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SCIENTIFIC DESCRIPTION OF THE SEA ICE COMPONENT IN THE COMMUNITY CLIMATE SYSTEM MODEL, VERSION THREE

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Preface

Based on the Climate System Model Version 1 (CSM1) simulations (Boville and Gent, 1998; Weatherly et al., 1998), the CSM Polar Climate Working Group recommended a number of improvements to the sea ice component (the sea ice component is referred to as the Community Sea Ice Model, or CSIM). The most recent versions of the Community Climate System Model (CCSM), Versions Two and Three, include improved versions of CSIM which satisfy all of those recommendations and include additional parameterizations, representing a major improvement over CSM1. The sea ice component of CCSM2, CSIM4, is described in Briegleb et al. (2002) on the CCSM web page, and the enhanced sea ice component of CCSM3, CSIM5, is presented here.

CSIM5 consists of: elastic-viscous-plastic dynamics (Hunke and Dukowicz, 1997), which includes the effects of metric terms (Hunke and Dukowicz, 2002), energy conserving thermodynamics with a resolved vertical temperature profile and an explicit brine pocket parameterization (Bitz and Lipscomb, 1999), Lagrangian ice thickness distribution (Thorndike et al., 1975; Bitz et al., 2001), linear remapping for thickness space evolution (Lipscomb, 2001), mechanical redistribution due to rafting and ridging (Hibler 1980), ice strength computed from energetics (Rothrock, 1975), lateral and bottom melt processes (McPhee, 1992), second order horizontal advection using remapping (Lipscomb and Hunke, 2004) and an albedo parameterization with implicit melt ponds. Five thickness categories adequately resolve the ice thickness distribution. Flux exchange with the atmosphere and ocean is evaluated over each thickness category and aggregated.

CSIM5 uses two-dimensional domain decomposition and time split thermodynamics and dynamics for efficient parallel performance, and is vectorized for efficient vector performance. The code is written using standard parallel and Fortran 90 constructs, and runs on several platforms.

This document presents the details of the CSIM5 physical justifications, fundamental equations, parameterizations, numerical approximations/algorithms and run-time output.

1. Introduction

The Community Climate System Model (CCSM) is a coupled climate model consisting of atmosphere, ocean, land, and sea ice components. The model includes a coupler which passes fluxes and state variables from one model component to another. The model has been designed to support experiments that contribute to the understanding of past, present and future climates.

Because of the great annual range of insolation at the top of the atmosphere in high latitudes, the ocean surface there loses heat to the atmosphere for months at a time, producing sub-freezing surface temperature and a sea ice cover. During the sunlit seasons, sea ice persists because of its relatively high surface albedo and its latent heat of formation. During the winter, sea ice interposes a layer of effective thermal insulation between the liquid ocean and the atmosphere. These effects keep the surface and lower atmosphere over high latitude oceans much colder than they would be in the absence of sea ice, thereby affecting the horizontal gradient of air temperature and the general circulation of the atmosphere. When external forcing perturbs the earth's heat budget, the sea ice cover may expand, contract, thicken or thin in such a way as to amplify the perturbation. Therefore it is important to include a realistic representation of sea ice processes in models that would produce accurate simulations of past, present and future climate.

The purpose of this document is to describe the sea ice component of the current version of the CCSM. This sea ice model is called the Community Sea Ice Model (CSIM), and its current version is CSIM5 which is being released as part of CCSM3. Emphasis is placed on relationships between the subcomponents of CSIM5, and on aspects of the model of interest to users who wish to perform simulations and diagnose the output. CSIM5 has evolved from earlier versions associated with the CCSM project. CSIM1, which was released as part of the Climate System Model version 1 (CSM1) resolved one category of sea ice thickness and included the following dependent variables: ice concentration, ice thickness, snow depth, surface temperature, ice temperature and ice velocity (Bettge, et al., 1996). The thermodynamic part of CSIM1 was based on Semtner (1976). The dynamics were modeled as a cavitating fluid (Flato and Hibler, 1992). Coupled experiments with CSM1 resulted in significant biases in sea ice thickness and other climate variables over the Arctic (Boville and Gent, 1998; Weatherly, et al., 1998).

The biases in CSIM1 resulted from the combined effects of the atmosphere, sea ice, ocean and land components of the model, and were not uniquely attributed to errors in CSIM1. Nevertheless, the biases called attention to simplifications and approximations in CSIM1 at a time when considerably more elaborate representations of ice dynamics and thermodynamics had been developed for regional ice-ocean models. Shortly after the release of CSIM1 the CCSM Polar Climate Working Group (PCWG) identified aspects of the sea ice model that should be given high priority in development efforts: (1) A plastic rheology with an elliptical yield curve; (2) Enhanced sea ice thermodynamics; (3) A multi-category ice thickness distribution; (4) Elimination of spurious ice convergence at the North Pole; (5) Compatibility of the grids used to model the sea ice and the ocean; (6) Efficient parallelization of the model code; (7) Development of an active ice-only version framework for testing the model without full coupling to the atmosphere and ocean model components. These seven high-priority objectives were achieved with the release in 2002 of CSIM4 as a component of CCSM2. A description of CSIM4 is available on the CCSM website (Briegleb, et al., 2002).

After the release of CSIM4 the PCWG decided to give high priority to developing an efficient vector version of the code, and to enhance some of the numerical algorithms in the model, resulting in CSIM5. In CSIM5 the second order horizontal advection scheme (Smolarkiewicz, 1984) has been replaced by a more accurate and numerically efficient remapping scheme (Lipscomb and Hunke, 2004). The dynamic boundary condition for marginal ice zones has been improved (Connolley et al. 2004). Salinity is now exchanged explicitly between the ocean and the sea ice model components. Finally, the albedo of snow and ice surfaces have been adjusted to accomodate changes in the simulation of the polar atmosphere by CCSM3.

This document is organized as follows. Section 2 provides an overview of CSIM5 by listing the state variables, fundamental equations, boundary conditions and a summary of the model physics. Section 3 presents the time and space discretizations of the fundamental equations. Section 4 is the longest section, providing detailed descriptions of the parameterizations and numerical approximations. Sections 5, 6, and 7 present the active ice-only framework, the output to history files, and a summary of CSIM5 respectively. In addition to the supported, default physics of the CSIM5 there are alternative physics options available, including developmental versions of the model. These options are summarized in the Appendix.

Additional information on how to obtain and run CSIM5 is available in the "CSIM User's Guide". For details on source code structure and its modification, see the "CSIM Code Reference". Both documents can be found on the CCSM web page under models (http://www.cesm.ucar.edu/models), as well as the present document.

2. Overview of the Community Sea Ice Model (CSIM)

2.1 State Variables

The **state variables** for the sea ice model are listed in Table 1. Where possible, we use conserved quantities as state variables.

Table 1. State Variables for the sea ice model. Subscript n, $\{n = 0, 1, 2, ..N\}$ refers to the n^{th} ice thickness category ($n = 0$ is open water), where N is the total number of categories. For CSIM5, $N = 5$. Subscript l, $\{l = 1, ..L\}$ refers to vertical level, with $L = 4$. For each category, ice thickness ($h_n = \Sigma_{l=1}^{L} V_{nl}/A_n$) lies within constant category thickness limits. Ice velocity u and the associated stress tensor σ (components i=1,2; j=1,2) are not resolved across the ice thickness distribution.

Symbol	Description
A_n	Sea ice area (fraction from 0 to 1)
V_{nl}	Sea ice volume (m^3 m^{-2})
E_{nl}	Sea ice internal energy (J m^{-2})
V_{sn}	Snow volume (m^3 m^{-2})
T_{sn}	Surface temperature of snow/ice (°C)
$\mathbf{u}$	Sea ice velocity (m s^{-1})
σ_{ij}	Stress tensor components (N m^{-1})

2.2 Fundamental Equations

The **fundamental equations** determine the spatial and temporal evolution of the state variables. We first give a rationale for using an ice thickness distribution before describing these equations.

Many properties of sea ice depend on ice thickness (Thorndike et al.,1975); for example, ice compressive strength, growth rate, surface temperature, turbulent and radiative flux exchange with the atmosphere. Two contrasting phenomena alter the distribution of ice thickness on a yearlong average: accretion and ablation against lead opening and ridging of ice. This competition was elegantly described by Thorndike et al. (1975): "thermodynamics seeks the mean and dynamics the extremes". The evolution of the thickness distribution is the historical integral of these two continuous processes.

Formally, the thickness distribution is described by the distribution function $g(h, x, t)$, where h is ice thickness (henceforth we suppress the explicit space and time dependence). $g(h)dh$ is the fraction of area covered by ice of thickness h to $h+dh$, normalized by $\int_0^\infty g(h)dh =$

1, the conservation of total area. The aggregate ice fraction is $A = \int_{0+}^{\infty} g(h)dh$, while the open water fraction is $A_0 = g(h = 0) = 1 - A$. The cumulative distribution function is $G(h) = \int_0^h g(h)dh$. The average of a quantity F that depends on ice thickness is referred to as the aggregate $\bar{F} = \frac{1}{A}\int_0^{\infty} F(h)g(h)dh$.

The evolution of g is governed by the distribution equation

$$\frac{\partial g}{\partial t} = -\frac{\partial}{\partial h}(\dot{h}g) + L(h,g) - \bigtriangledown \cdot (ug) + R(h,g,u) \tag{1}$$

where $\dot{h}$ is the rate of change in ice thickness due to vertical thermodynamic processes, $-\frac{\partial}{\partial h}(\dot{h}g)$ is the change in distribution due to thickness space transport, $L(h,g)$ is the change in distribution due to lateral melt/formation processes, $-\bigtriangledown\cdot(ug)$ is the change in distribution due to horizontal advection ($\bigtriangledown$ is the horizontal gradient operator and u is the velocity field over the thickness distribution) and $R(h,g,u)$ (ψ in Thorndike et al.,1975) is a redistribution function due to rafting and ridging processes.

To solve Eq. 1, a discrete set of N ice categories is assumed, delimited by the thicknesses $h_n^*, \{n = 0, 1, 2...N\}$ for which $h_0^* = 0$. Thus, Eq. 1 is integrated over the thickness limits for each category, resulting in a discrete set of N equations to be solved for the ice fraction in each category n:

$$A_n = \int_{h_{n-1}^*}^{h_n^*} g(h)dh, \tag{2}$$

where the total (aggregate) ice fraction $A = \sum_{n=1}^{N} A_n$. The first moment of the distribution function is the ice volume $V_n = \int_{h_{n-1}^*}^{h_n^*} hg(h)dh$, and any function F which is linear in the ice thickness (i.e. $F = F_0 + F_1 h$) results in $F_n = F_0 A_n + F_1 V_n$.

One way to solve Eq. 1 is to assume thickness is distributed uniformly within each category (Hibler, 1980; Flato and Hibler, 1995), resulting in Eulerian advection in thickness space due to sea ice growth and melt processes. Such advection is very diffusive unless a large number of categories are employed. A second way to solve Eq. 1 is to assume ice can vary in thickness within each category (Thorndike et al., 1975). This Lagrangian method is free from the diffusion of the Eulerian thickness advection, allowing for a smaller number of categories, as well as the resolution of the vertical temperature profile in snow and ice (Bitz et al., 2001). It is used for the present sea ice model, except for the linear remapping which uses a combination of both Eulerian and Langrangian methods.

Thus, the fundamental equations for the present sea ice model start with the discrete form of Eq. 1 for the ice fractions A_n, the first moment equations for the ice volume V_n, corresponding equations for the snow volume V_{sn}, equations for the vertically varying ice internal energy (from which the vertical temperature profile and heat transfer are evaluated), equations for the surface temperature required for the vertical heat transfer solution, and finally dynamic equations for the ice velocity u needed to evaluate horizontal advection and the ridging terms in the distribution equations.

The fundamental equations are as follows. For the category sea ice fraction and volume:

$$\frac{\partial A_n}{\partial t} = S_{TAn} - \bigtriangledown \cdot (uA_n) + S_{MAn} \qquad (n = 1, 2, ...5) \tag{3}$$

$$\frac{\partial V_n}{\partial t} = S_{TVn} - \bigtriangledown \cdot (uV_n) + S_{MVn} \qquad (n = 1, 2, ...5) \tag{4}$$

where terms S_T denote sources/sinks due to thermodynamic processes and thickness space transport, while terms S_M denote sources/sinks due to mechanical redistribution. The sea ice thickness h_n is derived from the fraction and volume as $h_n = V_n/A_n$.

To resolve vertical atmosphere/ocean heat exchange, and account as well for internal heat within the ice, the ice internal energy E_n (vertically varying) is governed by the conservation equation:

$$\frac{\partial E_n}{\partial t} = S_{TEn} - \bigtriangledown \cdot (uE_n) + S_{MEn} \qquad (n = 1, 2, ...5) \tag{5}$$

The internal sea ice energy E_n is proportional to the ice volume, $E_n = q_n V_n$, where the proportionality function q_n (termed the energy of melting, or enthalpy) is the internal energy per unit volume. The effects of brine pockets are represented explicitly through the temperature T_n and salinity S_n dependent energy of melting $q_n = q_n(T_n, S_n)$. The vertical temperature profile is inferred by solving for T_n in $q_n(T_n, S_n) = E_n/V_n$ over an ice thickness $h_n = V_n/A_n$, using a prescribed salinity profile S_n.

The conservation equation for snow volume V_{sn} is:

$$\frac{\partial V_{sn}}{\partial t} = S_{TVsn} - \bigtriangledown \cdot (uV_{sn}) + S_{MVsn} \qquad (n = 1, 2, ...5) \tag{6}$$

Snow thickness is derived from $h_{sn} = V_{sn}/A_n$. Snow energy per unit volume is $E_{sn} = q_s V_{sn}$, where the energy of melting of snow q_s is constant.

For each category, the heat equation governing vertical heat transfer over time interval t to t' corresponding to temperatures T_n and T'_n respectively, allowing for temperature and salinity dependent heat capacity c_i, thermal conduction and internal absorption of penetrating solar radiation, is given by:

$$\int_{T_n}^{T'_n} \rho_i c_i dT_n = \int_t^{t'} \left(\frac{\partial}{\partial z} k \frac{\partial T_n}{\partial z} + Q_{SW} \right) dt \qquad (n = 1, 2, ...5) \tag{7}$$

where sea ice is assumed to have a constant density ρ_i, z is the vertical coordinate within the sea ice, Q_{SW} is the absorbed shortwave flux, and the thermal conductivity k is that for either snow or ice. Modifications to the temperature profile resulting from heat transfer change the ice internal energy according to $E_n = q_n(T_n, S_n)V_n$.

The surface boundary conditions for the vertical heat transfer solution require surface temperature T_{sn} to satisfy the conservation equation

$$\frac{\partial AT_{sn}}{\partial t} = S_{TTsn} - \bigtriangledown \cdot (uA_nT_{sn}) + S_{MTsn} \qquad (n = 1, 2, ...5) \tag{8}$$

Evaluation of the thermodynamic source term S_{TTsn} at the surface requires calculation of surface snow and ice albedo, which are diagnostic functions of solar spectral interval, snow and ice thickness, and surface temperature.

For momentum conservation, sea ice is assumed to be a two-dimenstional continuum. The sea ice velocity u and stress tensor σ_{ij} are considered (along with related dynamic quantities) to be representative of the entire ice thickness distribution. Their governing equations are:

$$\bar{m}\frac{\partial u}{\partial t} = -\bar{m}fk \times u + \tau_a + \tau_o + \bar{m}g_e\nabla H_o + \nabla \cdot \sigma \tag{9}$$

where $\bar{m} = \rho_s\sum_{n=1}^{N} V_{sn} + \rho_i\sum_{n=1}^{N} V_n$, the non-linear u advection terms are ignored as they are negligibly small when the equations are scaled, f is the Coriolis parameter, k is the local vertical unit vector, τ_a and τ_o are air and water stresses respectively, g_e is the gravitational acceleration, H_o is the sea surface height and $\nabla \cdot \sigma$ is the force per unit area due to internal ice stress, where σ is the stress tensor. The stress tensor equations are:

$$\frac{\partial \sigma_{ij}}{\partial t} + \frac{e^2}{2T_{ew}}\sigma_{ij} + \frac{1-e^2}{4T_{ew}}\sigma_{kk}\delta_{ij} = \frac{P}{2T_{ew}\Delta'}\dot{\epsilon}_{ij} - \frac{P}{4T_{ew}}\delta_{ij} \quad (i,j = 1,2) \tag{10}$$

where $(i,j = 1,2)$ refer to the four components of the stress tensor, e is a constant ratio of major to minor axes of the elliptical yield curve, T_{ew} is a damping time scale for elastic waves, δ_{ij} is the Kronecker delta, P is the ice compressive strength (or mechanical pressure, a function of the thickness distribution), $\dot{\epsilon}_{ij}$ is the rate of strain tensor, in turn a function of velocity gradients, and Δ' is a function of the rate of strain tensor.

2.3 Boundary Conditions and Solution

Boundary conditions are represented vertically by atmospheric/oceanic forcing from the coupler, consisting of states and interfacial fluxes summarized in Table 2, and horizontally by no-slip $u \rightarrow 0$ along coastlines and $u \rightarrow u_o$ (ocean surface current) on the open ocean edge. The fundamental equations are solved subject to these boundary conditions, and selected states along with atmosphere and ocean fluxes are returned to the coupler as listed in Table 3.

The atmospheric states are used, along with ice surface temperature and roughness, to compute the bulk sensible/latent heat fluxes F_{SH}, F_{LH} and surface stress components τ_{ax}, τ_{ay}. The downwelling shortwave flux is partitioned directionally and spectrally into four components such that the total downwelling shortwave flux is $F_{SWDN} = F_{SWvsdr} + F_{SWvsdf} + F_{SWnidr} + F_{SWnidf}$, while the total downwelling longwave flux is F_{LWDN}.

Table 2. State Variables and Fluxes Received by Sea Ice Model from the Coupler

Symbol	Description	Units
	Atmospheric States	
z_a	reference height	m
u_a	x direction wind speed at z_a	m s^{-1}
v_a	y direction wind speed at z_a	m s^{-1}
θ_a	potential temperature at z_a	K
T_a	air temperature at z_a	K
q_a	specific humidity at z_a	kg kg^{-1}
ρ_a	air density at z_a	kg m^{-3}
	Atmosphere $\Rightarrow$ Sea Ice Fluxes (+ downwards)	
F_{SWvsdr}	direct, visible downwelling shortwave	W m^{-2}
F_{SWvsdf}	diffuse, visible downwelling shortwave	W m^{-2}
F_{SWnidr}	direct, near infrared downwelling shortwave	W m^{-2}
F_{SWnidf}	diffuse, near infrared downwelling shortwave	W m^{-2}
F_{LWDN}	downwelling longwave	W m^{-2}
F_{RN}	water flux due to rain	kg m^{-2} s^{-1}
F_{SNW}	water flux due to snow (liquid equivalent)	kg m^{-2} s^{-1}
	Ocean States	
T_{ocn}	sea surface temperature	K
S_{ocn}	sea surface salinity	ppt
u_o	x direction ocean surface current	m s^{-1}
v_o	y direction ocean surface current	m s^{-1}
$(\nabla H_o)_x$	x direction sea surface slope	m m^{-1}
$(\nabla H_o)_y$	y direction sea surface slope	m m^{-1}
	Ocean $\Rightarrow$ Sea Ice Fluxes (+ downwards)	
F_{Qoi}	freezing/melting potential	W m^{-2}

Based on the ice state, snow/ice directional and spectral albedos $\alpha_{vsdr}, \alpha_{vsdf}, \alpha_{nidr}, \alpha_{nidf}$ are evaluated, and used to compute the total absorbed shortwave in the ice F_{SW}. Of this total a portion I_{SW} penetrates below the surface and is either internally absorbed in the ice Q_{SW} or penetrates to the ocean below as F_{SWo}. The net longwave flux at the surface F_{LW} is the difference between the upwelling longwave F_{LWUP} and the absorbed downwelling longwave. The upwelling longwave is given by the the surface emission $\varepsilon\sigma_{sb}T_s^4$ (where ε is the snow/ice longwave emissivity and σ_{sb} is the Stefan-Boltzmann constant) and the reflection of downwelling longwave $(1-\varepsilon)F_{LWDN}$, while the absorbed downwelling longwave is εF_{LWDN}.

