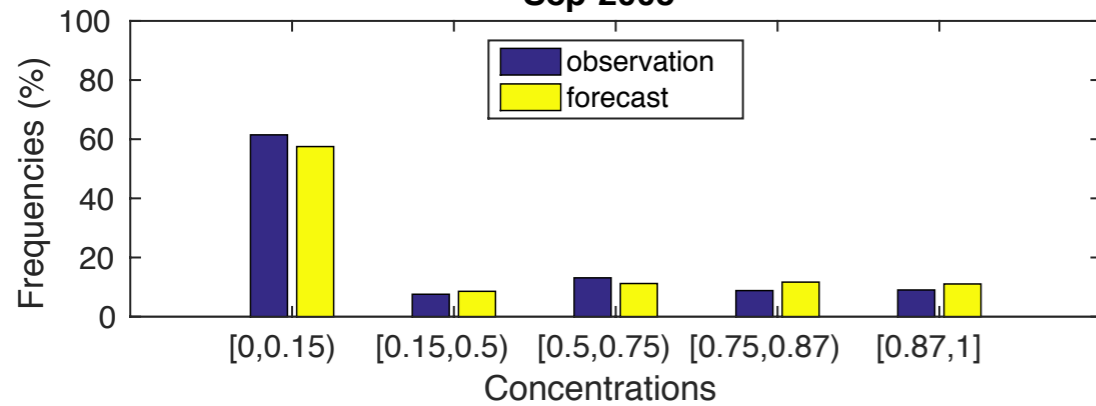
Mar-2003



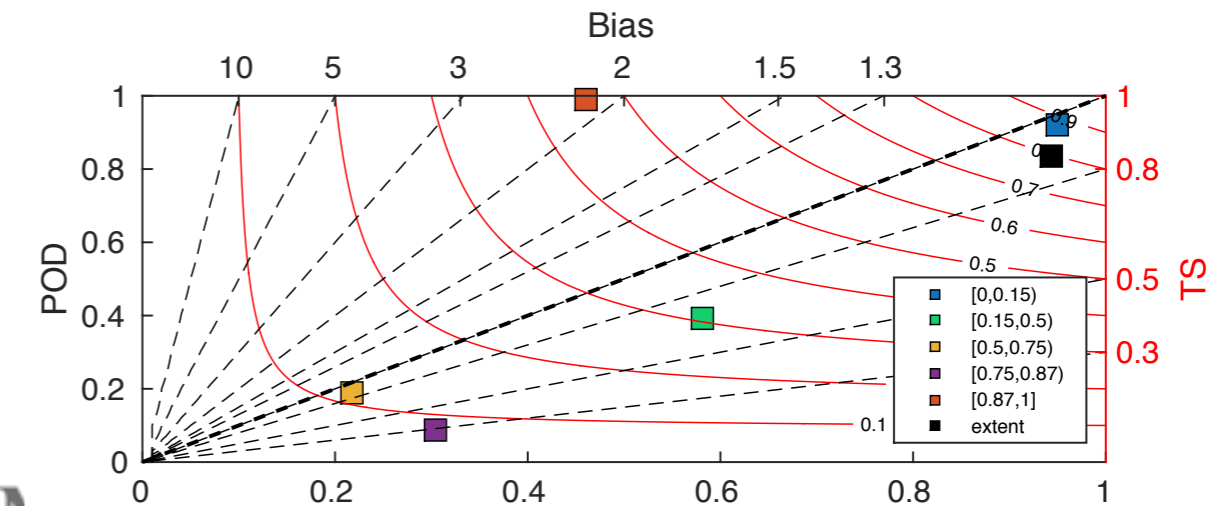