Rain F_{RN} is assumed to run off directly into the ocean, and thus contributes to the ocean-ice water flux F_{Wo}. Snow F_{SNW} is used to compute snow accumulation $dh_s/dt = F_{SNW}/\rho_s$. The latent heat flux F_{LH} is associated with an evaporative water flux to the atmosphere F_{EVAP}, and an associated salt flux $S_i\rho dh/dt$ with the ocean, where h is ice thickness and S_i is a constant sea ice reference salinity, which contributes to the ocean-ice salt flux F_{So}. This salt exchange with the ocean for sublimation/condensation is required to maintain a constant sea ice reference salinity.

The top boundary condition at surface temperature T_s is $F_{TOP}(T_s) = F_{SW} - I_{SW} + F_{LW} + F_{SH} + F_{LH} + kdT/dz$, while the lowest ice interface in contact with the ocean is at ocean freezing temperature. With these boundary conditions and internal shortwave heating, the heat equation can be solved. If $F_{TOP}(T_{melt}) > 0$, where T_{melt} is the snow/ice melting temperature, then snow/ice melt is computed by $F_{TOP}(T_s = T_{melt}) = qdh/dt$ as appropriate for either snow (if present) or ice (h is the thickness of snow or ice). Snow and ice melt is assumed to run off directly into the ocean, contributing to the ocean-ice water flux F_{Wo} by the amount $\rho dh/dt$ where ρh, are the density,thickness of snow or ice, respectively. If sea ice melts, then an additional salt flux of $S_i\rho dh/dt$ is exchanged with the ocean.

Ice formation occurs by three processes. Although these processes are distinguished in formation, no distinction is made between ice types. If the freezing/melting potential (F_{Qio} in Table 2.) is such that heat is required by the ocean to maintain the freezing temperature ($F_{Qoi} > 0$), then **frazil ice** formation occurs, at a rate $dV_f/dt = F_{Qoi}/\rho_i q_f$, where q_f is a heat of melting assuming the ice forms at $T = 0°C$ and $S = 0$. Frazil ice formation has an implied salt flux to the ice, sufficient to establish S_i as the mean salinity of the newly formed sea ice. If the freezing/melting potential indicates ocean heat is available to melt ice $F_{Qoi} < 0$, this heat is partitioned between lateral F_{SID} and bottom F_{BOT} heat fluxes according to the fraction of absorbed solar energy near the surface and in deeper water. The sea surface temperature T_{ocn} is used to compute heat fluxes F_{SID} and F_{BOT} (sea surface salinity S_{ocn} is not used). The heat flux for lateral and bottom melting is F_{Qio}, with associated water and salt fluxes $\rho_i dh/dt$ and $S_i\rho dh/dt$ to the ocean, respectively.

The bottom boundary condition is $F_{BOT} - kdT/dz = qdh/dt$. If bottom ice formation occurs (i.e. $dh/dt > 0$), this ice is termed **congelation ice**. If sufficient snow h_s overlies ice, the snow-ice interface can be depressed below sea level. Snow below sea level is converted into ice conserving mass and energy, and is termed **snow-ice**. For both of these processes, a salt flux of $S_i\rho dh/dt$ is exchanged with the ocean.

The ocean surface currents (u_o, v_o) and ice velocity (u, v) are used to compute ocean/ice stress (τ_{ox}, τ_{oy}). The tilt stress is computed from the gradient of the sea surface height

$((\nabla H_o)_x, (\nabla H_o)_y)$. With these stresses, sea ice velocity and ridging are computed.

Table 3. State Variables and Fluxes Sent from Sea Ice Model to the Coupler

Symbol	Description	Units
	Sea Ice States	
A	ice area	fraction (0 to 1)
T_s	surface temperature	K
α_{vsdr}	albedo (visible, direct)	fraction (0 to 1)
α_{vsdf}	albedo (visible, diffuse)	fraction (0 to 1)
α_{nidr}	albedo (near infrared, direct)	fraction (0 to 1)
α_{nidf}	albedo (near infrared, diffuse)	fraction (0 to 1)
	Sea Ice $\Rightarrow$ Atmosphere Fluxes (+ downwards)	
F_{LH}	latent heat flux	W m^{-2}
F_{SH}	sensible heat flux	W m^{-2}
F_{LWUP}	upwelling longwave	W m^{-2}
F_{EVAP}	evaporated water	kg m^{-2} s^{-1}
τ_{ax}	x direction atmosphere-ice stress	N m^{-2}
τ_{ay}	y direction atmosphere-ice stress	N m^{-2}
	Sea Ice $\Rightarrow$ Ocean Fluxes (+ downwards)	
F_{SWo}	shortwave transmitted to ocean	W m^{-2}
F_{Qio}	heat flux to ocean	W m^{-2}
F_{Wo}	water flux	kg m^{-2} s^{-1}
F_{So}	salt flux	kg m^{-2} s^{-1}
τ_{ox}	x direction ice-ocean stress	N m^{-2}
τ_{oy}	y direction ice-ocean stress	N m^{-2}
	Diagnostic Fields	
T_{ref}	reference temperature (2 m)	K
Q_{ref}	reference specific humidity (2 m)	kg/kg
F_{SW}	ice/ocean absorbed shortwave flux	W m^{-2}

2.4 Summary

Subject to initial and boundary conditions, Eqs. 3-10 constitute the fundamental equations for the sea ice model. The next two sections present the discretizations, parameterizations and numerical approximations used to solve the fundamental equations.

3. Discretization

3.1 Time

The time stepping loop in the sea ice model is split into two intervals for improved CCSM computational performance. The physical time step, Δt, represents the time interval from the beginning to the end of the time step, whereas the coupling time step is displaced from this and denotes the time interval at which information is sent to the coupler. The order of calculations in the sea ice model differs somewhat from the order presented in Section 4. For more information on the time stepping, see the CSIM Code Reference.

During the first half of the physical time step, forcing fields in Table 2 are received from the coupler and the vertical thermodynamics are calculated. At this point, the ice surface state and ice-atmosphere fluxes in Table 3 are returned to the coupler. When the atmospheric model receives the ice-atmosphere fluxes, it can run in parallel with the ice model during the second half of the physical time step.

During the second half of the physical time step, lateral thermodynamics, thickness space transport, dynamics and physical space transport, mechanical and thermodynamic redistribution and albedo calculations are done. Note that since the lateral thermodynamics (including exchange with underlying ocean) and the albedo calculation follow the send to the coupler, the ice-ocean fluxes and snow/ice albedos are offset by one time step with respect to the ice states and ice-atmosphere fluxes. The albedos are computed last to ensure that the atmospheric computation of the atmosphere-ice radiative fluxes received on the next time step use the **same** albedos as are used for the ice vertical thermodynamic calculation, thus ensuring energy conservation.

3.2 Thickness

The thickness distribution function $g(h)$ is integrated over N discrete thickness ranges or categories. Presently there are $N = 5$ ice thickness categories in the standard model, but arbitrary N is allowed. These categories are described in the next section. The state variables (listed in Table 1) of sea ice concentration, volume, energy, snow volume and surface temperature are discretized into $n = 1, 2, ...N$ categories. Henceforth, a subscript n will refer to the n^{th} thickness category.

3.3 Vertical

To compute vertical heat conduction through ice, ice thickness is divided into four vertical layers. This requires sea ice internal energy E to vary in the vertical over four evenly spaced layers in each thickness category. Temperatures are computed from E using the energy of melting and the ice volume in each layer. Internal temperatures are centered within each layer, while conductivities and energy fluxes are represented at layer interfaces. Temperature boundary conditions at the surface and base of ice are taken at the top and bottom interfaces respectively.

3.4 Horizontal

The horizontal grids used for the sea ice model are displaced pole grids, in which the South Pole is typically located at the geographic South Pole, but the North Pole may be located in any northern hemisphere land mass. The CCSM uses a grid with the North Pole in central Greenland. The grids are orthogonal curvilinear, so that vectors parallel to increasing longitude and latitude coordinates are perpendicular to one another. Two available resolutions are the standard gx1 (320 longitudes x 384 latitudes, $\sim 1.1^\circ \times 0.94^\circ$), and a coarse gx3 ($100$ longitudes x 116 latitudes, $\sim 3.6^\circ \times 1.6^\circ$). The grids south of the equator are regular latitude/longitude spherical coordinates.

Spatial discretization is that of a B-grid. All state variables except ice velocity and stress tensor components are taken at grid box mid-points (a tracer grid termed the T-grid), while velocities are defined at grid box corners (a velocity grid termed the U-grid). The stress tensor, rates of strain and viscosities are defined bilinearly across each grid cell using the values at the corners. This discretization tends to avoid decoupling problems associated with the B-grid.

Grid information is taken from a grid data file read in by the ice model at initialization. The fields on the U-grid are latitude, longitude and angle the grid makes with a geographic latitude line (ULAT, ULON and ANGLE respectively), and fields on the T-grid are the land/ocean mask and T cell widths on north and east sides (tmask, HTN and HTE respectively). Land points on the T-grid are designated by tmask, which is either 0 (land) or 1 (ocean). On the U-grid umask designates ocean points where ice velocities are possible, and is 0 if any of the four surrounding tmask points are 0 (i.e. land), and 1 otherwise. In other words, umask is zero for all coastal and land points. The T-grid longitudinal and latitudinal widths through cell centers are dxt and dyt respectively, while dxu and dyu are the analogous widths for the U-grid. ANGLET (χ^t) is ANGLE (χ^u) interpolated to the

T-grid. Specifically:

$$
\begin{aligned}
dxt_{ij} &= \frac{1}{2}(HTN_{ij} + HTN_{ij-1}) \\
dyt_{ij} &= \frac{1}{2}(HTE_{ij} + HTE_{i-1j}) \\
dxu_{ij} &= \frac{1}{2}(HTN_{ij} + HTN_{i+1j}) \\
dyu_{ij} &= \frac{1}{2}(HTE_{ij} + HTE_{ij+1}) \\
A^t_{ij} &= dxt_{ij}\ dyt_{ij} \\
A^u_{ij} &= \frac{1}{4}(A^t_{ij} + A^t_{i+1j} + A^t_{ij+1} + A^t_{i+1j+1}) \\
\chi^t_{ij} &= \frac{1}{4}(\chi^u_{ij} + \chi^u_{i-1j} + \chi^u_{ij-1} + \chi^u_{i-1j-1})
\end{aligned}
\tag{13}
$$

where the ij are the grid longitude and latitude indices respectively, A^t_{ij},A^u_{ij} are the grid box areas on the T-grid and U-grid respectively, and for the χ^t calculation, χ^u values are adjusted if any differ by more than $180°$.

3.5 Domain Decomposition

The horizontal computational grid is domain decomposed in two dimensions for parallelization. The global domain of dimensions $imt_{global} \times jmt_{global}$ (for example, the gx1 grid has $imt_{global} = 320$ longitude points and $jmt_{global} = 384$ latitude points) is divided into integral NX longitude by NY latitude subdomains of dimensions $(imt_{local} = imt_{global}/NX + 2n_{ghost} + 1) \times (jmt_{local} = jmt_{global}/NY + 2n_{ghost} + 1)$, where imt_{global}/NX, jmt_{global}/NY must be integers. Each subdomain has a physical portion indexed as $[ilo : ihi, jlo : jhi]$ with n_{ghost} boundary cells outside. Periodic boundary conditions are applied, with boundary routines performing communications between subdomains when running parallel. Global scatter and gather routines distribute information from the global domain to the subdomains and back.

We note that since the thermodynamic calculations involve one grid point at a time, a purely thermodynamic model integration is independent of the domain decompostion (i.e. the exact values of NX,NY), while the dynamic calculation depends upon domain boundary conditions. Hence an integration with active dynamics is dependent upon the exact values of NX and NY.

4. Parameterizations and Numerical Approximations

Sections 2 and 3 introduced the sea ice model state variables, fundamental equations, boundary conditions, solutions and discretizations. In the present section we elaborate both on the parameterizations necessary to represent various forcing terms in the fundamental equations and the details of the numerical solutions. For many of the subsections to follow, the processes are described (sometimes implicitly) for a particular thickness category n. For exchange with the atmosphere and the ocean however, only aggregate quantities (i.e. those summed over the thickness distribution) are used.

4.1 Thickness Distribution

The number of ice thickness categories N (see Table 1) used in the ice model results from a trade off between the desire to resolve thin ice that is important in ocean-atmosphere heat exchange and feedback processes against computational cost. Bitz et al. (2001) showed that five thickness categories with adequate thin ice resolution are sufficient to represent the first order effects of an ITD. We therefore chose $N = 5$, with thickness boundaries given in Table 4, based on the category limit formula of Lipscomb (2001). While $N = 5$ is a convenient value for climate modeling, it is not hardwired in the code; other values can be used, including $N = 1$. Also, while the category 1 lower limit is 0, for thermodyanmic stability the minimum ice thickness in category 1 is h_{min}. If ice thickness in this category falls below h_{min}, it is reshaped using $h_{min}A_1' = h_1 A_1$, where A_1' is the adjusted category 1 ice fraction.

Table 4. Ice Thickness Distribution (N=5)

n	Range (m)
0	0
1	0^+ - 0.65
2	0.65 - 1.39
3	1.39 - 2.47
4	2.47 - 4.60
5	> 4.60

In the Lagrangian method of ice thickness distribution used here, there is no process (thermodynamic or dynamic) that absolutely prevents ice from outgrowing its thickness range for a given category. While it is true that the incremental linear remapping used to evaluate thickness space transport in most cases prevents ice from outgrowing its category thickness limits, an occasional adjustment is still necessary, which is termed "thermodynamic redistribution". This process contributes to the thermodynamic source terms S_T in the conservation equations (see Section 1).

Any category of ice which outgrows its upper thickness limit is combined with the next thickness category. The combination is done preserving ice area, volume, energy and snow volume. Any category of ice which melts below its lower thickness limit, will be combined with the next lower category in a manner similar to outgrowth just described, except for the thinnest ice category. For ice in category 1 that melts below its lower limit, the ice is reshaped so its thickness equals the minimum, with its concentration adjusted to conserve ice volume.

Small amounts of either open water or ice area can be created due to numerical diffusion associated with horizontal advection. To reduce the possibility of roundoff error corrupting the ice state owing to very small amounts of sea ice, any ice category whose area is less than an adjustable minimum (typically 5×10^{-6}) is added to the nearest ice filled category, if one is available. Small amounts of open water (typically less than 1×10^{-6}) are eliminated by increasing ice concentration equivalently in the thinnest ice category. Any remaining small ice areas in a grid box are set to zero in such a way that ice and snow volume are conserved across the entire hemisphere's ice pack (i.e. renormalization factors are applied across the entire hemispheric ice pack to compensate exactly for setting small ice areas and volumes to zero).

4.2 Thermal Properties

The ice area (A_n) and volume (V_n) were introduced in the Section 2. The ice internal energy (E_n) is proportional to the ice volume:

$$E_n = q_n V_n \tag{14}$$

where the proportionality function q_n is termed the energy of melting, or enthalpy. q_n is the internal energy of the ice per unit volume. It is derived from the basic thermodynamic relation between the applied heat Q for the given heat capacity of sea ice c_i and the resulting temperature change from T to T':

$$Q = \int_T^{T'} \rho_i c_i dT \tag{15}$$

where the ice density ρ_i is a constant (see List of Physical Constants). Treating ice density as a constant is a limitation of the model. During the melt season, a layer of deteriorated ice 5-10 cm thick is often observed at the top surface, with a density of 500 kg m^{-3} or less. Beneath this deteriorated ice, multiyear ice contains air-filled pores that can reduce its density to 700-800 kg m^{-3} in the upper 30-50 cm (Bitz, 2000). A constant ice density implies that all drained brine pockets are filled with melt water and not air.

The storage of latent heat in brine pockets is accounted for explicitly by using the heat capacity of Bitz and Lipscomb (1999), originally from Ono (1967):

$$c_i(T,S) = c_0 + \frac{L_i \mu S}{T^2} \tag{16}$$

where c_0 (J kg^{-1} °C^{-1}) is the heat capacity for pure water ice, L_i (J kg^{-1}) is the latent heat of fusion of ice, S (ppt) is the ice salinity, T (°C) is the temperature, and μ (°C ppt^{-1}) is the empirical constant in the melting temperature (T_{melt}) and salinity relation:

$$T_{melt} = -\mu S. \tag{17}$$

(See List of Physical Constants.) For each category n, Eq. 15 is evaluated using this heat capacity from temperature T to the melting temperature T_{melt} at salinity S:

$$q_n(T,S) = -\rho_i c_0 (T_{melt} - T) - \rho_i L_i \left(1 + \frac{\mu S}{T}\right). \tag{18}$$

The enthalpy q_n is defined to be negative, so that $|q_n|$ is the amount of energy required to melt a unit volume of sea ice of salinity S and temperature T. With this sign convention, a positive amount of heat $|q_n(T,S)|$ must be applied to raise the ice temperature from T to T_{melt} at salinity S, resulting in a rise of internal energy from $E_n < 0$ to zero.

For snow, the heat required to change its temperature below melting is small compared to the latent heat of fusion, and thus for simplicity is ignored. Further, snow is fresh and therefore has zero salinity. Hence, the amount of energy required to melt a unit volume of snow is given by:

$$q_s = -\rho_s L_i \tag{19}$$

where ρ_s is the constant snow density. Note that since q_s is a constant, the snow internal energy is proportional to V_{sn}, so that an explicit snow internal energy is not a state variable. When required, the snow internal energy is computed from:

$$E_{sn} = q_s V_{sn}. \tag{20}$$

The snow/ice surface temperature is an important quantity for determining the heat and mass exchange between the atmosphere and the snow/ice surface. The surface temperature T_{sn} for the n^{th} category varies rapidly with changing forcing conditions, and because it is used as an initial condition for the thermodynamic surface energy calculation, it is treated as a state variable. The area-weighted surface temperature is used for conservation and transport, in both thickness and physical space.

Snow and ice thickness and ice temperature are not state variables, but they can be diagnosed as follows. Ice and snow thickness are computed from the ice area and volume and snow volume, respectively, as:

$$h_n = V_n / A_n \quad h_{sn} = V_{sn} / A_n. \tag{21}$$

Ice temperature can be diagnosed from the energy of melting. As discussed in Section 4.6 on vertical heat conduction, the ice is divided vertically into a number of layers. For each layer there is an internal energy and volume. From these an energy of melting (q_n) can be computed for each layer (Eq. 14), and hence a layer temperature from the solution to the quadratic equation (Eq. 18)

$$\rho_i c_0 T^2 - (q_n + \rho_i c_0 T_{melt} + \rho_i L_i)T - \rho_i L_i \mu S = 0. \tag{22}$$

The solution yields one temperature below T_{melt} and another above T_{melt}, which is discarded.

4.3 Input from the Coupler

Fields received by the ice model from the coupler are shown in Table 2. They include atmospheric/oceanic states and fluxes. Atmospheric states must be available to the ice model because with an ice thickness distribution, it is necessary for the ice model, rather than for the coupler (as in CSM1), to compute fluxes over each ice thickness category and aggregate them.

All of the fields received are on the T-grid (see Section 3.4). However, the vector fields of surface wind, surface ocean current and tilt are projected onto geographical latitude-longitude directions. These vectors are first rotated to the displaced pole grid directions using a T-grid rotation angle(χ^t) calculated from a U-grid rotation angle (χ^u) provided by a grid input dataset. The rotated surface wind is then used on the T-grid to calculate atmosphere/ice fluxes, including stresses (see Section 4.5). The resulting atmosphere/ice stress, as well as the ocean surface current and tilt, are then bilinearly interpolated with area weights to the U-grid for use in the dynamics (see Section 4.10).

Specifically, let (u_g^t, v_g^t) represent a vector field of components (u, v), where the subscript "g" refers to geographic, and the superscripts "t" and "u" to the T-grid and U-grid respectively. Similarly, let a subscript "dp" refer to the displaced pole grid. All the vector fields received from the coupler are then (u_g^t, v_g^t) fields. For these to be useful on the displaced pole grid, they must first be rotated as follows:

$$\begin{aligned} (u_{dp}^t)_{ij} &= (u_g^t)_{ij} cos(\chi_{ij}^t) + (v_g^t)_{ij} sin(\chi_{ij}^t) \\ (v_{dp}^t)_{ij} &= -(u_g^t)_{ij} sin(\chi_{ij}^t) + (v_g^t)_{ij} cos(\chi_{ij}^t) \end{aligned} \tag{23}$$

where (ij) are the longitude/latitude indices of the displaced pole grid. The atmosphere winds in the (u_{dp}^t, v_{dp}^t) form can be used to directly compute atmosphere/ice stresses. However, these stresses, as well as the ocean currents and tilts, are required to be on the U-grid for the dynamic calculation. Therefore, the following interpolation from the T-grid to the U-grid is required:

$$\begin{aligned} (u_{dp}^u)_{ij} &= \frac{1}{4}(A_{ij}^t (u_{dp}^t)_{ij} + A_{i+1j}^t (u_{dp}^t)_{i+1j} + A_{ij+1}^t (u_{dp}^t)_{ij+1} + A_{i+1j+1}^t (u_{dp}^t)_{i+1j+1})/A_{ij}^u \\ (v_{dp}^u)_{ij} &= \frac{1}{4}(A_{ij}^t (v_{dp}^t)_{ij} + A_{i+1j}^t (v_{dp}^t)_{i+1j} + A_{ij+1}^t (v_{dp}^t)_{ij+1} + A_{i+1j+1}^t (v_{dp}^t)_{i+1j+1})/A_{ij}^u \end{aligned} \tag{24}$$

where A^t_{ij} is the T-grid box area, and A^u_{ij} is the U-grid box area.

The freezing/melting potential F_{Qoi} is calculated in the ocean model and received by the ice model as input. In the ice model, three forms of ice are distinguished: **frazil** (which forms directly in the ocean surface layer), **congelation** (which forms at the ice base), and **snow-ice** (which forms by flooding of snow-covered ice). Frazil ice formation is determined by the ocean model, and the other two by the ice model.

If the ocean surface layer temperature (T_o) falls below freezing (at fixed temperature T_{of}), frazil ice forms such that the heat flux F_{Qoi} restores the ocean temperature to freezing:

$$F_{Qoi} = \rho_o c_o h_o (T_{of} - T_o)/\Delta t \tag{25}$$

where $\rho_o c_o$ is the product of ocean density and heat capacity, h_o is the surface layer thickness, Δt is the coupling time step for the ocean (i.e. the time between exchanges of data with the coupler, usually one day). If $(T_o < T_{of})$ then $F_{Qoi} > 0$ and frazil ice forms (note that all CCSM fluxes are positive downwards).

For completeness, we discuss the salinity rejected in frazil ice formation, although it has no effect on the sea ice model. The salinity adjustment ΔS in the ocean model due to brine rejection is:

$$\Delta S = (S_o - S_i) F_{Qoi} \Delta t / (\rho_o h_o L_i) \tag{26}$$

where S_o is the constant reference ocean salinity, S_i is the constant reference salinity of sea ice, ρ_o is the constant ocean density, and L_i is the latent heat of fusion of sea ice. Note that the ocean freezing temperature T_{of} is kept constant independent of salinity, and the latent heat of fusion L_i is identical in the ocean and sea ice models.

The total downwards shortwave flux is the sum of the four components, which are defined in Table 2:

$$F_{SWDN} = F_{SWvsdr} + F_{SWvsdf} + F_{SWnidr} + F_{SWnidf}. \tag{27}$$

For the rest of the document, Δt will represent both the physical and the coupling time step for the sea ice model, which for CSIM5 is one hour. It is possible to run the model with a physical time step for the dynamics smaller than the coupling time step.

4.4 Snow and Ice Albedo

Snow and ice albedos are important for computing the absorption of shortwave radiation in the snow/ice system, and hence snow/ice albedo feedback (Curry et al., 1995). The physics of this absorption and scattering is very complex (Ebert and Curry, 1993; Grenfell et al., 1994), but here it is simplied significantly. The snow and ice albedo formulas are basically those of CCSM2, which have been lowered somewhat to yield better sea ice simulation in CCSM3. The albedo depends upon spectral band, snow thickness, ice thickness and surface temperature.

Snow and ice spectral albedos (visible = vs, wavelength $< 0.7\mu m$ and near-infrared = ni, wavelength $> 0.7\mu m$) are distinguished, as both snow and ice spectral reflectivities are significantly higher in the vs band than in the ni band. This two-band separation represents the basic spectral dependence. Thus, we ignore the near-infrared spectral structure, with generally decreasing reflectivity with increasing wavelength (Ebert and Curry, 1993).

The zenith angle dependence of snow and ice is ignored (Ebert and Curry, 1993; Grenfell et al., 1994), and therefore the distinction between downwelling direct (dr) and diffuse (df) shortwave radiation. The error in this approximation is probably no larger than .05 . Solar elevation angles in the polar regions are low. The zenith angle dependence for the albedo affects the downwelling direct radiation, while clear skies are relatively rare in polar regions. Horizontal variations in snow/ice topography are also ignored, which affect scattering and transmission into the surface through shadowing effects and through variations in the angle of the surface above the horizon.

Snow albedo depends strongly on snow age (i.e. grain size, Grenfell et al., 1994), and on surface temperature (i.e. melting or non-melting conditions, (Ebert and Curry, 1993). Sea ice albedo depends on ice thickness (Allison et al., 1993), as well as the presence of melt ponds (Ebert and Curry, 1993). In addition, snow only partially covers a surface if there are strong topographic variations (Allison et al., 1993).

Here we ignore the dependence of snow albedo on age, but retain the melting/non-melting distinction and thickness dependence. Dry snow spectral albedos for the n^{th} category are:

$$\begin{aligned} \alpha^s_{vsdfn}(dry) &= 0.96 \\ \alpha^s_{nidfn}(dry) &= 0.68 \end{aligned} \tag{28}$$

(Note that these are less than the corresponding CCSM2 albedos at 0.98 and 0.70 respectively.) These values are roughly consistent with those of Grenfell et al. (1994) and Ebert and Curry (1993). In the case of the measurements of Grenfell et al. (1994), these dry snow albedos are slightly lower than clear sky values for low sun and for limited cloud cover, corresponding to the spring-time high values prior to significant melt (Curry et al., 2001). These albedos are only slightly higher than those for late summer conditions with early snow fall under cloudy skies.

To represent melting snow albedos, the surface temperature is used. Springtime warming produces a rapid transition from sub-zero to melting temperatures, while late fall values transition more slowly to sub-zero conditions. This is approximated by a temperature dependence out to $-1°$C. Let T_{snc} represent the snow/ice surface temperature for category

n in °C. If $T_{snc} = T_{sn} - T_{melt} \geq -1$°C then

$$\begin{aligned} \Delta T_s =& T_{snc} + 1.0 \\ \alpha^s_{vsdfn}(melt) =& \alpha^s_{vsdfn}(dry) - 0.10\Delta T_s \\ \alpha^s_{nidfn}(melt) =& \alpha^s_{nidfn}(dry) - 0.15\Delta T_s \end{aligned} \tag{29}$$

The lowest albedos at 0°C are .86 and .53 for visible and near-ir respectively, lower by about .02 than those of Ebert and Curry (1993) (the corresponding CCSM2 albedos are .88 and .56 respectively). If the surface temperature $T_{snc} < -1$°C, the dry snow albedos are used; otherwise the melt albedos.

For bare non-melting sea ice, albedo depends on thickness and spectral band. If $h_n < 0.5$ m then

$$\begin{aligned} \alpha_{vsdfn}(dry) =& \alpha_o(1 - fh) + \alpha_{vsdfn}(thick)fh \\ \alpha_{nidfn}(dry) =& \alpha_o(1 - fh) + \alpha_{nidfn}(thick)fh \end{aligned} \tag{30}$$

where α_o is the open ocean diffuse albedo (as for snow and sea ice, the zenith angle dependence of the ocean surface is also ignored),

$$fh = min(tan^{-1}(c_{fh}\ h_i)/tan^{-1}(c_{fh}\ 0.5), 1.0) \tag{31}$$

$c_{fh} = 4$ (for CCSM2, $c_{fh} = 5$), and the thick, non-melting sea ice albedos are:

$$\begin{aligned} \alpha_{vsdfn}(thick) =& 0.73 \\ \alpha_{nidfn}(thick) =& 0.33 \end{aligned} \tag{32}$$

which are the asymptotic values for ice thicker than 0.5 m (corresponding CCSM2 values are 0.78 and 0.36 respectively). These expressions represent a rough fit to the data of Allison et al. (1993), with the limiting cases for zero ice thickness that of the open ocean albedo α_o, and for ice thicker than 0.5 m that of the thick ice case of Ebert and Curry (1993). The inverse tangent functional form approximates the theoretical dependence of ice albedo on thickness.

For bare melting sea ice, melt ponds can significantly lower the area averaged albedo. This effect is crudely approximated by the following temperature dependence. If $T_{snc} \geq -1$°C, where $T_{snc} = T_{sn} - T_{melt}$, then

$$\begin{aligned} \alpha_{vsdfn}(melt) =& \alpha_{vsdfn}(dry) - 0.075\Delta T_s \\ \alpha_{nidfn}(melt) =& \alpha_{nidfn}(dry) - 0.075\Delta T_s \end{aligned} \tag{33}$$

This results in minimum spectral albedos of .655 and .255 for visible and near-ir respectively, or a rough broad band albedo (summertime spectral ratios of visible and near-ir of .53 and .47 respectively) of .467 (corresponding spectral albedos for CCSM2 were .705 and .285 respectively, and a broadband of .508). As for the case of snow, if the surface temperature $T_{snc} < -1$°C, the dry sea ice albedos are used; otherwise the melt albedos.

The horizontal fraction of surface covered with snow is

$$f_{sn} = \frac{h_{sn}}{h_{sn} + 0.02} \tag{34}$$

This expression is approximately in keeping with snow depth dependence of albedo from Ebert and Curry (1993), and from measurements of albedo on snow covered Antarctic sea ice Allison et al. (1993). We arrived at the value of .02 m in the denominator to achieve a good agreement with SHEBA data. This means that 2 cm of snow will cover 50% of the horizontal sea-ice area; the rest is assumed to be bare sea-ice.

Combining ice and snow albedos by averaging over the horizontal coverage results in

$$\begin{aligned} \alpha_{vsdfn} &= \alpha_{vsdfn}(1 - f_{sn}) + f_{sn}\alpha^{s}_{vsdfn} \\ \alpha_{nidfn} &= \alpha_{nidfn}(1 - f_{sn}) + f_{sn}\alpha^{s}_{nidfn} \end{aligned} \tag{35}$$

As noted above, the direct albedos are assumed identical to the diffuse. These formulas are limited for thin, bare melting sea-ice to be greater than the ocean albedo α_o.

This crude albedo, when compared with SHEBA measurements (Curry et al., 2001), is able to approximately represent the major albedo regimes of springtime pre-melt dry snow, melting snow cover, dry bare ice, bare ice with melt ponds, and early fall freeze with light snow. However, the parameterization is arbitrary and inconsistent across the snow/ice thermal and physical state, angle, spectral, and snow/ice thickness dependencies. It does not allow for any consistent addition of organic and/or non-organic materials, and is not consistent with internal heating and penetration into the underlying ocean. There is no allowance for snow aging nor for explicit melt ponds. While a first step, there is much room for improvement in this albedo parameterization.

For diagnostic purposes, it is useful to have an aggregate broad band surface albedo for the history file (see Section 6):

$$\alpha_{bb} = .29\alpha_{vsdr} + .24\alpha_{vsdf} + .31\alpha_{nidr} + .16\alpha_{nidf} \tag{36}$$

The relative weights are only rough estimates of typical surface flux in each spectral band and incident angle. A broad band albedo consistent with the downwelling shortwave fluxes listed in Table 2 and the total shortwave flux in Eq. 27 could easily be computed in future versions.

4.5 Ice to Atmosphere Flux Exchange

Atmospheric states and downwelling fluxes, along with surface states and properties, are used to compute atmosphere-ice shortwave and longwave fluxes, stress, sensible and latent heat fluxes. Surface states are temperature T_{sn} and albedos α_{vsdrn}, α_{vsdfn}, α_{nidrn}, α_{nidfn} (see Section 4.4), while surface properties are longwave emissivity ε and aerodynamic roughness z_i (note that these properties in general vary with ice thickness, but are here assumed constant). Additionally, certain flux temperature derivatives required for the ice temperature calculation are computed, as well as reference diagnostic surface air temperature and specific humidity.

The following formulae are used for the n^{th} category for the absorbed shortwave fluxes and upwelling longwave flux:

$$F_{SWvsn} = F_{SWvsdr}(1 - \alpha_{vsdrn}) + F_{SWvsdf}(1 - \alpha_{vsdfn}) \tag{37}$$

$$F_{SWnin} = F_{SWnidr}(1 - \alpha_{nidrn}) + F_{SWnidf}(1 - \alpha_{nidfn}) \tag{38}$$

$$F_{SWn} = F_{SWvsn} + F_{SWnin} \tag{39}$$

$$F_{LWUPn} = -\varepsilon\sigma_{sb}T_{sn}^4 + (1 - \varepsilon)F_{LWDN} \tag{40}$$

The downwelling shortwave flux and albedos distinguish between visible $(vs, \lambda < 0.7\mu m)$, near-infrared $(ni, \lambda > 0.7\mu m)$, direct (dr) and diffuse (df) radiation for each category. Note that the upwelling longwave flux has a reflected component from the downwelling longwave whenever $\varepsilon < 1$.

For the n^{th} category stress components, sensible and latent heat flux, the following bulk formulae are used (NCAR CSM Flux Coupler, 1996):

$$\tau_{axn} = \rho_a r_{mn} u_n^* u_a \tag{41}$$

$$\tau_{ayn} = \rho_a r_{mn} u_n^* v_a \tag{42}$$

$$F_{SHn} = \rho_a c_a r_{hn} u_n^* (\theta_a - T_{sn}) \tag{43}$$

$$F_{LHn} = \rho_a L_s r_{en} u_n^* (q_a - q_s(T_{sn})) \tag{44}$$

where:

$$q_s(T_{sn}) = (q_1/\rho_a)e^{-q_2/T_{sn}} \tag{45}$$

$$c_a = C_p(1 + C_{pvir} q_s(T_{sn})) \tag{46}$$

$$C_{pvir} = (C_{pwv}/C_p) - 1. \tag{47}$$

L_s in Eq. 44 is the latent heat of sublimation from ice to vapor. $q_s(T)$ is the surface saturation specific humidity for either ice or ocean at temperature T in Kelvins (the values of q_1, q_2 for ice were kindly supplied by Xubin Zeng of the University of Arizona), C_p is the specific heat of dry air and C_{pwv} of water vapor (see List of Physical Constants for values of constants). The exchange coefficients for momentum, sensible and latent heat for each category are r_{mn}, r_{hn}, and r_{en} respectively.

The bulk formulae are based on Monin-Obukhov similarity theory. Among boundary layer scalings, this is the most well tested (Large, 1998). It is based on the assumption that in the surface layer (typically the lowest tenth of the atmospheric boundary layer), but away from the surface roughness elements, only the distance from the boundary and the surface kinematic fluxes are important in the turbulent exchange. The fundamental turbulence scales that are formed from these quantities are the friction velocity u_n^*, the temperature and moisture fluctuations θ_n^* and q_n^* respectively, and the Monin-Obukhov length scale L_n:

$$u_n^* = r_{mn} V_{mag} \tag{48}$$

$$\theta_n^* = r_{hn}(\theta_a - T_{sn}) \tag{49}$$

$$q_n^* = r_{en}(q_a - q_s(T_{sn})) \tag{50}$$

$$L_n = u_n^{*3}/(\kappa F_n) \tag{51}$$

with

$$V_{mag} = max(1.0, \sqrt{u_a^2 + v_a^2}), \tag{52}$$

to prevent zero or small fluxes under quiescent wind conditions, κ is von Karman's constant (0.4), and F_n is the bouyancy flux, defined as:

$$F_n = \frac{u_n^*}{g_e}\{\frac{\theta_n^*}{\theta_{vn}} + \frac{q_n^*}{z_v^{-1} + q_a}\} \tag{53}$$

with g_e the gravitational acceleration and the virtual potential temperature $\theta_v = \theta_a(1 + z_v q_a)$ where $z_v = \rho_{wv}/\rho_a - 1$.

Similarity theory holds that the vertical gradients of mean horizontal wind, potential temperature and specific humidity are universal functions of stability parameter $\zeta = z/L$, where z is height above the surface (ζ is positive for a stable surface layer and negative for an unstable surface layer). These universal similarity functions are determined from observations in the atmospheric boundary layer (Hogstrom, 1988) though no single form is widely accepted. Integrals of the vertical gradient relations result in the familiar logarithmic mean profiles, from which the exchange coefficients can be defined, where $\zeta_n = z_a/L_n$:

$$r_{mn} = r_0\{1 + \frac{r_0}{\kappa}(ln(z_a/z_{ref}) - \chi_m(\zeta_n))\}^{-1} \tag{54}$$

$$r_{hn} = r_0\{1 + \frac{r_0}{\kappa}(ln(z_a/z_{ref}) - \chi_h(\zeta_n))\}^{-1} \tag{55}$$

$$r_{en} = r_{hn} \tag{56}$$

with the neutral coefficient r_0 over ice:

$$r_0 = \frac{\kappa}{ln(z_{ref}/z_i)}, \tag{57}$$

and over ocean:

$$r_0 = (.0027/V_{mag} + .000142 + .0000764 V_{mag})^{1/2} \tag{58}$$

where z_{ref} is a reference height (presently 10 m). Note that the square of r_0 is often referred to as the neutral drag coefficient C_d, and that for the sea ice aerodynamic roughness value in the List of Physical Constants, $C_d = r_0^2 = 1.6 \times 10^{-3}$. The flux profile functions (integrals of the similarity functions mentioned above) for momentum m and heat/moisture h are:

$$\chi_m(\zeta_n) = \chi_h(\zeta_n) = -5\zeta_n \tag{59}$$

for stable conditions ($\zeta_n > 0$). For unstable conditions ($\zeta_n < 0$):

$$\chi_m(\zeta_n) = ln\{(1 + X_n(2 + X_n))(1 + X_n^2)/8\} - 2tan^{-1}(X_n) + 0.5\pi \tag{60}$$

$$\chi_h(\zeta_n) = 2ln\{(1 + X_n^2)/2\} \tag{61}$$

with

$$X_n = (max((1 - 16\zeta_n)^{1/2}), 1.)^{1/2}. \tag{62}$$

The stability parameter ζ_n is a function of the turbulent scales and thus the fluxes, so an iterative solution is necessary. The coefficients are initialized with their neutral value r_0, from which the turbulent scales, stability, and then flux profile functions can be evaluated. This order is repeated for five interations to ensure convergence to an acceptible solution.

The surface temperature derivatives required by the ice temperature calculation are evaluated as:

$$\frac{\partial F_{LWUPn}}{\partial T_{sn}} = -4\varepsilon\sigma_{sb}T_{sn}^3 \tag{63}$$

$$\frac{\partial F_{SHn}}{\partial T_{sn}} = -\rho_a c_a r_{hn} u_n^* \tag{64}$$

$$\frac{\partial F_{LHn}}{\partial T_{sn}} = -\rho_a L_s r_{en} u_n^* \frac{\partial q_s(T_{sn})}{\partial T_{sn}} \tag{65}$$

where the small temperature dependencies of c_a, the exchange coefficients r_{hn} and r_{en} and velocity scale u_n^* are ignored.