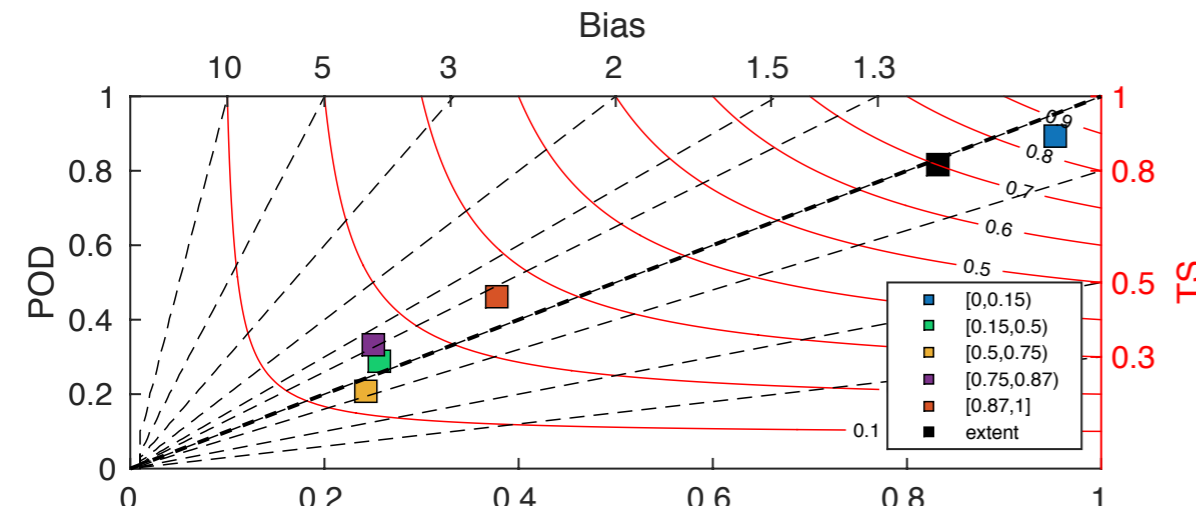
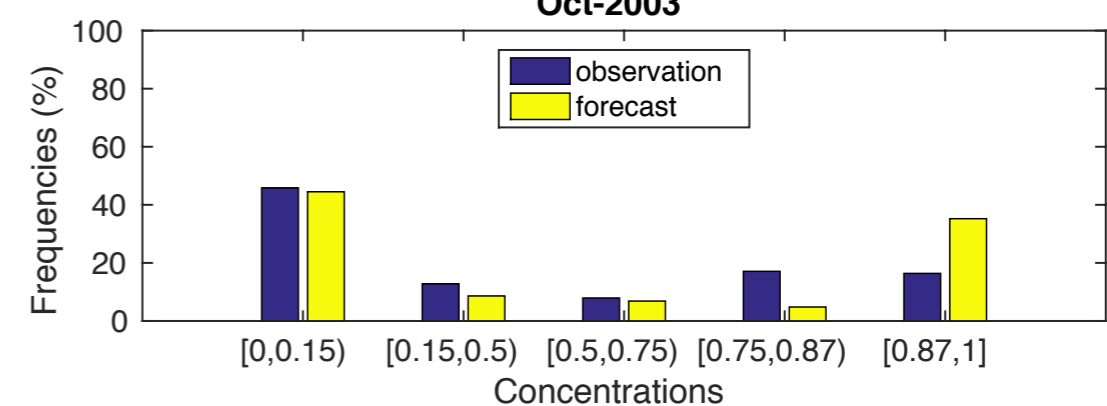
Jul-2003