For diagnostic purposes, an air temperature (T_{REFn}) and specific humidity (Q_{REFn}) are computed at the reference height of $z_{2m} = 2m$. We define $\zeta = \pm z_{2m}/z_a$, where the $\pm$ refers to stable/unstable conditions respectively. Using Eq. 62 along with Eq. 59 (stable) and Eq. 61 (unstable), we define χ_2. Hence for temperature:

$$f_{int} = (r_{hn}/\kappa)(-ln(z_{2m}/z_{ref}) - \chi_{hn} + \chi_2) \tag{66}$$

$$T_{refn} = \theta_a + (\theta_a - T_{sn})f_{int} \tag{67}$$

$$T_{refn} = T_{refn} - .01z_{2m} \tag{68}$$

For specific humidity we have:

$$f_{int} = (r_{en}/\kappa)(-ln(z_{2m}/z_{ref}) - \chi_{hn} + \chi_2) \tag{69}$$

$$Q_{refn} = q_a + (q_a - q_{sn})f_{int}. \tag{70}$$

4.6 Vertical Heat Conduction

Vertical heat conduction follows Maykut and Untersteiner (1971) and Bitz and Lipscomb (1999). This section is also drawn from Bitz (2000) with modifications to match the notation used in this report, and with minor modifications for assumptions unique to this model.

To represent the vertical transfer of heat through the ice, we allow the ice internal energy (Eq. 14) to vary with level z, where z is vertical depth measured positive downwards from the ice surface. The number of layers of ice (L) in each category is four, with each layer thickness $\Delta h_n = h_n/L$ where h_n is from Eq. 21. The internal energy for each layer can be solved for an equivalent layer temperature (Eq. 22). Vertical heat transfer is then calculated for $(l = 1 ... L)$ vertical layers in the sea ice and one layer of overlying snow (if present). A staggered vertical grid is used, with temperature and salinity defined at layer midpoints and conductivity defined at layer interfaces. Layers at the top and bottom are referred to as surface layers, and those away from the surfaces as interior layers. In this section, the superscript is reserved for the time index m, and the category index n is implied; the subscript s on T denotes the surface and the subscript 0 denotes the snow layer.

The invariant vertical salinity profile is represented by

$$S(w) = 1.6\left\{1 - \cos\left(\pi w^{\frac{0.407}{0.573+w}}\right)\right\} \tag{71}$$

with the normalized coordinate w calculated for each category as

$$w = z/h_n, 0 \leq w \leq 1 \tag{72}$$

Thus the salinity profile is independent of category. The profile varies from 0 ppt at ice surface to 3.2 ppt at ice base; salinity values at w of 0.25, 0.50 and 0.75 are 1.5, 2.8 and 3.1 ppt respectively. The profile is meant to represent multi-year ice for which surface melt has reduced salinity near the top layers with respect to the lower. Note that this salinity profile need not be consistent with the reference salinity used in ice-ocean salt exchange (S_i; see the List of Physical Constants). The vertically averaged salinity from the above profile is 2.3 ppt.

The heat content change over the time interval t to t' corresponding to temperatures T and T', respectively, allowing for temperature dependent heat capacity, thermal conduction and internal absorption of penetrating solar radiation, is given by (see Eqs. 15,18):

$$\int_T^{T'} \rho_i c_i dT = \rho_i c_0 (T' - T)\left(1 + \frac{L_i \mu S}{c_0 T' T}\right) = \int_t^{t'} \left(\frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + Q_{SW}\right) dt \tag{73}$$

where c_i is from Eq. 16, Q_{SW} is the absorbed shortwave flux, and the thermal conductivity k is either that for snow or ice. For snow, $k = k_s$ is a constant, while for ice:

$$k(S,T) = k_{fi} + \frac{\beta S}{T} \tag{74}$$

where k_{fi} and β are empirical constants from Untersteiner (1961). Q_{SW} is given by:

$$Q_{SW} = -\frac{d}{dz}\{I_{0vs}e^{-\kappa_{vs}z} + I_{0ni}e^{-\kappa_{ni}z}\} \tag{75}$$

and I_{0vs}, I_{0ni} are the fractions of absorbed shortwave radiation in the visible and near-ir that penetrate the surface, respectively, given by (see Eq. 37):

$$I_{0vs} = 0.70F_{SWvsn}(1 - f_{sn}) \tag{76}$$

$$I_{0ni} = 0.0 \tag{77}$$

where f_{sn}, the horizontal fraction of surface covered by snow, is given by Eq. 34. It is assumed that no shortwave radiation penetrates the snow covered surface. The ice spectral extinction coefficients κ_{vs} and κ_{ni} are for the visible and near-infrared bands respectively (Gary Maykut, personal communication). To compute the penetration factors (.70 and .0) for the visible and near-ir radiation respectively, a surface layer of 5 cm thick was assumed. However, for the surface energy balance calculation (see Section 4.6.1, Eq. 83) the surface layer thickness is not explicitly used. Note that there is no distinction made between direct or diffuse shortwave: in effect, we assume shortwave radiation penetrating the surface is diffuse. Also, the functional form of penetration in Eq. 75 is that for pure absorption; in effect, we assume all scattering occurs at the surface while the rest of the ice is purely absorbing.

The heat equation (Eq. 73) is discretized using a backwards-Euler, space-centered scheme. Using the staggered grid with T_l representing the layer temperature and k_l representing conductivity at the layer interfaces, for interior layers we have

$$\rho_i c_0(T_l^{m+1} - T_l^m)\left(1 + \frac{L_i \mu S_l}{c_0 T_l^{m+1} T_l^m}\right) = \frac{\Delta t}{\Delta h^m}\left(k_{l+1}^m \frac{T_{l+1}^{m+1} - T_l^{m+1}}{\Delta h^m} - k_l^m \frac{T_l^{m+1} - T_{l-1}^{m+1}}{\Delta h^m} + I_l^m\right), \tag{78}$$

where $\Delta h^m = h^m/L$, the conductivity is

$$k_l^m = k\left(\frac{S_l + S_{l+1}}{2}, \frac{T_l^m + T_{l+1}^m}{2}\right), \tag{79}$$

and the absorbed solar radiation is

$$I_l^m = I_{0vs}(e^{-\kappa_{vs} l \Delta h^m} - e^{-\kappa_{vs}(l+1)\Delta h^m}) + I_{0ni}(e^{-\kappa_{ni} l \Delta h^m} - e^{-\kappa_{ni}(l+1)\Delta h^m}). \tag{80}$$

For a purely implicit backward scheme, k should be evaluated at the $m+1$ time level. However, when k is evaluated at time level m, experiments show that the solution is stable and converges to the same solution one gets when evaluating k at $m+1$.

The discrete heat equation for the surface layers is modified slightly from Eq. 78 to maintain second-order accuracy for $\partial T/\partial z$. The equation for the bottom layer ($l = L$) is

$$\begin{aligned} &\rho_i c_0(T_L^{m+1} - T_L^m)\left(1 + \frac{L_i \mu S_L}{c_0 T_L^{m+1} T_L^m}\right) = \\ &\frac{\Delta t}{\Delta h^m}\left(3k_{L+1}\frac{T_b - T_L^{m+1}}{\Delta h^m} - \frac{1}{3}k_{L+1}\frac{T_b - T_{L-1}^{m+1}}{\Delta h^m} - k_L^m \frac{T_L^{m+1} - T_{L-1}^{m+1}}{\Delta h^m} + I_L^m\right), \end{aligned} \tag{81}$$

where the $L+1$ interface in contact with the underlying ocean is assumed to be at temperature $T_b = T_{of}$ and salinity S_b (given by Eqs. 71,72 with $w = 1$), and therefore conductivity $k_{L+1} = k(S_b, T_b)$. The equations for the top surface depend on the surface conditions, of which there are four possibilities, as outlined in Table 5. If the snow depth is too small, numerical solutions are unreliable, and hence the insulating effects of snow are ignored for depths below h_{smin}.

Table 5. Top Surface Boundary Cases

	Snow Accumulated	Melting
case I	yes	no
case II	no	no
case III	yes	yes
case IV	no	yes

4.6.1 Surface Boundary Conditions

Water fluxes from the atmosphere are received by the ice model in the form of rain F_{RN} and snow F_{SNW} (see Table 2). Presently, the rain is assumed to run off into the ocean without modification to the snow and ice, and adds to the water flux into the ocean F_{Wo} (see Table 3). Unless otherwise noted, all fluxes have an implied time step index m.

Snow (see Eq. 21) is assumed to accumulate on the surface as:

$$h_{sn}^{m+1/2} = h_{sn}^{m} + F_{SNW}\Delta t/\rho_s. \tag{82}$$

The boundary condition at the ice surface results from a balance of fluxes

$$F_{TOPn}(T_{sn}) = F_{SWvsn} - I_{0vs} + F_{SWnin} - I_{0ni} + \varepsilon F_{LWDN} - \varepsilon\sigma_{sb}T_{sn}^4 + F_{SHn} + F_{LHn} + k_n\left.\frac{dT_n}{dz}\right|_{z=0}, \tag{83}$$

where z is the vertical depth measured downwards from the ice surface.

If $F_{TOPn}(T_{sn}) \leq 0$ then no surface melting occurs. If $F_{TOPn}(T_{sn}) > 0$ then $T_{sn} = T_{melt}$ and surface melting when snow is present proceeds according to

$$F_{TOPn}(T_{sn}) = q_s\frac{dh_{sn}}{dt}. \tag{84}$$

When no snow is present surface melting proceeds according to

$$F_{TOPn}(T_{sn}) = q_n(T_n, S)\frac{dh_n}{dt}, \tag{85}$$

where T_n and S are the ice temperature and salinity of the top ice layer, respectively. The boundary condition at the ice bottom is

$$-F_{BOT} - k_n\frac{\partial T_n}{\partial z} = q_n(T_n, S)\frac{dh_n}{dt} \tag{86}$$

where F_{BOT} is the heat flux from the ocean (see section 4.8, Eq. 131), k_n is the ice thermal conductivity, and T_n,S are the temperature and salinity of the bottom ice layer, respectively.

Four cases must be distinguished to solve for vertical heat conduction, based on presence or absence of snow, and whether melting or non-melting conditions pertain.

4.6.2 Case I: Snow accumulation with no melting

The discrete heat equation for the uppermost snow layer (recall that the subcript 0 on T denotes the snow layer) is:

$$\rho_s c_s (T_0^{m+1} - T_0^m) = \frac{\Delta t}{h_s^m}\left[k_1^m \frac{T_1^{m+1} - T_0^{m+1}}{(\Delta h^m + h_s^m)/2} - \alpha k_s \frac{T_0^{m+1} - T_s^{m+1}}{h_s^m} - \beta k_s \frac{T_1^{m+1} - T_s^{m+1}}{h_s^m}\right]. \tag{87}$$

The heat equation solver is formulated for the general case where the heat capacity of snow c_s may be specified, although here is set to 0. The parameters α and β are defined to give second-order accurate spatial differencing for $\partial T/\partial z$ across the changing layer spacing at the snow/ice boundary;

$$\begin{aligned}\alpha &= \frac{h_s^m + \Delta h^m/2}{h_s^m/2}\;\frac{2}{h_s^m + \Delta h^m} h_s^m \\ \beta &= \frac{-h_s^m/2}{h_s^m + \Delta h^m/2}\;\frac{2}{h_s^m + \Delta h^m} h_s^m.\end{aligned} \tag{88}$$

The conductivity at the snow–ice interface is found by equating conductive fluxes above and below the interface;

$$k_1^m = \frac{2k_s k(S_1, T_1^m)}{h_s^m k(S_1, T_1^m) + \Delta h^m k_s}\;\frac{h_s^m + \Delta h^m}{2}. \tag{89}$$

Because T_s is below melting, a flux boundary condition is used, and an additional equation is required in the coupled set:

$$F_o(T_s^{m+1}) + \alpha k_s \frac{T_0^{m+1} - T_s^{m+1}}{h_s^m} + \beta k_s \frac{T_1^{m+1} - T_s^{m+1}}{h_s^m} = 0, \tag{90}$$

where $F_o(T_s^{m+1})$ is the sum of all terms on the right-hand side of Eq. 83 except $k\partial T/\partial z$. The net surface flux $F_o(T_s^{m+1})$ is approximated as linear in T_s^{m+1}; thus

$$F_o(T_s^{m+1}) \sim F_o(T_s^m) + \left.\frac{\partial F_o}{\partial T_s}\right|_{T_s^m} (T_s^{m+1} - T_s^m). \tag{91}$$

with

$$\left.\frac{\partial F_o}{\partial T_s}\right|_{T_s^m} = \left.\frac{\partial F_{LWUP}}{\partial T_s}\right|_{T_s^m} + \left.\frac{\partial F_{SH}}{\partial T_s}\right|_{T_s^m} + \left.\frac{\partial F_{LH}}{\partial T_s}\right|_{T_s^m} \tag{92}$$

(see Eqs. 63-65).

To simplify our set of equations, we define

$$\hat{c}_l^{m+1} = \rho_i \left(c_0 + \frac{L_i \mu S}{T_l^{m+1} T_l^m}\right), \tag{93}$$

where the hat implies that $\hat{c}_l^{m+1}$ depends on T_l^m as well as on T_l^{m+1}, and

$$\chi_l^{m+1} = \frac{\Delta t}{\Delta h^m} \frac{1}{\hat{c}_l^{m+1}}. \tag{94}$$

Also, let

$$k_l = \frac{k_l^m}{\Delta h^m}. \tag{95}$$

for $l \geq 2$ and

$$k_0 = \frac{k_s}{h_s^m} \tag{96}$$

$$k_1 = \frac{k_1^m}{(\Delta h^m + h_s^m)/2} \tag{97}$$

and suppress the index m for I_l^m, so that for interior layers $(l = 1...L-1)$,

$$T_l^{m+1} - T_l^m = \chi_l^{m+1} \left[k_{l+1}(T_{l+1}^{m+1} - T_l^{m+1}) - k_l(T_l^{m+1} - T_{l-1}^{m+1}) + I_l\right] \tag{98}$$

and at the bottom layer

$$\begin{aligned} T_L^{m+1} - T_L^m = \chi_L^{m+1} \Big[& 3k_b(T_b - T_L^{m+1}) - \frac{1}{3}k_b(T_b - T_{L-1}^{m+1}) \\ & -k_L(T_L^{m+1} - T_{L-1}^{m+1}) + I_L \Big] \end{aligned} \tag{99}$$

where $k_b = k_{L+1}/\Delta h^m$. The equation describing the snow layer is written

$$\rho_s c_s (T_0^{m+1} - T_0^m) = \frac{\Delta t}{h_s^m} \left[k_1(T_1^{m+1} - T_0^{m+1}) - \alpha k_0(T_0^{m+1} - T_s^{m+1}) - \beta k_0(T_1^{m+1} - T_s^{m+1})\right]. \tag{100}$$

Finally, the flux boundary condition becomes

$$F_o(T_s^m) + \left.\frac{\partial F_o}{\partial T_s}\right|_{T_s^m} (T_s^{m+1} - T_s^m) = -\alpha k_0(T_0^{m+1} - T_s^{m+1}) - \beta k_0(T_1^{m+1} - T_s^{m+1}). \tag{101}$$

The complete set of coupled equations for case I can be written with all of the terms that explicitly depend on temperature at the $m+1$ time step gathered on the right-hand side:

$$\begin{aligned} -F_o(T_s^m) + \left.\frac{\partial F_o}{\partial T_s}\right|_{T_s^m} T_s^m &= T_s^{m+1} \left(\left.\frac{\partial F_o}{\partial T_s}\right|_{T_s^m} - \alpha k_0 - \beta k_0 \right) \\ &\quad + T_0^{m+1}\alpha k_0 + T_1^{m+1}\beta k_0 \\ \rho_s c_s T_0^m &= T_s^{m+1} \left(-\frac{\Delta t}{h_s^m}\right)(\alpha k_0 + \beta k_0) \\ &\quad + T_0^{m+1}\left(\rho_s c_s + \frac{\Delta t}{h_s^m}(\alpha k_0 + k_1)\right) \\ &\quad + T_1^{m+1}\frac{\Delta t}{h_s^m}(\beta k_0 - k_1) \\ T_l^m + \chi_l^{m+1} I_l &= T_{l-1}^{m+1}(-\chi_l^{m+1} k_l) \\ &\quad + T_l^{m+1}(1 + \chi_l^{m+1} k_l + \chi_l^{m+1} k_{l+1}) \\ &\quad + T_{l+1}^{m+1}(-\chi_l^{m+1} k_{l+1}) \\ T_L^m + \chi_L^{m+1} I_L + \frac{8}{3}\chi_L^{m+1} k_b T_b &= T_{L-1}^{m+1}\left(-\frac{1}{3}\chi_L^{m+1} k_b - \chi_L^{m+1} k_L\right) \\ &\quad + T_L^{m+1}(1 + 3\chi_L^{m+1} k_b + \chi_L^{m+1} k_L). \end{aligned} \tag{102}$$

These equations are subsequently related to the following abbreviated form

$$\begin{aligned}
r_s &= T_s^{m+1} b_s + T_0^{m+1} c_s + T_1^{m+1} d_s \\
r_0 &= T_s^{m+1} a_0 + T_0^{m+1} b_0 + T_1^{m+1} c_0 \\
r_1 &= T_0^{m+1} a_1 + T_1^{m+1} b_1 + T_2^{m+1} c_1 \\
&\vdots \\
r_L &= T_{L-1}^{m+1} a_L + T_L^{m+1} b_L.
\end{aligned} \tag{103}$$

The first two rows can be combined to eliminate the coefficient on T_1^{m+1} in the first row, allowing the set to be written in tridiagonal form:

$$r = \begin{bmatrix} r_s c_0 - r_0 d_s \\ r_0 \\ r_1 \\ \vdots \end{bmatrix} \qquad A = \begin{bmatrix} b_s c_0 - a_0 d_s & c_s c_0 - b_0 d_s & & \\ a_0 & b_0 & c_0 & \\ & a_1 & b_1 & c_1 \\ & & & \ddots \end{bmatrix} \qquad T = \begin{bmatrix} T_s^{m+1} \\ T_0^{m+1} \\ T_1^{m+1} \\ \vdots \end{bmatrix}. \tag{104}$$

Because the matrix A depends on χ_l^{m+1}, which in turn depends on T_l^{m+1}, the system of equations is solved iteratively. An initial guess is used for the temperature dependence of χ_l^{m+1}, and then χ_l^{m+1} is updated successively after each iteration. Under most conditions the method approaches a solution in less than four iterations with a maximum error tolerance of ΔT_{err} for T_l with an initial guess of $T_l^{m+1} = T_l^m$.

4.6.3 Case II: Snow free with no melting

Nearly the same method applies when the ice is snow free, except one less equation is needed to describe the evolution of the temperature profile. The equation for the uppermost ice layer is written

$$\begin{aligned}
&\rho_i c_0 (T_1^{m+1} - T_1^m) \left(1 + \frac{L_i \mu S_1}{c_0 T_1^{m+1} T_1^m}\right) \\
&= \frac{\Delta t}{\Delta h^m} \left(k_2^m \frac{T_2^{m+1} - T_1^{m+1}}{\Delta h^m} - 3k_1^m \frac{T_1^{m+1} - T_s^{m+1}}{\Delta h^m} + \frac{1}{3} k_1^m \frac{T_2^{m+1} - T_s^{m+1}}{\Delta h^m} + I_1^m \right),
\end{aligned} \tag{105}$$

where $k_1^m = k(S_1, T_1^m)$. After the definitions from Eqs. 93–95 are applied, Eq. 105 becomes

$$T_1^{m+1} - T_1^m = \chi_1^{m+1} \left[k_2 (T_2^{m+1} - T_1^{m+1}) - 3k_1 (T_1^{m+1} - T_s^{m+1}) + \frac{1}{3} k_1 (T_2^{m+1} - T_s^{m+1}) + I_1^m \right]. \tag{106}$$

The flux boundary condition follows after linearizing $F_o(T_s^{m+1})$ in T_s^{m+1}:

$$F_o(T_s^m) + \left.\frac{\partial F_o}{\partial T_s}\right|_{T_s^m} (T_s^{m+1} - T_s^m) = -3k_1 (T_1^{m+1} - T_s^{m+1}) + \frac{1}{3} k_1 (T_2^{m+1} - T_s^{m+1}). \tag{107}$$

The complete set of coupled equation includes Eqs. 102 for layers 2 to L with the following two equations for the surface and upper ice layer:

$$
\begin{aligned}
-F_o(T_s^m) + \left.\frac{\partial F_o}{\partial T_s}\right|_{T_s^m} T_s^m &= T_s^{m+1}\left(\left.\frac{\partial F_o}{\partial T_s}\right|_{T_s^m} - k_1\frac{8}{3}\right) + T_1^{m+1}3k_1 + T_2^{m+1}(-k_1/3) \\
T_1^m + \chi_1^{m+1} I_1^m &= T_s^{m+1}\left(-\chi_1^{m+1}k_1\frac{8}{3}\right) \\
&\quad + T_1^{m+1}(1+\chi_1^{m+1}k_2 + 3\chi_1^{m+1}k_1) \\
&\quad + T_2^{m+1}(-\chi_1^{m+1}k_2 - \frac{1}{3}\chi_1^{m+1}k_1),
\end{aligned}
\tag{108}
$$

which can be written

$$
\begin{aligned}
r_s &= T_s^{m+1}b_s + T_1^{m+1}c_s + T_2^{m+1}d_s \\
r_1 &= T_s^{m+1}a_1 + T_1^{m+1}b_1 + T_2^{m+1}c_1.
\end{aligned}
\tag{109}
$$

These two equations can be combined to eliminate the coefficient on T_2^{m+1}, allowing the set to be written in tridiagonal form:

$$
r = \begin{bmatrix} r_s c_1 - r_1 d_s \\ r_1 \\ r_2 \\ \vdots \end{bmatrix} \qquad A = \begin{bmatrix} b_s c_1 - a_1 d_s & c_s c_1 - b_1 d_s & & \\ a_1 & b_1 & c_1 & \\ & a_2 & b_2 & c_2 \\ & & & \ddots \end{bmatrix} \qquad T = \begin{bmatrix} T_s^{m+1} \\ T_1^{m+1} \\ T_2^{m+1} \\ \vdots \end{bmatrix}. \tag{110}
$$

As for case I, this system of equations must be solved iteratively.

4.6.4 Case III: Snow accumulation with melting

Case III describes melting conditions in the presence of a snow layer at the surface. Here a temperature boundary condition is used, which simplifies the solution because the first row in Eqs. 102 is not needed and $T_s = T_{melt}$ in the second row. Hence the complete set of coupled equations is identical to Eqs. 102 for layers 1 to L, with the addition of an equation for the snow layer,

$$
\rho_s c_s T_0^m + T_{melt}\frac{\Delta t}{h_s}(\alpha+\beta)k_0 = T_0^{m+1}\left[\rho_s c_s + \frac{\Delta t}{h_s}(k_1 + \alpha k_0)\right] - T_1^{m+1}\frac{\Delta t}{h_s}(k_1 - \beta k_0). \tag{111}
$$

This set of equations can be written in tridiagonal form, without the need to eliminate any terms, as was required in cases I and II. However, the solution must still be iterated.

4.6.5 Case IV: No snow with melting

Like case III, case IV describes melting conditions, but here the sea ice is snow free. Hence, the first two rows of Eqs. 102 are not needed, and $T_s = T_{melt}$ for $l = 1$. The set of coupled equations comprises those from Eqs. 102 for layers 2 to L and the following equation for layer 1:

$$
T_1^m + \chi_1^{m+1} I_1^m + T_{melt}\chi_1^{m+1}k_1\frac{8}{3} = T_1^{m+1}\left(1+\chi_1^{m+1}k_2 + 3\chi_1^{m+1}k_1\right) + T_2^{m+1}\left(-\chi_1^{m+1}k_2 - \frac{1}{3}\chi_1^{m+1}k_1\right). \tag{112}
$$

As in case III, this set of equations can immediately be written in the tridiagonal form and solved iteratively.

4.7 Thickness Changes: Top, Bottom and Snow-to-Ice Conversion

The energy of melting of snow is a constant, given by Eq. 19, while the energy of melting for each layer l of sea ice depends on the temperature and salinity (T_l and S_l, respectively) of the layer according to Eqs. 17 and 18:

$$q_l = -\rho_i c_0(-\mu S_l - T_l) - \rho_i L_i \left(1 + \frac{\mu S_l}{T_l}\right). \tag{113}$$

The energy balance at the top and bottom surfaces determines the top melt and bottom melt and growth rates of the sea ice. From the top surface flux balance in Eq. 83, if $F_{\text{TOP}}(T_{sn}) > 0$, then the upper surface is fixed at the melting temperature and F_{TOP} is used for melting, according to

$$\delta h_s|_{\text{melt}} = \frac{F_{\text{TOP}}(T_{melt})\Delta t}{q_s} \tag{114}$$

where $\delta h_s|_{\text{melt}}$ is the change in snow thickness due to melt. For bare ice:

$$\delta h|_{\text{melt}} = \frac{F_{\text{TOP}}(T_{melt})\Delta t}{q_1}, \tag{115}$$

where $\delta h|_{\text{melt}}$ is the change in the top layer thickness due to ice melt, and q_s and q_1 are the energy of melting of the snow and the top layer of the ice, respectively. If all the snow is melted in one time step, any residual heat is applied to melt the ice. Snow and ice melt water is assumed to drain to the ocean below without any effect on the intervening snow and ice.

Sublimation of snow occurs when $F_{LH} < 0$ (regardless of T_s), according to

$$\delta h_s|_{\text{sub}} = -\frac{F_{LH}\Delta t}{(q_s - \rho_s L_v)} \tag{116}$$

where $\delta h_s|_{\text{sub}}$ is the change in snow thickness due to sublimation, and for ice according to

$$\delta h|_{\text{sub}} = -\frac{F_{LH}\Delta t}{(q_1 - \rho_i L_v)} \tag{117}$$

if the ice is snow free, where $\delta h|_{\text{sub}}$ is the change in the top layer ice thickness due to sublimation. The same set of equations applies when $F_{LH} > 0$ for condensation on the ice or snow.

The bottom-surface energy balance is (see section 4.8)

$$\delta h|_{\text{basal}} = \frac{(-F_{BOT} - k\frac{\partial T}{\partial z})\Delta t}{q_b}, \tag{118}$$

where $\delta h|_{\text{basal}}$ is the change in the lowest ice layer thickness due to basal freezing or melting, F_{BOT} is the heat supplied to the ice from the underlying ocean (see section 4.8, Eq. 131), the basal heat conduction is

$$k\frac{\partial T}{\partial z} = 3k_b\frac{T_b - T_L^{m+1}}{\Delta h^m} - \frac{1}{3}k_b\frac{T_b - T_{L-1}^{m+1}}{\Delta h^m} \tag{119}$$

accurate to second order, where the subscript "b" refers to ice base (i.e. $k_b = k_{L+1}$; see discussion following Eq. 81), and

$$q_b = \begin{cases} q_L; & \delta h|_{basal} < 0 \\ -\rho_i c_0(-\mu S_b - T_b) - \rho_i L_i\left(1 + \frac{\mu S_b}{T_b}\right); & \delta h|_{basal} > 0. \end{cases} \tag{120}$$

$\delta h|_{\text{basal}} > 0$ represents formation of congelation ice, and is treated as a (negative) water flux to the ocean; while $\delta h|_{\text{basal}} < 0$ represents basal ice melting which is added to the water flux to the ocean (see section 4.13).

Snow to ice conversion is allowed. This occurs if the snow layer overlying the sea ice becomes thick enough to depress the snow-ice interface below freeboard (the ocean surface). The interface height is:

$$z_{int} = h - (\rho_s h_s + \rho_i h)/\rho_o. \tag{121}$$

If $z_{int} < 0$, then an amount of snow in depth equal to $-z_{int}\rho_i/\rho_s$ is removed from the snow layer and added to the ice. It is assumed that ocean water floods the depressed snow, and then converts it into ice of thickness $-z_{int}$. The energy of melting of the newly formed ice is: $q_{flood} = q_s\rho_i/\rho_s$.

The energy of melting of the ice and snow layers needs to be adjusted when the layer spacing changes after growth/melt, sublimation/condensation, and flooding. The adjusted energy of melting is

$$q_l' = \begin{cases} \sum_{k=1}^{L} w_{k,1}q_k - q_{flood}\frac{z_{int}}{\Delta h'}; & l = 1 \\ \sum_{k=1}^{L} w_{k,l}q_k; & 1 < l < L \\ \sum_{k=1}^{L} w_{k,L}q_k + q_b\ \max(\frac{\delta h|_{\text{basal}}}{\Delta h'}, 0); & l = L. \end{cases} \tag{122}$$

where $w_{k,l}$ are weights computed from the relative overlap of layer l with each layer k from the old layer spacing and $\Delta h'$ is the new layer spacing.

As discussed in section 4.3, and further in section 4.13, for the purposes of ocean-ice salt exchange, sea ice is assumed to have a constant reference salinity S_i. This implies that any ice formation or melt must be accompanied by a salt exchange with the ocean in order to maintain the constant ice reference salinity (apart from frazil ice formation whose salinity exchange with the ocean is accounted for in the ocean model itself- see section 4.3).

Let $\delta V = \delta hA + h\delta A$ be the change in ice volume due either to a δh change in ice thickness or a δA change in ice area. The following equations give the salt flux exchange with the ocean for ice melt, net congelation (or basal) at ice base, net sublimation and condensation, and snow ice formation respectively:

$$F_S(ice\ melt) = +\rho_i S_i \delta V_{ice\ melt}/\Delta t \tag{123}$$

$$F_S(cong\ ice) = -\rho_i S_i \delta V_{cong\ ice}/\Delta t \tag{124}$$

$$F_S(sub\ cond) = -\rho_i S_i \delta V_{sub\ cond}/\Delta t \tag{125}$$

$$F_S(snow\ ice) = -\rho_i S_i \delta V_{snow-ice}/\Delta t \tag{126}$$

When ice melts, salinity is given to the ocean (positive flux), but for non-frazil ice formation salinity must be drawn out of the ocean. This is because the associated water flux exchange for non-frazil ice formation with the ocean is fresh.

4.8 Lateral Formation, Side and Bottom Melt Fluxes

Lateral formation and melt occurs depending on the sign of the freezing/melting potential F_{Qoi} (see Section 4.3, Eq. 25).

For **positive** F_{Qoi}, frazil ice formation occurs. A volume of frazil ice $V_f = F_{Qoi}\Delta t/q_f$ is formed, where $q_f = -\rho_i L_i$ is the enthalpy of frazil ice formation. If there is open water (i.e. $A_0 > 0$), then the thickness of the newly formed frazil ice is $h_f = V_f/A_0$. If $h_f < h_{min}$, new ice of thickness h_{min} is assumed to form over area $A_0' = V_f/h_{min}$, else $A_0' = A_0$, where primes denote updated quantities. This new ice is added to category 1 so long as $h_f < h_1^\star$, the thickest ice allowed for that category. If the new ice is thicker than this limit, or if there is no open water (i.e. $A_0 = 0$), then the new ice is distributed evenly over all categories. The lower flux boundary condition is F_{BOT}=0.

For new frazil ice evenly distributed over all categories, $V_{surp} = V_f - A_0 h_1^\star$ is the surplus ice volume over category 1, after which $V_f = A_0 h_1^\star$, so that:

$$\begin{aligned} V_n' &= V_n + V_{surp}A_n/A \\ E_n'(z) &= E_n(z) + (V_{surp}A_n/AL)q_f \end{aligned} \tag{127}$$

where A is the total ice concentration over all categories and E_n is adjusted for each level.

For new frazil ice added to category 1, for which even distribution of ice to thicker categories may have occured, we have:

$$\begin{aligned} A_1' &= A_1 + A_0' \\ V_1' &= V_1 + V_f \\ T_{s1}' &= (T_{s1}A_1 + T_{of}A_0')/A_1' \\ E_1'(z) &= E_1(z) + (V_f/L)q_f \end{aligned} \tag{128}$$

where E_1' is adjusted for each level.

For **negative** F_{Qoi}, heat is available to melt ice. This flux is partitioned into heat available for side melt and bottom melt based on first assuming F_{Qoi} is dominated by shortwave radiation, and then assuming shortwave radiation absorbed in the ocean surface layer above the mean ice thickness causes side melting and below it causes basal melting. For the mean ice thickness $\bar{h} = V/A$, where V and A are the total ice volume and area respectively (see Section 4.13):

$$f_{bot} = Re^{-\bar{h}/\zeta_1} + (1-R)e^{-\bar{h}/\zeta_2} \tag{129}$$

$$f_{sid} = 1 - f_{bot} \tag{130}$$

where $R = 0.68$, $\zeta_1 = 1.2\ m^{-1}$, $\zeta_2 = 28\ m^{-1}$ (Paulson and Simpson, 1977) and f_{bot} and f_{sid} are the fractions of bottom and side melt flux available, respectively. Thus the maximum fluxes available for melt are $f_{bot}F_{Qoi}$ and $f_{sid}F_{Qoi}$. The actual amount used for bottom melting, F_{BOT}, is based on boundary layer theory (Mcphee, 1992).

$$F_{BOT} = max(-\rho_o c_o c_h u^* \Delta T, f_{bot}F_{Qoi}) \tag{131}$$

where the empirical drag coefficient c_h=0.006, and

$$\Delta T = max(T_o - T_{of}, 0) \tag{132}$$

$$u^* = min\{\sqrt{(\tau_{ox}^2 + \tau_{oy}^2)/\rho_o}, u_{min}\} \tag{133}$$

with u_{min} the minimum allowed skin friction velocity u^*, and the ocean-ice stresses τ_{ox} and τ_{oy} are from Eq. 157.

The heat flux for lateral melt is the product of the vertically averaged, aggregate energy of melting of snow and ice, E_{tot}, with the interfacial melting rate M_a and the total floe perimeter p_f per unit floe area A_f.

$$E_{tot} = \rho_s L_s V_s + \sum_{n=1}^{N}\sum_{n=1}^{L_n} q_{nl} V_n / L_n \tag{134}$$

where q_{nl} is the energy of melting for layer l, and L_n is the number of layers of sea ice for category n. The interfacial melting rate is taken from the empirical expression of Maykut and Perovich (1987) based on Marginal Ice Zone Experiment observations: $M_a = m_1 \Delta T^{m_2}$, where $m_1 = 1.6 \times 10^{-6}$m s^{-1} deg^{m_2} and $m_2 = 1.36$. The lead-ice perimeter depends on the ice floe distribution and geometry. For a mean floe diameter d and number of floes n_f, $p_f = n_f \pi d$ and the floe area $A_f = n_f \eta_{lm} d^2$ (Rothrock and Thordike, 1984; Bitz 2000). As the heat flux for lateral melt is $E_{tot}(p_f/A_f)M_a$, the actual amount used is:

$$F_{SID} = max(\frac{E_{tot}\pi}{\eta_{lm} d} m_1 \Delta T^{m_2}, f_{sid}F_{Qoi}) \tag{135}$$

where $\eta_{lm} = 0.66$ (Rothrock and Thordike, 1984). Based partially on tuning and partially on the results of floe distribution measurements, the mean floe diameter of d=300 m was chosen. The ice area, volume, snow volume, and ice energy are all reduced by side melt in time Δt by the fraction $R_{side} = |\frac{F_{SID}\Delta t}{E_{tot}}|$.

The heat flux available that is actually used by the ice model is:

$$F_{Qio} = F_{BOT} + F_{SID} \tag{136}$$

4.9 Linear Remapping

In this section we evaluate the first right-hand side term in the distribution equation (Eq. 1) due to thermodynamic processes, which can be thought of as a transport in thickness space:

$$\frac{\partial g}{\partial t} = -\frac{\partial}{\partial h}(\dot{h}g) \tag{137}$$

where $\dot{h} = dh/dt$. To evaluate this term we use the linear remapping method of Lipscomb (2001). The method is similar to the 1D version of the incremental remapping algorithm of Dukowicz and Baumgardner (2000).

The linear remapping method uses the integral form of the distribution equation. Integrating the above transport equation between $h^*_{n-1}(t)$ and $h^*_n(t)$, where the boundary thicknesses $h^*_n, \{n = 0, 1, 2...N\}$ are time dependent (i.e. following the motion), results in

$$\int_{h^*_{n-1}(t)}^{h^*_n(t)} \frac{\partial g}{\partial t} dh + \int_{h^*_{n-1}(t)}^{h^*_n(t)} \frac{\partial}{\partial h}(\dot{h}g) dh = 0. \tag{138}$$

Evaluating the second term yields

$$\int_{h^*_{n-1}(t)}^{h^*_n(t)} \frac{\partial}{\partial h}(\dot{h}g) dh = g(h^*_n) \left.\frac{dh^*}{dt}\right|_n - g(h^*_{n-1}) \left.\frac{dh^*}{dt}\right|_{n-1}. \tag{139}$$

The integrated transport equation can be rewritten as

$$\frac{d}{dt} \int_{h^*_{n-1}(t)}^{h^*_n(t)} g(h) dh = 0, \tag{140}$$

or using Eq. 2

$$\frac{dA_n}{dt} = 0. \tag{141}$$

This equation can be interpreted in a Lagrangian sense as a conservation equation, where the time dependent limits are the boundaries of the Lagrangian volume following the motion in thickness space. Differentiating the volume ($V_n = A_n h_n$) with time we have

$$\frac{dV_n}{dt} = A_n \frac{dh_n}{dt}. \tag{142}$$

The linear remapping method calculates the new Lagrangian boundaries, linearly approximates the distribution function, and transfers ice area and volume to restore the original thickness boundaries. $g(h)$ is approximated by a series of linear piecewise continuous functions, and ice is transferred in small increments between categories.

The original ice thickness in category n is h_n^m. After the thermodynamic (both vertical and lateral) changes to ice thickness are computed, the new ice thickness is h_n^{m+1}. Let the growth rate of ice thickness in category n be $f_n = \dot{h}_n = (h_n^{m+1} - h_n^m)/\Delta t$. The $m+1$ growth rates at h_n^{*m} are estimated by interpolating between neighboring values of f_n:

$$f_n^* = f_n + \frac{(f_{n+1} - f_n)}{(h_{n+1}^m - h_n^m)}(h_n^{*m} - h_n^m) \tag{143}$$

The new boundary locations are then: $h_n^{*m+1} = h_n^{*m} + f_n^* \Delta t$. Note that in principle the boundaries can shift any distance, but we require here that $h_{n-1}^{m+1} < h_n^{*m+1} < h_{n+1}^{m+1}$.

If any category has no ice (i.e. $A_n = 0$), while an adjacent category does ($A_{n+1} \neq 0$), the boundary is adjusted by the same amount as the thickness in the non-zero category: $h_n^{*m+1} = h_n^{*m} + f_{n+1}^* \Delta t$.

We approximate each Lagrangian volume linearly as $g_n(h) = g_{0n} + g_{1n} h$. To evaluate the coefficients $\{g_{0n}, g_{1n}\}$, we make use of the area and volume constraints, the requirement of a positive distribution function, and the minimum and maximum boundary conditions. The area and volume constraints are

$$\int_{h_{n-1}^{*m}}^{h_n^{*m}} g(h)dh = \int_{h_{n-1}^{*m+1}}^{h_n^{*m+1}} g(h)dh = A_n^m \tag{144}$$

$$\int_{h_{n-1}^{*m+1}}^{h_n^{*m+1}} hg(h)dh = A_n^m h_n^{m+1} \tag{145}$$

Note that the area is conserved following the motion (i.e A_n^m is a constant) but the volume changes to $A_n^m h_n^{m+1}$.