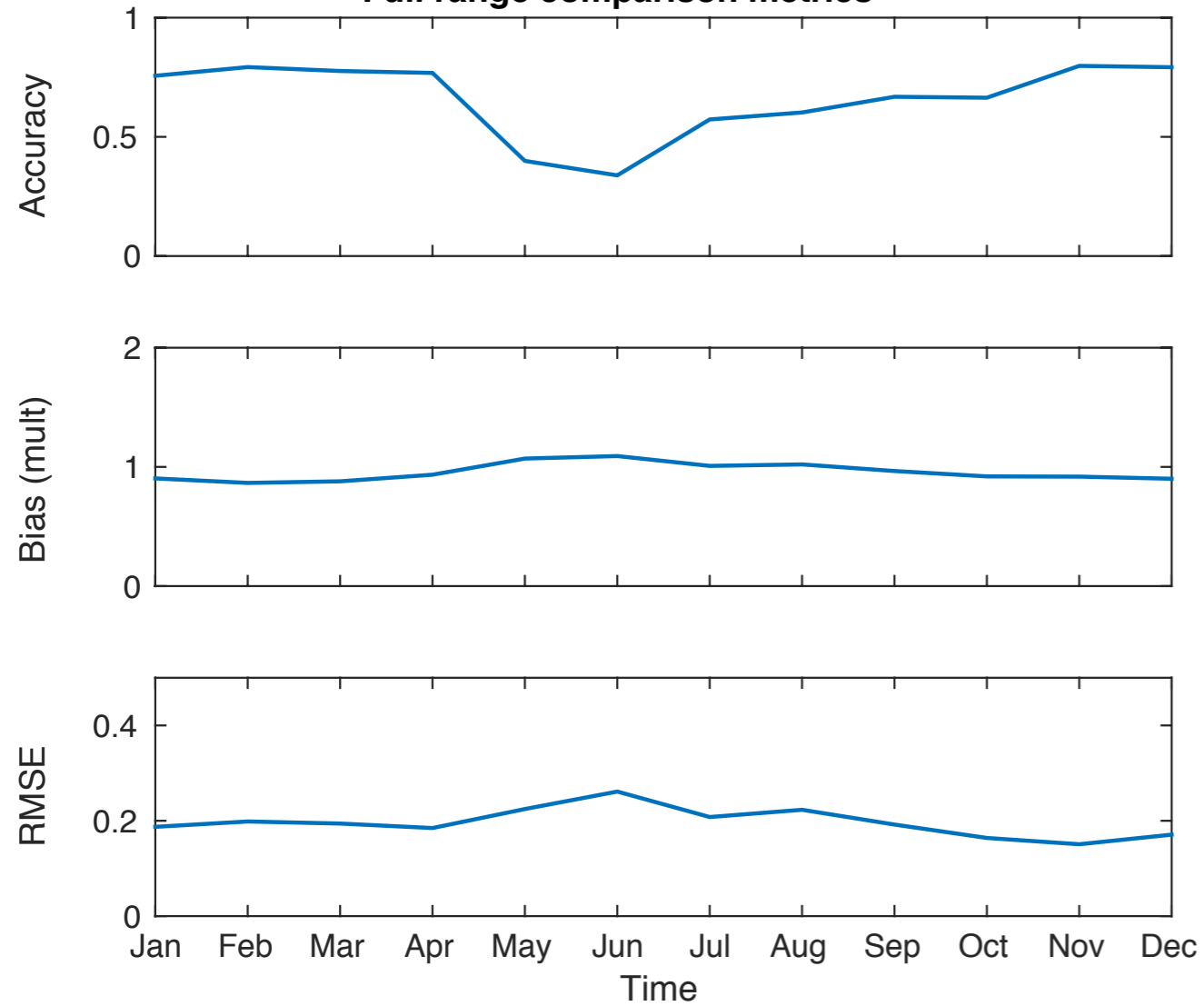
Sep-2003



Oct-2003



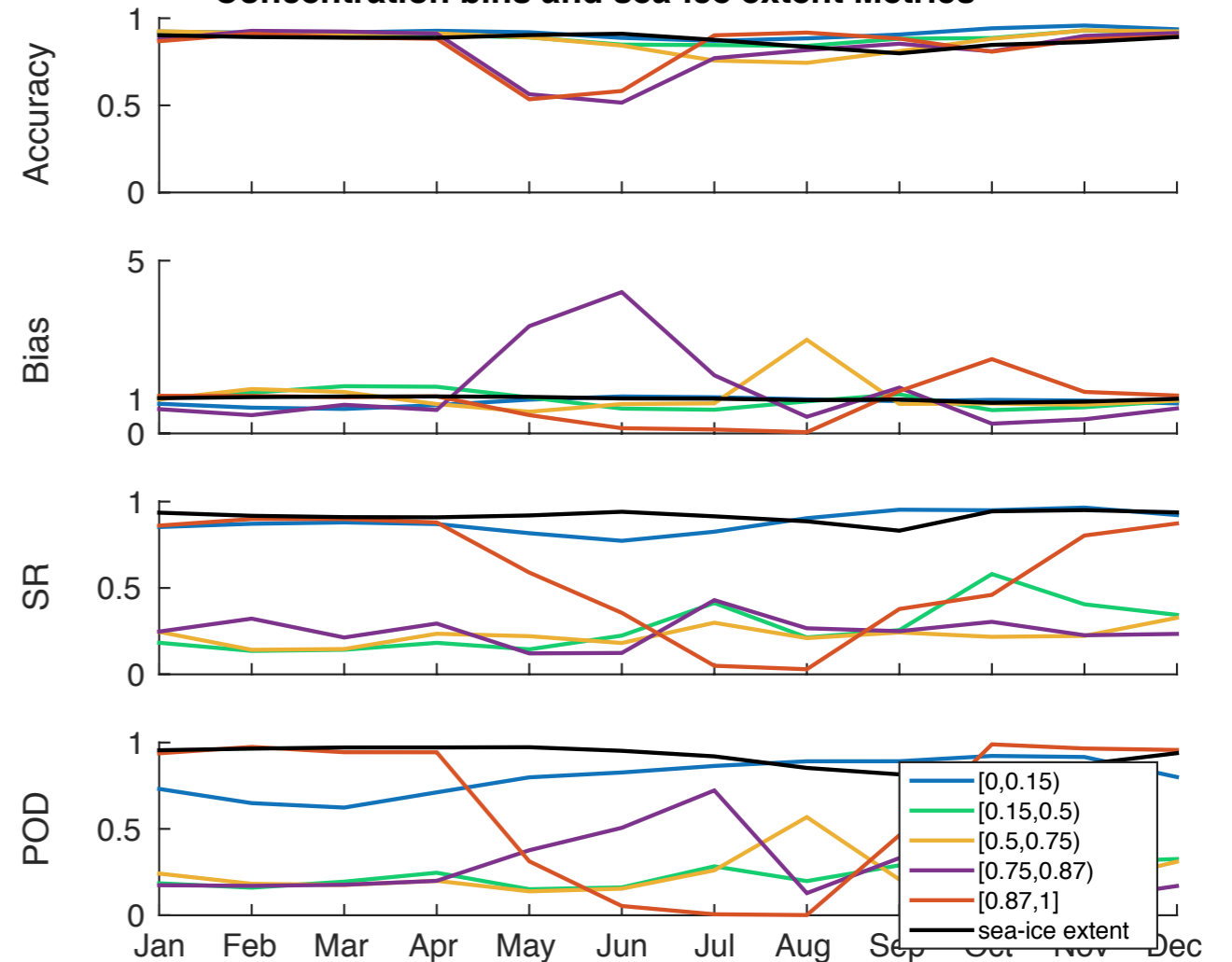
### Full range comparison metrics



- Late spring and summer months are the least accurate and have the highest RMSE
- Bias (mult) alone does not provide useful information

- Bad accuracy is driven by high concentration inaccuracy
- The bias shows the disparity of each bin (the mult Bias looks good because of compensation)
- SR and POD confirm the model's inaccuracy in late spring and summer months

### Concentration bins and sea-ice extent Metrics



# Conclusion from sea ice concentration comparison

- Sea-ice extent is well matched all year long
- Concentration is well matched year round besides in the summer during which forecast is weaker (larger error in observations as well though)
- Thermodynamics needs to be improved? (can't wait for the column physics package release)
- A similar analysis with different bin size (e.g. equal bin size) provides similar conclusions



# Conclusion

- We developed a sea-ice model capable of representing sea-ice fractures and lead openings.
- The model is running and simulates reasonable results.
- Performance and frequency diagrams provide quantitative insight into the validation of multi-category variables. It has the advantage to be easy to read, interpret and implement
- We created a git repository with the code performing the comparison. It is easy to adapt for any model and is available for sea-ice concentration and thickness
- Work in progress for sea ice displacement validation and higher resolution runs