Let us redefine the n^{th} category boundaries at time step $m+1$ as $h_L = h_{n-1}^{*m+1}$ and $h_R = h_n^{*m+1}$ (henceforth we drop the time and category indices). A positive linear function is constructed such that the area and volume integrals over the Lagrangian volume are satisfied. We transform variables from h to η (a relative coordinate): for each Lagrangian volume $\eta = h - h_L$ and $\eta_n = h_n - h_L$ so that $g(\eta) = g_0 + g_1 \eta$, where η ranges from 0 to $\eta_R = h_R - h_L$. Note that the h_n in $\eta_n = h_n - h_L$ is the $m+1$ value h_n^{m+1}.

$$A_n = \int_0^{\eta_R} g(\eta)d\eta = \frac{1}{2}{\eta_R}^2 g_1 + \eta_R g_0 \tag{146}$$

$$A_n h_n = \int_0^{\eta_R} (\eta + h_L)(g_1 \eta + g_0)d\eta = h_L A_n + \frac{1}{3}{\eta_R}^3 g_1 + \frac{1}{2}{\eta_R}^2 g_0 \tag{147}$$

We have two linear equations for g_{0n} and g_{1n} for the n^{th} category

$$\frac{1}{2}{\eta_R}^2 g_{1n} + \eta_R g_{0n} = A_n \tag{148}$$

$$\frac{1}{3}{\eta_R}^3 g_{1n} + \frac{1}{2}{\eta_R}^2 g_{0n} = A_n \eta_n \tag{149}$$

which have the solution

$$g_{1n} = \frac{12A_n}{{\eta_R}^3}(\eta_n - \frac{\eta_R}{2}) \tag{150}$$

$$g_{0n} = \frac{6A_n}{{\eta_R}^2}(\frac{2\eta_R}{3} - \eta_n). \tag{151}$$

Note that the sign of g_{1n} is determined by $\eta_n - \frac{\eta_R}{2} = h_n - h_L - \frac{h_R - h_L}{2} = h_n - \frac{1}{2}(h_L + h_R)$. When h_n is greater than the Lagrangian midpoint, the slope is positive; when it is less, it is negative.

As g is linear, its maximum and minimum lie at the boundaries $\eta = 0$ and $\eta = \eta_R$

$$g(0) = \frac{6A_n}{{\eta_R}^2}(\frac{2\eta_R}{3} - \eta_n) \tag{152}$$

and

$$g(\eta_R) = \frac{6A_n}{{\eta_R}^2}(\eta_n - \frac{\eta_R}{3}) \tag{153}$$

For $g(\eta)$ to be positive, both boundary values must be positive. $g(0)$ is less than zero when $(\frac{2\eta_R}{3} - \eta_n) < 0$, or $\eta_n > \frac{2\eta_R}{3}$, and $g(\eta_R)$ is less than zero when $\eta_n < \frac{\eta_R}{3}$, i.e. whenever η_n lies outside the central third of the Lagrangian thickness range. As just noted, whenever h_n is greater than the range mid point, the slope $g_{1n} > 0$; if it is greater than $h_L + \frac{2}{3}(h_R - h_L)$, the slope is so great that the minimum value at $g(0)$ falls below zero; and conversely for $h_n < h_L + \frac{1}{3}(h_R - h_L)$, for which the slope becomes so negative as to require $g(\eta_R)$ to be less than zero.

For the case when h_n falls in the first third of the Lagrangian thickness range, we redefine the upper limit to h'_R as: $h_n = h_L + \frac{1}{3}(h'_R - h_L)$ or $h'_R = 3h_n - 2h_L$ and set $g = 0$ between h'_R and h_R. Similarly, when h_n lies in the upper third of the Lagrangian thickness range, the lower limit is redefined to h'_L as: $h_n = h'_L + \frac{2}{3}(h_R - h'_L)$ or $h'_L = 3h_n - 2h_R$ and set $g = 0$ between h_L and h'_L. In either case, the solutions for g_{0n} and g_{1n} can still be used as long as the appropriate boundaries are defined.

Once $g(h)$ is constructed for each category, the thickness distribution is remapped to the original category boundaries by transferring the appropriate area ΔA_n and volume ΔV_n. If the displaced boundary h_n^{*m+1} has moved to the right, then:

$$\begin{aligned} \Delta A_n^{m+1} &= \int_{h_n^{*m}}^{h_n^{*m+1}} g_n(h)dh \\ \Delta V_n^{m+1} &= \int_{h_n^{*m}}^{h_n^{*m+1}} h g_n(h)dh \end{aligned} \tag{154}$$

If the displaced boundary has moved to the left, the limits of integration are reversed.

The thinnest (i.e. $n = 1$) and thickest (i.e. $n = N$) categories have special minimum and maximum boundary conditions respectively. For category 1, if sea ice is growing in open water at a positive rate f_0, shift h_1^* to the right by $f_0 \Delta t$. If sea ice is not growing in open water, approximate the growth rate as $f_0 = f_1$. If $f_0 < 0$, reduce the ice area by the integral of $g(h)$ from 0 to $-f_0 \Delta t$, leaving the ice volume fixed, as ice volume cannot cross the left boundary. For the right boundary, h_N^* varies with h_N. As $g(h)$ is linear, setting $h_N^* = 3h_N - 2h_{N-1}^*$ ensures $g(h_N^*) = 0$.

Snow volume, internal energy of ice and surface temperature are affected by thickness space transport as follows. Assuming that within each category snow depth varies linearly with ice thickness, the snow volume transport is proportional to the ice volume transport (see Section 2.2). The internal energy of ice is $E_n = q_n V_n$, where q_n is the energy of melting of ice, a function of ice temperature and salinity. The internal energy of ice is proportional to the volume of ice, so that the new internal energy at $m+1$ is $E_n^{m+1} = q_n^{m+1} V_n^{m+1}$, where q_n^{m+1} is the $m+1$ energy of melting after the vertical thermodynamic heat transfer has been computed. The surface temperature T_{sn}^m changes with area due to thickness space transport.

4.10 Velocity

Pack ice is composed of rigid ice floes, ranging in size from order 1 m to greater than 10 km. The characteristics of motion for the pack ice are discontinuous slippage near shore, near rigid motion under considerable wind forcing (i.e. nearly rate-independent stress), small or zero tensile strength for both uniaxial and two dimensional dilation, and high compressive strength.

To model this material, the resolved sea-ice is considered to be a highly fractured, closely packed, isotropic medium in which inter-floe forces are contact stresses. The resolved spatial scales are considered to be much larger than the scale of inhomogeneities (i.e. floes). We consider only aggregate ice motion (across the ITD) in each grid box. Thus, we model sea-ice as a two dimensional continuum, whose momentum conservation is described by

$$\bar{m}\frac{\partial u}{\partial t} = -\bar{m} f k \times u + \tau_a + \tau_o + \bar{m} g_e \nabla H_o + \nabla \cdot \sigma \tag{155}$$

where $\bar{m}$ is the total mass of snow and ice per unit area given by

$$\bar{m} = \rho_s \sum_{n=1}^{N} V_{sn} + \rho_i \sum_{n=1}^{N} V_n, \tag{156}$$

the non-linear u advection terms are ignored as they are negligibly small when the equations are scaled, f is the Coriolis parameter, k is the local vertical unit vector, τ_a and τ_o are forces due to air and water stresses respectively, g_e is the gravitational acceleration, H_o is the sea surface slope and $\nabla \cdot \sigma$ is the force due to internal ice stress.

The air-ice stress τ_a is described in Section 4.5 (Eqs. 41,42 and following) with aggregate

$\tau_a = (1/A)\Sigma_{n=1}^{N} \tau_{an} A_n$. The ocean-ice stress τ_o is in the form of a nonlinear drag law

$$\tau_o = \rho_o c_d |u_o - u|(u_o - u) \tag{157}$$

where c_d is the ocean-ice drag coefficient, ρ_o is the density of seawater and u_o is the surface ocean current. Note that while the drag coefficient c_d varies with ice thickness for actual ice floes, here we assume it to be constant.

We note the following modification to the usual two dimensional continuum equation used above for momentum conservation (Eq. 155), that better approximates point-wise free drift theory. That theory states the actual (rather than cell-averaged) ice velocity should be uniform under uniform forcing, regardless of ice area. As written, Eq. 155 will give different solutions under free drift (i.e. $\nabla \cdot \sigma = 0$) for different sea ice areas but constant ice thickness (i.e. $\bar{m}$ is proportional to sea ice area for constant thickness). The modification is to multiply the air-ice and ocean-ice stresses by the concentration A, specifically, use $\sum_{n=1}^{N} A_n \tau_{an} + A\tau_o$ in place of the $\tau_a + \tau_o$ terms in Eq. 155, so that the solutions to the equation better approximate velocity for the sea ice itself, rather than cell-average. For a complete description of this modification, see Hunke and Dukowicz (2003) and Connolley et.al (2004).

The internal stress tensor σ is a linear vector function of a vector argument, which gives the internal force on the material for a specified direction. The general constitutive law for sea-ice relates the stress to the rate of strain, and generally includes elastic (linear and reversible) and plastic (non-linear and irreversible) components.

The CSM1 sea ice model assumes a cavitating fluid rheology, in which both elastic, shear and tensile stresses are ignored (Bettge et al. 1996), and the ocean-ice stress is linearized. The model suffers numerical grid convergence difficulties near the North Pole (Weatherly et al. 1998). The cavitating fluid rheology is useful in some circumstances, but is limited especially due to lack of shear stresses. A more realistic and generally accepted rheology is the viscous-plastic, or VP (Hibler 1979; Kreyscher et al., 2000).

The VP rheology derives from the general stress-strain relation for viscous fluids

$$\sigma_{ij} = 2\eta\dot{\epsilon}_{ij} + (\zeta - \eta)\dot{\epsilon}_{kk}\delta_{ij} - \frac{P}{2}\delta_{ij} \tag{158}$$

where the stress tensor is σ_{ij} ($i, j =$ component indices), the compressive strength P, δ_{ij} is the Kronecker delta and the rate of strain tensor is

$$\dot{\epsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{159}$$

the total linear rate of strain (divergence) is $\dot{\epsilon}_{kk} = \dot{\epsilon}_{11} + \dot{\epsilon}_{22}$ and $\dot{\epsilon}_{12} = \dot{\epsilon}_{21}$ is the shear rate of strain component. ζ and η are bulk (i.e. linear) and shear viscosities respectively. The general form of this plastic rheology satisfies the condition that the deformational part of the plastic stress tensor vanishes for constant u and that for solid body rotation no stress is produced. For the viscous flow to be dissipative both ζ and η must be positive.

The **plastic** assumption is that the flow obeys an idealized plastic behavior, namely, that stressed ice is motionless until a yield stress is obtained, after which the flow is irreversible and rate independent. The principal stress states for plastic deformation lie on the yield curve specified by a normalized convex yield function, while the (irreversible) deformation is given by the normal flow rule.

The VP rheology assumes an elliptical yield curve of specified ratio of major to minor axes e. In terms of the stress and strain rate (or deformation) invariants, obtained by diagonalizing and transforming the symmetric stress (σ_I,σ_{II}) and rate of strain tensors ($\dot{\epsilon}_I$,$\dot{\epsilon}_{II}$) into pure compression (I) and shear (II) components, we have the yield function

$$Y(\sigma_I, \sigma_{II}) = \frac{\left(\sigma_I + \frac{P}{2}\right)^2}{\left(\frac{P}{2}\right)^2} + \frac{\sigma_{II}^2}{\left(\frac{P}{2e}\right)^2} = 1 \tag{160}$$

which is chosen to lie in the lower left quadrant of principal stress space, corresponding to very small tensile stress but with finite compressional and shear stresses (the latter if e is relatively small). With the normal flow rule $\dot{\epsilon}_I = \lambda \frac{\partial Y}{\partial \sigma_I}$, and $\dot{\epsilon}_{II} = \lambda \frac{\partial Y}{\partial \sigma_{II}}$, the unknown λ can be computed, resulting in

$$\sigma_I = \zeta \dot{\epsilon}_I - P/2 \tag{161}$$

$$\sigma_{II} = \eta \dot{\epsilon}_{II} \tag{162}$$

$$\zeta = P/2\Delta \tag{163}$$

$$\eta = \zeta/e^2 \tag{164}$$

where

$$\dot{\epsilon}_I = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \tag{165}$$

$$\dot{\epsilon}_{II} = \{(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y})^2 + (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})^2\}^{\frac{1}{2}} \tag{166}$$

(Stern et al., 1995), and

$$\Delta = \left[\dot{\epsilon}_I^2 + \dot{\epsilon}_{II}^2/e^2\right]^{1/2}. \tag{167}$$

In the limit of zero strain rate (i.e. rigid solid with no deformation), the viscosities become infinite. To regularize this behavior, the viscosities are bounded for sufficiently small strain rates so that the sea ice moves as a linear viscous fluid undergoing slow creep. Minimum viscosities are set to prevent non-linear instabilities.

The **elastic viscous plastic** (EVP) rheology (Hunke and Dukowicz 1997) derives from the simple stress-strain relation for small strains: $\sigma_{ij} = E\epsilon_{ij}$, where E (analogous to Young's modulus) is related to ice strength such that it increases as ice strength increases. Writing this relation in terms of rate of strain, and ignoring the non-linear advection as in the momentum equation, gives $\frac{1}{E}\frac{\partial \sigma_{ij}}{\partial t} = \dot{\epsilon}_{ij}$. The stress tensor equation (Eq. 158) for viscous fluids can be solved in terms of the rate of strain, as

$$\dot{\epsilon}_{ij} = \frac{1}{2\eta}\sigma_{ij} + \frac{(\eta-\zeta)}{4\eta\zeta}\sigma_{kk}\delta_{ij} + \frac{P}{4\zeta}\delta_{ij} \tag{168}$$

Combining these two rates of strain (elastic and plastic) to yield the total rate of strain results in

$$\dot{\epsilon}_{ij} = \frac{1}{E}\frac{\partial \sigma_{ij}}{\partial t} + \frac{1}{2\eta}\sigma_{ij} + \frac{\eta-\zeta}{4\eta\zeta}\sigma_{kk}\delta_{ij} + \frac{P}{4\zeta}\delta_{ij} \tag{169}$$

In the limit $E \to \infty$ this rate of strain equation asymptotes to the pure VP rheology, while for $\eta, \zeta \to \infty$, the purely elastic rheology is recovered. Hence, as $\eta, \zeta \to \infty$ under conditions of very small strain rate, the elastic term controls the solution behavior, and represents a regularization of the VP rheology. Note that the elastic term in the stress tensor equation requires that the stress tensor components become prognostic variables in EVP. This is in contrast to VP for which the stress tensor components are diagnostic. The elastic parameter E is given in terms of the bulk viscosity and a damping time scale for elastic waves T_{ew} and time step Δt

$$E = \zeta / T_{ew} \tag{170}$$

$$T_{ew} = E_0 \Delta t. \tag{171}$$

where E_0 is a constant less than 1. The momentum and stress tensor equations are

$$\bar{m}\frac{\partial u}{\partial t} + c'u - \bar{m}fv = c'u_o + \tau_{ax} - \bar{m}g_e\frac{\partial H_o}{\partial x} + F_x \tag{172}$$

$$\bar{m}\frac{\partial v}{\partial t} + c'v + \bar{m}fu = c'v_o + \tau_{ay} - \bar{m}g_e\frac{\partial H_o}{\partial y} + F_y \tag{173}$$

$$\frac{\partial \sigma_{ij}}{\partial t} + \frac{e^2}{2T_{ew}}\sigma_{ij} + \frac{1-e^2}{4T_{ew}}\sigma_{kk}\delta_{ij} = \frac{P}{2T_{ew}\Delta'}\dot{\epsilon}_{ij} - \frac{P}{4T_{ew}}\delta_{ij} \tag{174}$$

where $c' = \rho_o c_d |u_o - u|$ and $\Delta' = max(\Delta, \Delta_{min})$ and if $\Delta < \Delta_{min}$, then P is replaced by $P\Delta/\Delta_{min}$, which prevents residual ice motion due to spatial variations in P for extremely small or zero rates of strain, where $\Delta_{min} = 10^{-13}A^t$, A^t is the T-grid area, section 3.4. The stress divergence terms F_x and F_y are evaluated for a general orthogonal curvilinear coordinate system subject to the contraints that the discretization be dissipative and includes grid curvature effects (see Hunke and Dukowicz, 2002).

It is convenient to introduce the divergence D_D, the horizontal tension D_T and shearing D_S strain rates defined by:

$$D_D = \dot{\epsilon}_{11} + \dot{\epsilon}_{22} \tag{175}$$

$$D_T = \dot{\epsilon}_{11} - \dot{\epsilon}_{22} \tag{176}$$

$$D_S = 2\dot{\epsilon}_{12} \tag{177}$$

Letting $\sigma_1 = \sigma_{11} + \sigma_{22}$ and $\sigma_2 = \sigma_{11} - \sigma_{22}$, Eqs. 174 can be alternatively expressed as:

$$\frac{1}{E}\frac{\partial \sigma_1}{\partial t} + \frac{\sigma_1}{2\zeta} + \frac{P}{2\zeta} = D_D \tag{178}$$

$$\frac{1}{E}\frac{\partial \sigma_2}{\partial t} + \frac{\sigma_2}{2\eta} = D_T \tag{179}$$

$$\frac{1}{E}\frac{\partial \sigma_{12}}{\partial t} + \frac{\sigma_{12}}{2\eta} = \frac{1}{2}D_S. \tag{180}$$

where we note that $\sigma_I = \sigma_1/2$, $\sigma_{II} = \sqrt{\sigma_2^2/4 + \sigma_{12}^2}$, $\dot\epsilon_I = D_D$, $\dot\epsilon_{II} = \sqrt{D_T^2 + D_S^2}$, the above definitions of ζ and η are unchanged, while Δ is:

$$\Delta = \left[D_D^2 + \frac{1}{e^2}\left(D_T^2 + D_S^2\right)\right]^{1/2}. \tag{181}$$

Multiplying the momentum equations by u and v respectively and summing, one can form the kinetic energy equation. Using the product rule, $u_i\frac{\partial \sigma_{ij}}{\partial x_j} = \frac{\partial}{\partial x_j}(u_i\sigma_{ij}) - \sigma_{ij}\frac{\partial u_i}{\partial x_j} = \frac{\partial}{\partial x_j}(u_i\sigma_{ij}) - \sigma_{ij}\dot\epsilon_{ij}$, where repeated indices imply summation over $(1,2)$. The area integral of the $\frac{\partial}{\partial x_j}(u_i\sigma_{ij})$ term will vanish on lateral boundaries where $u_i = 0$, or for open ocean where $\sigma_{ij} = 0$. Thus, area integrals of the kinetic energy equation over the ice will result in dissipative internal stresses so long as D, defined by:

$$D = \int (\sigma_{11}\dot\epsilon_{11} + 2\sigma_{12}\dot\epsilon_{12} + \sigma_{22}\dot\epsilon_{22})\, dA, \tag{182}$$

is positive definite. Using the definitions of D_D, D_T and D_S we have:

$$D = \int \left[\frac{1}{2}\sigma_1 D_D + \frac{1}{2}\sigma_2 D_T + \sigma_{12} D_S\right] dA$$

In steady state ($\frac{\partial \sigma}{\partial t} \to 0$), Eqs 178, 179 and 180 reduce to the viscous-plastic constitutive law. Using the steady state forms for σ_1, σ_2 and σ_{12}, D can be written as:

$$D = \frac{1}{2}\int P\,(\Delta - D_D)\, dA \geq 0, \tag{183}$$

which ensures that the work done by the internal stress is dissipative. Note that D is a scalar invariant independent of coordinate system.

To include grid curvature effects, consider an orthogonal, curvilinear coordinate system, and let u and v represent velocity components along nondimensional coordinates ξ_1 and ξ_2 (0 to 1 across a grid box), with scale factors (the physical lengths of the grid box sides) h_1 and h_2, respectively. The rate of strain components are then:

$$\dot\epsilon_{11} = \frac{1}{h_1}\left(\frac{\partial u}{\partial \xi_1} + \frac{v}{h_2}\frac{\partial h_1}{\partial \xi_2}\right) \tag{184}$$

$$\dot\epsilon_{22} = \frac{1}{h_2}\left(\frac{\partial v}{\partial \xi_2} + \frac{u}{h_1}\frac{\partial h_2}{\partial \xi_1}\right) \tag{185}$$

$$\dot{\epsilon}_{12} = \frac{1}{2}[\frac{h_1}{h_2}\frac{\partial}{\partial \xi_2}\left(\frac{u}{h_1}\right) + \frac{h_2}{h_1}\frac{\partial}{\partial \xi_1}\left(\frac{v}{h_2}\right)]. \tag{186}$$

Using these expressions in the dissipation D (Eq. 182) and integrating by parts, orthogonal curvilinear forms for the stress divergence are derived:

$$F_x = \frac{1}{2}\left[\frac{1}{h_1}\frac{\partial \sigma_1}{\partial \xi_1} + \frac{1}{h_1 h_2^2}\frac{\partial}{\partial \xi_1}\left(h_2^2 \sigma_2\right) + \frac{2}{h_1^2 h_2}\frac{\partial}{\partial \xi_2}\left(h_1^2 \sigma_{12}\right)\right] \tag{187}$$

$$F_y = \frac{1}{2}\left[\frac{1}{h_2}\frac{\partial \sigma_1}{\partial \xi_2} - \frac{1}{h_1^2 h_2}\frac{\partial}{\partial \xi_2}\left(h_1^2 \sigma_2\right) + \frac{2}{h_1 h_2^2}\frac{\partial}{\partial \xi_1}\left(h_2^2 \sigma_{12}\right)\right]. \tag{188}$$

The discretization of the velocity and stress tensor components is termed the "bilinear discretization" (Hunke and Dukowicz, 2002). For example, the velocity components are expressed in terms of the grid box vertex values and non-dimensional coordinates (0 to 1 across the grid box) as:

$$u\left(\xi_1,\xi_2\right) = u^{ne}\xi_1\xi_2 + u^{nw}\left(1-\xi_1\right)\xi_2 + u^{sw}\left(1-\xi_1\right)\left(1-\xi_2\right) + u^{se}\xi_1\left(1-\xi_2\right) \tag{189}$$

$$v\left(\xi_1,\xi_2\right) = v^{ne}\xi_1\xi_2 + v^{nw}\left(1-\xi_1\right)\xi_2 + v^{sw}\left(1-\xi_1\right)\left(1-\xi_2\right) + v^{se}\xi_1\left(1-\xi_2\right). \tag{190}$$

where the four grid box velocities are referred to as ne, nw, sw, se for northeast, northwest, southwest and southeast respectively. Note that velocity is continuous across cell edges (for example, $u_{ij}^{ne} = u_{i+1j}^{nw}$, where ij now represent grid indices). The stress tensor components, associated with velocity gradients through strain rates, are discontinuous, with each cell having four corner values for stress. This method of discretization suppresses B-grid checkerboard solutions, because it is not technically B-grid, since we do not have one grid box center value for the stress tensor components.

The scale factors are evaluated at grid center by averaging the two appropriate sides ($h_1 = \bar{h}_1,\ h_2 = \bar{h}_2$), while scale factor spatial derivatives are simple differences in the grid side lengths ($\partial h_1/\partial \xi_2 = \Delta_2 h_1,\ \partial h_2/\partial \xi_1 = \Delta_1 h_2$). Using the bilinear discretization, the strain rate terms are evaluated as follows:

Divergence

$$D_D^{ne} = \frac{1}{\bar{h}_1\bar{h}_2}\left[\bar{h}_2\left(u^{ne}-u^{nw}\right) + \Delta_1 h_2 u^{ne} + \bar{h}_1\left(v^{ne}-v^{se}\right) + \Delta_2 h_1 v^{ne}\right]$$

$$D_D^{nw} = \frac{1}{\bar{h}_1\bar{h}_2}\left[\bar{h}_2\left(u^{ne}-u^{nw}\right) + \Delta_1 h_2 u^{nw} + \bar{h}_1\left(v^{nw}-v^{sw}\right) + \Delta_2 h_1 v^{nw}\right]$$

$$D_D^{se} = \frac{1}{\bar{h}_1\bar{h}_2}\left[\bar{h}_2\left(u^{se}-u^{sw}\right) + \Delta_1 h_2 u^{se} + \bar{h}_1\left(v^{ne}-v^{se}\right) + \Delta_2 h_1 v^{se}\right]$$

$$D_D^{sw} = \frac{1}{\bar{h}_1\bar{h}_2}\left[\bar{h}_2\left(u^{se}-u^{sw}\right) + \Delta_1 h_2 u^{sw} + \bar{h}_1\left(v^{nw}-v^{sw}\right) + \Delta_2 h_1 v^{sw}\right]$$

Tension

$$D_T^{ne} = \frac{1}{\bar{h}_1\bar{h}_2}\left[\bar{h}_2\left(u^{ne}-u^{nw}\right) - \Delta_1 h_2 u^{ne} - \bar{h}_1\left(v^{ne}-v^{se}\right) + \Delta_2 h_1 v^{ne}\right]$$

$$D_T^{nw} = \frac{1}{\bar{h}_1\bar{h}_2}\left[\bar{h}_2\left(u^{ne} - u^{nw}\right) - \Delta_1 h_2 u^{nw} - \bar{h}_1\left(v^{nw} - v^{sw}\right) + \Delta_2 h_1 v^{nw}\right]$$

$$D_T^{se} = \frac{1}{\bar{h}_1\bar{h}_2}\left[\bar{h}_2\left(u^{se} - u^{sw}\right) - \Delta_1 h_2 u^{se} - \bar{h}_1\left(v^{ne} - v^{se}\right) + \Delta_2 h_1 v^{se}\right]$$

$$D_T^{sw} = \frac{1}{\bar{h}_1\bar{h}_2}\left[\bar{h}_2\left(u^{se} - u^{sw}\right) - \Delta_1 h_2 u^{sw} - \bar{h}_1\left(v^{nw} - v^{sw}\right) + \Delta_2 h_1 v^{sw}\right]$$

Shearing

$$D_S^{ne} = \frac{1}{\bar{h}_1\bar{h}_2}\left[\bar{h}_1\left(u^{ne} - u^{se}\right) - \Delta_2 h_1 u^{ne} + \bar{h}_2\left(v^{ne} - v^{nw}\right) - \Delta_1 h_2 v^{ne}\right]$$

$$D_S^{nw} = \frac{1}{\bar{h}_1\bar{h}_2}\left[\bar{h}_1\left(u^{nw} - u^{sw}\right) - \Delta_2 h_1 u^{nw} + \bar{h}_2\left(v^{ne} - v^{nw}\right) - \Delta_1 h_2 v^{nw}\right]$$

$$D_S^{se} = \frac{1}{\bar{h}_1\bar{h}_2}\left[\bar{h}_1\left(u^{ne} - u^{se}\right) - \Delta_2 h_1 u^{se} + \bar{h}_2\left(v^{se} - v^{sw}\right) - \Delta_1 h_2 v^{se}\right]$$

$$D_S^{sw} = \frac{1}{\bar{h}_1\bar{h}_2}\left[\bar{h}_1\left(u^{nw} - u^{sw}\right) - \Delta_2 h_1 u^{sw} + \bar{h}_2\left(v^{se} - v^{sw}\right) - \Delta_1 h_2 v^{sw}\right]$$

For the divergence of the stress tensor, we note that for the ne corner of grid box ij, there are contributions from the four surrounding grid boxes (i.e. northeast corner of ij, southeast corner of $ij+1$, northwest corner of $i+1j$ and the southwest corner of $i+1j+1$). The terms below show these contributions to the stress divergence for the northeast corner of grid box ij. Hence, for the contributions of each grid box, the corner designations of σ are relative to that box.

Contribution of the σ_1 term to F_x

$$\begin{aligned}
\frac{1}{\bar{h}_1\bar{h}_2}\Bigg\{&\left[-\frac{\bar{h}_2}{4}\left(\frac{1}{3}\left(\sigma_1^{ne} + \sigma_1^{nw}\right) + \frac{1}{6}\left(\sigma_1^{se} + \sigma_1^{sw}\right)\right) - \frac{\Delta_1 h_2}{2}\left(\frac{1}{9}\sigma_1^{ne} + \frac{1}{18}\left(\sigma_1^{nw} + \sigma_1^{se}\right) + \frac{1}{36}\sigma_1^{sw}\right)\right]_{ij} \\
&+ \left[\frac{\bar{h}_2}{4}\left(\frac{1}{3}\left(\sigma_1^{ne} + \sigma_1^{nw}\right) + \frac{1}{6}\left(\sigma_1^{se} + \sigma_1^{sw}\right)\right) - \frac{\Delta_1 h_2}{2}\left(\frac{1}{9}\sigma_1^{nw} + \frac{1}{18}\left(\sigma_1^{sw} + \sigma_1^{ne}\right) + \frac{1}{36}\sigma_1^{se}\right)\right]_{i+1j} \\
&+ \left[-\frac{\bar{h}_2}{4}\left(\frac{1}{3}\left(\sigma_1^{se} + \sigma_1^{sw}\right) + \frac{1}{6}\left(\sigma_1^{ne} + \sigma_1^{nw}\right)\right) - \frac{\Delta_1 h_2}{2}\left(\frac{1}{9}\sigma_1^{se} + \frac{1}{18}\left(\sigma_1^{ne} + \sigma_1^{sw}\right) + \frac{1}{36}\sigma_1^{nw}\right)\right]_{ij+1} \\
&+ \left[\frac{\bar{h}_2}{4}\left(\frac{1}{3}\left(\sigma_1^{se} + \sigma_1^{sw}\right) + \frac{1}{6}\left(\sigma_1^{ne} + \sigma_1^{nw}\right)\right) - \frac{\Delta_1 h_2}{2}\left(\frac{1}{9}\sigma_1^{sw} + \frac{1}{18}\left(\sigma_1^{nw} + \sigma_1^{se}\right) + \frac{1}{36}\sigma_1^{ne}\right)\right]_{i+1j+1}\Bigg\}
\end{aligned}$$

Contribution of the σ_1 term to F_y

$$\begin{aligned}
\frac{1}{\bar{h}_1\bar{h}_2}\Bigg\{&\left[-\frac{\bar{h}_1}{4}\left(\frac{1}{3}\left(\sigma_1^{ne} + \sigma_1^{se}\right) + \frac{1}{6}\left(\sigma_1^{nw} + \sigma_1^{sw}\right)\right) - \frac{\Delta_2 h_1}{2}\left(\frac{1}{9}\sigma_1^{ne} + \frac{1}{18}\left(\sigma_1^{nw} + \sigma_1^{se}\right) + \frac{1}{36}\sigma_1^{sw}\right)\right]_{ij} \\
&+ \left[-\frac{\bar{h}_1}{4}\left(\frac{1}{3}\left(\sigma_1^{nw} + \sigma_1^{sw}\right) + \frac{1}{6}\left(\sigma_1^{ne} + \sigma_1^{se}\right)\right) - \frac{\Delta_2 h_1}{2}\left(\frac{1}{9}\sigma_1^{nw} + \frac{1}{18}\left(\sigma_1^{sw} + \sigma_1^{ne}\right) + \frac{1}{36}\sigma_1^{se}\right)\right]_{i+1j} \\
&+ \left[\frac{\bar{h}_1}{4}\left(\frac{1}{3}\left(\sigma_1^{ne} + \sigma_1^{se}\right) + \frac{1}{6}\left(\sigma_1^{nw} + \sigma_1^{sw}\right)\right) - \frac{\Delta_2 h_1}{2}\left(\frac{1}{9}\sigma_1^{sw} + \frac{1}{18}\left(\sigma_1^{ne} + \sigma_1^{sw}\right) + \frac{1}{36}\sigma_1^{nw}\right)\right]_{ij+1}
\end{aligned}$$

$$+\left[\frac{\bar h_1}{4}\left(\frac{1}{3}\left(\sigma_1^{nw}+\sigma_1^{sw}\right)+\frac{1}{6}\left(\sigma_1^{ne}+\sigma_1^{se}\right)\right)\;-\;\frac{\Delta_2 h_1}{2}\left(\frac{1}{9}\sigma_1^{sw}+\frac{1}{18}\left(\sigma_1^{nw}+\sigma_1^{se}\right)+\frac{1}{36}\sigma_1^{ne}\right)\right]_{i+1j+1}\Bigg\}$$

Contribution of the σ_2 term to F_x

$$\frac{1}{\bar h_1\bar h_2}\Bigg\{\left[-\frac{\bar h_2}{4}\left(\frac{1}{3}\left(\sigma_2^{ne}+\sigma_2^{nw}\right)+\frac{1}{6}\left(\sigma_2^{se}+\sigma_2^{sw}\right)\right)\;+\;\frac{\Delta_1 h_2}{2}\left(\frac{1}{9}\sigma_2^{ne}+\frac{1}{18}\left(\sigma_2^{nw}+\sigma_2^{se}\right)+\frac{1}{36}\sigma_2^{sw}\right)\right]_{ij}$$

$$+\left[\frac{\bar h_2}{4}\left(\frac{1}{3}\left(\sigma_2^{ne}+\sigma_2^{nw}\right)+\frac{1}{6}\left(\sigma_2^{se}+\sigma_2^{sw}\right)\right)\;+\;\frac{\Delta_1 h_2}{2}\left(\frac{1}{9}\sigma_2^{nw}+\frac{1}{18}\left(\sigma_2^{sw}+\sigma_2^{ne}\right)+\frac{1}{36}\sigma_2^{se}\right)\right]_{i+1j}$$

$$+\left[-\frac{\bar h_2}{4}\left(\frac{1}{3}\left(\sigma_2^{se}+\sigma_2^{sw}\right)+\frac{1}{6}\left(\sigma_2^{ne}+\sigma_2^{nw}\right)\right)\;+\;\frac{\Delta_1 h_2}{2}\left(\frac{1}{9}\sigma_2^{se}+\frac{1}{18}\left(\sigma_2^{ne}+\sigma_2^{sw}\right)+\frac{1}{36}\sigma_2^{nw}\right)\right]_{ij+1}$$

$$+\left[\frac{\bar h_2}{4}\left(\frac{1}{3}\left(\sigma_2^{se}+\sigma_2^{sw}\right)+\frac{1}{6}\left(\sigma_2^{ne}+\sigma_2^{nw}\right)\right)\;+\;\frac{\Delta_1 h_2}{2}\left(\frac{1}{9}\sigma_2^{sw}+\frac{1}{18}\left(\sigma_2^{nw}+\sigma_2^{se}\right)+\frac{1}{36}\sigma_2^{ne}\right)\right]_{i+1j+1}\Bigg\}$$

Contribution of the σ_2 term to F_y

$$\frac{1}{\bar h_1\bar h_2}\Bigg\{\left[\frac{\bar h_1}{4}\left(\frac{1}{3}\left(\sigma_2^{ne}+\sigma_2^{se}\right)+\frac{1}{6}\left(\sigma_2^{nw}+\sigma_2^{sw}\right)\right)\;-\;\frac{\Delta_2 h_1}{2}\left(\frac{1}{9}\sigma_2^{ne}+\frac{1}{18}\left(\sigma_2^{nw}+\sigma_2^{se}\right)+\frac{1}{36}\sigma_2^{sw}\right)\right]_{ij}$$

$$+\left[\frac{\bar h_1}{4}\left(\frac{1}{3}\left(\sigma_2^{nw}+\sigma_2^{sw}\right)+\frac{1}{6}\left(\sigma_2^{ne}+\sigma_2^{se}\right)\right)\;-\;\frac{\Delta_2 h_1}{2}\left(\frac{1}{9}\sigma_2^{nw}+\frac{1}{18}\left(\sigma_2^{sw}+\sigma_2^{ne}\right)+\frac{1}{36}\sigma_2^{se}\right)\right]_{i+1j}$$

$$+\left[-\frac{\bar h_1}{4}\left(\frac{1}{3}\left(\sigma_2^{ne}+\sigma_2^{se}\right)+\frac{1}{6}\left(\sigma_2^{nw}+\sigma_2^{sw}\right)\right)\;-\;\frac{\Delta_2 h_1}{2}\left(\frac{1}{9}\sigma_2^{sw}+\frac{1}{18}\left(\sigma_2^{ne}+\sigma_2^{sw}\right)+\frac{1}{36}\sigma_2^{nw}\right)\right]_{ij+1}$$

$$+\left[-\frac{\bar h_1}{4}\left(\frac{1}{3}\left(\sigma_2^{nw}+\sigma_2^{sw}\right)+\frac{1}{6}\left(\sigma_2^{ne}+\sigma_2^{se}\right)\right)\;-\;\frac{\Delta_2 h_1}{2}\left(\frac{1}{9}\sigma_2^{sw}+\frac{1}{18}\left(\sigma_2^{nw}+\sigma_2^{se}\right)+\frac{1}{36}\sigma_2^{ne}\right)\right]_{i+1j+1}\Bigg\}$$

Contribution of the σ_{12} term to F_x

$$\frac{1}{\bar h_1\bar h_2}\Bigg\{\left[-\frac{\bar h_1}{2}\left(\frac{1}{3}\left(\sigma_{12}^{ne}+\sigma_{12}^{se}\right)+\frac{1}{6}\left(\sigma_{12}^{nw}+\sigma_{12}^{sw}\right)\right)\;+\;\Delta_2 h_1\left(\frac{1}{9}\sigma_{12}^{ne}+\frac{1}{18}\left(\sigma_{12}^{nw}+\sigma_{12}^{se}\right)+\frac{1}{36}\sigma_{12}^{sw}\right)\right]_{ij}$$

$$+\left[-\frac{\bar h_1}{2}\left(\frac{1}{3}\left(\sigma_{12}^{nw}+\sigma_{12}^{sw}\right)+\frac{1}{6}\left(\sigma_{12}^{ne}+\sigma_{12}^{se}\right)\right)\;+\;\Delta_2 h_1\left(\frac{1}{9}\sigma_{12}^{nw}+\frac{1}{18}\left(\sigma_{12}^{sw}+\sigma_{12}^{ne}\right)+\frac{1}{36}\sigma_{12}^{se}\right)\right]_{i+1j}$$

$$+\left[\frac{\bar h_1}{2}\left(\frac{1}{3}\left(\sigma_{12}^{ne}+\sigma_{12}^{se}\right)+\frac{1}{6}\left(\sigma_{12}^{nw}+\sigma_{12}^{sw}\right)\right)\;+\;\Delta_2 h_1\left(\frac{1}{9}\sigma_{12}^{sw}+\frac{1}{18}\left(\sigma_{12}^{ne}+\sigma_{12}^{sw}\right)+\frac{1}{36}\sigma_{12}^{nw}\right)\right]_{ij+1}$$

$$+\left[\frac{\bar h_1}{2}\left(\frac{1}{3}\left(\sigma_{12}^{nw}+\sigma_{12}^{sw}\right)+\frac{1}{6}\left(\sigma_{12}^{ne}+\sigma_{12}^{se}\right)\right)\;+\;\Delta_2 h_1\left(\frac{1}{9}\sigma_{12}^{sw}+\frac{1}{18}\left(\sigma_{12}^{nw}+\sigma_{12}^{se}\right)+\frac{1}{36}\sigma_{12}^{ne}\right)\right]_{i+1j+1}\Bigg\}$$

Contribution of the σ_{12} term to F_y

$$\frac{1}{\bar h_1\bar h_2}\Bigg\{\left[-\frac{\bar h_2}{2}\left(\frac{1}{3}\left(\sigma_{12}^{ne}+\sigma_{12}^{nw}\right)+\frac{1}{6}\left(\sigma_{12}^{se}+\sigma_{12}^{sw}\right)\right)\;+\;\Delta_1 h_2\left(\frac{1}{9}\sigma_{12}^{ne}+\frac{1}{18}\left(\sigma_{12}^{nw}+\sigma_{12}^{se}\right)+\frac{1}{36}\sigma_{12}^{sw}\right)\right]_{ij}$$

$$+ \left[\frac{\bar{h}_2}{2} \left(\frac{1}{3} (\sigma_{12}^{ne} + \sigma_{12}^{nw}) + \frac{1}{6} (\sigma_{12}^{se} + \sigma_{12}^{sw}) \right) + \Delta_1 h_2 \left(\frac{1}{9} \sigma_{12}^{nw} + \frac{1}{18} (\sigma_{12}^{sw} + \sigma_{12}^{ne}) + \frac{1}{36} \sigma_{12}^{se} \right) \right]_{i+1j}$$

$$+ \left[-\frac{\bar{h}_2}{2} \left(\frac{1}{3} (\sigma_{12}^{se} + \sigma_{12}^{sw}) + \frac{1}{6} (\sigma_{12}^{ne} + \sigma_{12}^{nw}) \right) + \Delta_1 h_2 \left(\frac{1}{9} \sigma_{12}^{se} + \frac{1}{18} (\sigma_{12}^{ne} + \sigma_{12}^{sw}) + \frac{1}{36} \sigma_{12}^{nw} \right) \right]_{ij+1}$$

$$+ \left[\frac{\bar{h}_2}{2} \left(\frac{1}{3} (\sigma_{12}^{se} + \sigma_{12}^{sw}) + \frac{1}{6} (\sigma_{12}^{ne} + \sigma_{12}^{nw}) \right) + \Delta_1 h_2 \left(\frac{1}{9} \sigma_{12}^{sw} + \frac{1}{18} (\sigma_{12}^{nw} + \sigma_{12}^{se}) + \frac{1}{36} \sigma_{12}^{ne} \right) \right]_{i+1j+1} \Bigg\}$$

Eqns. 172,173, 178,179 and 180 are solved simultaneously over elastic time step $\Delta t_e = \Delta t / N_e < T_{ew} < \Delta t$ where N_e is the number of elastic subcycle time steps. The velocities are solved for on the U-grid (i.e. on the corners of the T-grid cells; see section 3.4). The left hand side terms are treated implicitly, the right hand side terms explicitly, and the rate of strain $\dot{\epsilon}_{ij}$ and Δ are updated each elastic time step. The definition of the elastic parameter E in terms of the bulk viscosity, as well as the updating of the rate of strain tensor each elastic time step, eliminates any linearization error associated with viscosities which are lagged over the time step.

4.11 Advection

Horizontal advection in Eqs. 3-6, and 8 is evaluated after the incremental remapping algorithm of Lipscomb and Hunke (2004), following closely the work of Dukowicz and Baumgardner (2000), designated DB in this section. The following description is adapted from the more detailed CICE documentation (Hunke and Lipscomb, 2004).

The remapping algorithm is second-order accurate in space, except where field gradients are limited to preserve monotonicity (that is, to avoid creating unphysical extrema or ripples in the transported fields). One benefit of remapping is that it preserves the monotonicity of tracers: quantities that obey the advection equation $dh/dt = \partial h / \partial t + u \cdot \nabla h = 0$, where h is a tracer. Using the conservation equations 3-6 and 8, along with the definitions of ice/snow thickness in Eq. 21 and enthalpy in Eq. 14, one can show that thickness, enthalpy, and surface temperature are tracers.

The remapping algorithm can be summarized as follows: Given values of the ice area and tracer fields in each grid cell, construct linear approximations and limit the field gradients to preserve monotonicity. Given ice velocities at grid cell corners, identify departure regions for the transports across each cell edge, and then divide these departure regions into triangles and compute the coordinates of the triangle vertices. Integrate these fields over the departure triangles to obtain the area, volume, and energy transports across each cell edge. Transfer these quantities across cell edges and update the state variables.

Since all scalar fields are transported by the same velocity field, identifying departure regions is done only once per time step. The other three steps are repeated for each field in each thickness category. These steps are described below.

4.11.1 Reconstructing area and tracer fields

First, using the known values of the state variables, the ice area and tracer fields are reconstructed in each grid cell as linear functions of x and y. For each field we compute the value at the cell center (i.e., at the origin of a 2D Cartesian coordinate system defined for that grid cell), along with gradients in the x and y directions. The gradients are limited to preserve monotonicity. When integrated over a grid cell, the reconstructed fields must have mean values equal to the known state variables denoted for example by $\bar{a}$ for fractional area. The mean values are not, in general, equal to the values at the cell center. For example, the mean ice area must equal the value at the centroid, which may not lie at the cell center.

Consider first the fractional ice area. For each thickness category we construct a field $a(r)$ whose mean is $\bar{a}$, where $r = (x, y)$ is the position vector relative to the cell center. That is, we require

$$\int_A a\, dA = \bar{a}\, A, \tag{191}$$

where $A = \int_A dA$ is the grid cell area. This equation is satisfied if $a(r)$ has the form

$$a(r) = \bar{a} + \alpha \, \langle \nabla a \rangle \cdot (r - \bar{r}), \tag{192}$$

where $\langle \nabla a \rangle$ is a centered estimate of the area gradient within the cell, α is a limiting coefficient that enforces monotonicity, and $\bar{r}$ is the cell centroid:

$$\bar{r} = \frac{1}{A} \int_A r\, dA.$$

It follows from Eq. 192 that the ice area at the cell center ($r = 0$) is

$$a_c = \bar{a} - a_x \overline{x} - a_y \overline{y},$$

where $a_x = \alpha(\partial a/\partial x)$ and $a_y = \alpha(\partial a/\partial y)$ are the limited gradients in the x and y directions, respectively, and the components of $\bar{r}$, $\overline{x} = \int_A x\, dA/A$ and $\overline{y} = \int_A y\, dA/A$. Ice thickness, snow thickness and enthalpy fields are treated analogously.

We preserve monotonicity by van Leer limiting. Let $\bar{\phi}(i, j)$ denote the mean value of some field in grid cell (i, j). We first compute centered gradients of $\bar{\phi}$ in the x and y directions, then check whether these gradients give values of ϕ within cell (i, j) that lie outside the range of $\bar{\phi}$ in the cell and its eight neighbors. Let $\bar{\phi}_{\max}$ and $\bar{\phi}_{\min}$ be the maximum and minimum values of $\bar{\phi}$ over the cell and its neighbors, and let $\phi_{\max}$ and $\phi_{\min}$ be the maximum and minimum values of the reconstructed ϕ within the cell. Since the reconstruction is linear, $\phi_{\max}$ and $\phi_{\min}$ are located at cell corners. If $\phi_{\max} > \bar{\phi}_{\max}$ or $\phi_{\min} < \bar{\phi}_{\min}$, we multiply the unlimited gradient by $\alpha = \min(\alpha_{\max}, \alpha_{\min})$, where

$$\alpha_{\max} = (\bar{\phi}_{\max} - \bar{\phi})/(\phi_{\max} - \bar{\phi}),$$

$$\alpha_{\min} = (\bar{\phi}_{\min} - \bar{\phi})/(\phi_{\min} - \bar{\phi}).$$

Otherwise the gradient need not be limited.

4.11.2 Locating departure triangles

The locating of departure triangles is discussed in detail by DB. The basic idea is that the contents of a quadrilateral departure region are transported to the target grid cell. Consider a departure quadrilateral north and west of the target grid cell whose motion is towards the southeast. The neighboring grid cells are labeled by compass directions: NW, N, NE, W, and E. The four vectors point along the velocity field at the cell corners, and the departure region is formed by joining the starting points of these vectors. Instead of integrating over the entire departure region, it is convenient to compute transports across cell edges. We identify departure regions for the north and east edges of each cell, which are also the south and west edges of neighboring cells. There are 20 triangles that can contribute transports across the north edge of a grid cell.

This scheme was designed for rectangular grids. Grid cells in CCSM3 actually lie on the surface of a sphere and must be projected onto a plane. Many such projections are possible. The projection used in CCSM3 approximates spherical grid cells as quadrilaterals in the plane tangent to the sphere at a point inside the cell. The quadrilateral vertices are $(N/2, E/2)$, $(-N/2, W/2)$, $(-S/2, -W/2)$, and $(S/2, -E/2)$, where N, S, E, and W are the lengths of the cell edges on the spherical grid. The quadrilateral area, $(N + S)(E + W)/4$, is a good approximation to the true spherical area. However, cell edges in this projection are not orthogonal (i.e., they do not meet at right angles) as on the spherical grid. This means that when vectors are translated from cell corners to cell centers, we must take care that the departure points in the cell-center coordinate system lie inside the grid cell contributing the transport. Otherwise, monotonicity may be violated, because van Leer limiting does not apply outside the grid cell.

Most grids cells are nearly rectangular. On the 1° displaced-pole grid used in CCSM3, the maximum angle is about 1°. Vector transformations may therefore be omitted on most grids with little loss of accuracy. We have retained them, however, because they ensure exact monotonicity at little added cost.

We made one other change in the DB scheme for locating triangles. In their paper, departure points are defined by projecting cell corner velocities directly backward. That is,

$$x'_D = -u' \, \Delta t, \tag{193}$$

where x'_D is the location of the departure point relative to the cell corner and the primes denote vectors defined in the cell-corner basis. This approximation is only first-order accurate. The accuracy is increased to second-order by correcting the velocity with a midpoint approximation before finding the departure point.

4.11.3 Integrating transports

Next, we integrate the reconstructed fields over the departure triangles to find the total transports of area, volume, and energy across each cell edge. Area transports are easy to compute since the area is linear in x and y. Given a triangle with vertices $x_i = (x_i, y_i)$, $i \in \{1, 2, 3\}$, the triangle area is

$$A_T = \frac{1}{2} \left|(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)\right|. \tag{194}$$

The integral I_1 of any linear function $f(r)$ over a triangle is given by

$$I_1 = A_T f(x_0), \tag{195}$$

where $x_0 = (x_0, y_0)$ is the triangle midpoint,

$$x_0 = \frac{1}{3} \sum_{i=1}^{3} x_i. \tag{196}$$

To compute the area transport, we evaluate the area at the midpoint,

$$a(x_0) = a_c + a_x x_0 + a_y y_0, \tag{197}$$

and multiply by A_T. By convention, northward and eastward transports are positive, while southward and westward transports are negative.

Eq. 197 cannot be used for volume transports, because the reconstructed volumes are quadratic functions of position. (They are products of two linear functions, area and thickness.) The integral of a quadratic polynomial over a triangle requires function evaluations at three points,

$$I_2 = \frac{A_T}{3} \sum_{i=1}^{3} f\left(x_i'\right), \tag{198}$$

where $x_i' = (x_0 + x_i)/2$ are points lying halfway between the midpoint and the three vertices. Eq. 198 does not work for ice and snow energies, which are cubic functions—products of area, thickness, and enthalpy. Integrals of a cubic polynomial over a triangle can be evaluated using a four-point formula:

$$I_3 = A_T \left[-\frac{9}{16} f(x_0) + \frac{25}{48} \sum_{i=1}^{3} f(x_i'') \right] \tag{199}$$

where $x_i'' = (3x_0 + 2x_i)/5$.

4.11.4. Updating state variables

Finally, we use these transports to compute new values of the state variables in each ice category and grid cell. The new fractional ice areas $a'_{in}(i,j)$ are given by

$$a'_{in}(i,j) = a_{in}(i,j) + \frac{F_{Ea}(i-1,j) - F_{Ea}(i,j) + F_{Na}(i,j-1) - F_{Na}(i,j)}{A(i,j)} \tag{200}$$

where $F_{Ea}(i,j)$ and $F_{Na}(i,j)$ are area transports across the east and north edges, respectively, of cell (i,j), and $A(i,j)$ is the grid cell area. All transports added to one cell are subtracted from a neighboring cell; thus Eq. 200 conserves total ice area.

The new ice volumes and internal energies are computed analogously. New thicknesses are given by the ratio of volume to area, and enthalpies by the ratio of energy to volume. Tracer monotonicity is ensured because

$$h' = \frac{\int_A a\, h\, dA}{\int_A a\, dA},$$

$$q' = \frac{\int_A a\, h\, q\, dA}{\int_A a\, h\ dA},$$

where h' and q' are the new-time thickness and enthalpy, given by integrating the old-time ice area, volume, and energy over a Lagrangian departure region with area A. That is, the new-time thickness and enthalpy are weighted averages over old-time values, with non-negative weights a and ah. Thus the new-time values must lie between the maximum and minimum of the old-time values.

4.12 Mechanical Redistribution

Mechanical redistribution of ice thickness due to rafting and ridging processes is treated in this section. Specifically, the redistribution function R of Eq. 1 (ψ in Thorndike et al. 1975) is parameterized, and then used to evaluate the S_M source/sink terms in Eqs. 3-6,8. For example, the integral of R over the n^{th} category gives the ice fraction source

$$S_{MAn} = \int_{h_n^*}^{h_{n+1}^*} R(h,g,u)dh. \tag{201}$$

The redistribution function R depends on the the ice thickness h, the distribution function $g(h)$, and the velocity field u, specifically the invariants of the strain rate tensor (see Section 4.10, Eqs 159,165,166). These invariants are the divergence $\dot{\epsilon}_I$ and the shear $\dot{\epsilon}_{II}$. These two invariants are also used in terms of magnitude $|\dot{\epsilon}| = (\dot{\epsilon}_I^2 + \dot{\epsilon}_{II}^2)^{1/2}$ and strain rate angle $\theta = \tan^{-1}(\dot{\epsilon}_{II}/\dot{\epsilon}_I)$. (Note that $\theta = 0°$ refers to pure divergence, $\theta = 45°$ uniaxial extension, $\theta = 90°$ pure shear, $\theta = 135°$ to uniaxial compression, and $\theta = 180°$ pure convergence).

Two strong constraints on the redistribution follow from the first two moments of the distribution equation, corresponding to area and volume conservation:

$$\int_0^{h_{max}} R dh = \dot{\epsilon}_I \tag{202}$$

$$\int_0^{h_{max}} hRdh = 0. \tag{203}$$

The first follows from the conservation of total area, since the import or export of area ($\dot{\epsilon}_I$) must be balanced by a redistribution. The second follows from the requirement that mechanical redistribution cannot change the ice volume.

The parameterization of lead opening and mechanical redistribution follows from the theoretical formulation of Thorndike et al. (1975):

$$R = |\dot{\epsilon}|[\alpha_0(\theta)\delta(h) + \alpha_r(\theta)w_r(h,g)], \tag{204}$$

where $\delta(h)$(the delta function) is the opening mode, and $w_r(h,g)$ is the ridging mode. Note that the conservation of area requires:

$$\int_0^{\infty} w_r(h,g)dh = -1. \tag{205}$$

The coefficients $|\dot{\epsilon}|\alpha_0(\theta)$ and $|\dot{\epsilon}|\alpha_r(\theta)$ are known as the lead opening and closing rates, respectively, and they are related such that their difference equals the ice divergence, $|\dot{\epsilon}|\alpha_0(\theta) - |\dot{\epsilon}|\alpha_r(\theta) = \dot{\epsilon}_I$.

Two aspects of the mechanical redistribution must be considered: the relation between the distribution function and the deformation/compressive strength used in the ice dynamics, and the redistribution source terms in the conservation equations.

For the first, we follow an energetics argument by Rothrock (1975). The deformational work done on the ice is equated to known sinks of energy in ridge building:

$$\sigma_I\dot{\epsilon}_I + \sigma_{II}\dot{\epsilon}_{II} = R_{pot} + R_{fric} \tag{206}$$

where R_{pot} is the rate of mechanical production of gravitational potential energy per unit area, and R_{fric} is the rate of frictional energy loss per unit area. We write

$$\sigma_I\dot{\epsilon}_I + \sigma_{II}\dot{\epsilon}_{II} = (1 + \frac{R_{fric}}{R_{pot}})R_{pot} = ZR_{pot} \tag{207}$$

where Z is the ratio of total energy dissipated to potential energy gain. Ice thickness h has potential energy relative to sea level of $P_e = C_{pe}h^2$, where $C_{pe} = \frac{1}{2}\frac{\rho_i}{\rho_o}(\rho_o - \rho_i)g_e$ Integrating over the entire thickness distribution results in the total potential energy

$$P_e = C_{pe}\int_0^{\infty} h^2 g(h)dh \tag{208}$$

The rate of gain of P_e is

$$\frac{dP_e}{dt} = C_{pe}\int_0^{\infty} h^2\frac{dg(h)}{dt}dh. \tag{209}$$

From the distribution equation, using $\nabla \cdot ug = g\nabla \cdot u + u \cdot \nabla g$:

$$\frac{dg}{dt} = -\frac{\partial}{\partial h}(\dot{h}g) + L(h,g) - g\nabla \cdot u + R. \tag{210}$$

Thus the total rate of change of potential energy is

$$\frac{dP_e}{dt} = -\int_0^\infty h^2 \frac{\partial}{\partial h}(\dot{h}g)dh + \int_0^\infty h^2 L(h,g)dh - P_e \nabla \cdot u + \int_0^\infty h^2 R dh. \tag{211}$$

The first two terms on the right hand side refer to gain/loss due to thermodynamic and thickness transport processes, the third to large scale divergence, while the last term is due to mechanical redistribution. Using Eq. 204 for R we have

$$R_{pot} = |\dot{\epsilon}|\alpha_r(\theta)C_{pe}\int_0^\infty h^2 w_r(h,g)dh. \tag{212}$$

We can therefore define

$$P = ZC_{pe}\int_0^\infty h^2 w_r(h,g)dh \tag{213}$$

so that

$$\sigma_I\dot{\epsilon}_I + \sigma_{II}\dot{\epsilon}_{II} = |\dot{\epsilon}|\alpha_r(\theta)P \tag{214}$$

where P is the compressive strength used in the dynamics (Eq. 163).

Further, from the viscous/plastic relations of σ_I and σ_{II} in terms of $\dot{\epsilon}_I$ and $\dot{\epsilon}_{II}$, and the bulk/shear viscosity definitions (see Section 4.10, Eqs. 161,162,167), we have:

$$\sigma_I\dot{\epsilon}_I + \sigma_{II}\dot{\epsilon}_{II} = \zeta\dot{\epsilon}_I^2 - \frac{P}{2}\dot{\epsilon}_I + \eta\dot{\epsilon}_{II}^2 \tag{215}$$

or

$$\sigma_I\dot{\epsilon}_I + \sigma_{II}\dot{\epsilon}_{II} = P|\dot{\epsilon}|\{\frac{1}{2|\dot{\epsilon}|}(\Delta - \dot{\epsilon}_I) \tag{216}$$

so that the ridging mode $\alpha_r(\theta)$ in terms of the strain rate angle is

$$\alpha_r(\theta) = -\frac{1}{2}cos(\theta) + \frac{1}{2}\sqrt{cos^2(\theta) + \frac{sin^2(\theta)}{e^2}} \tag{217}$$

$$|\dot{\epsilon}|\alpha_r(\theta) = \frac{1}{2}(\Delta - \dot{\epsilon}_I). \tag{218}$$

which is the result for the elliptical yield curve with aspect ratio e found by Hibler (1980).

Flato and Hibler (1995) separated the expression in Eq. 218 into the sum of two terms representing the energy dissipation from ridge building by shear and convergence:

$$|\dot{\epsilon}|\alpha_r(\theta) = C_s\frac{1}{2}(\Delta - |\dot{\epsilon}_I|) - \min(\dot{\epsilon}_I, 0), \tag{219}$$

with the factor C_s added so the shearing component can be altered by varying C_s between 0 (all energy dissipation by shear is lost to sliding) and 1 (all energy dissipation by shear is used to build ridges). Eqs. 218 and 219 are equivalent when $C_s = 1$. The experiments of Flato and Hibler (1995) indicate that $C_s = 0.5$ is appropriate to produce the concentrations of ridged ice observed in the Arctic. However, Bitz et al. (2001) found that a coupled model tended to predict too much ridged ice with $C_s = 0.5$. Bitz et al. (2001) tested the model with $C_s = 0$ and found better agreement with observations. Unfortunately, the parameter C_s depends on e and P, and none of these values is well constrained by observations. It is some consolation that Bitz et al. (2001) found that C_s, and hence the precise concentration of ridged ice, has relatively little affect on the *climate* of the Arctic. Our standard model uses a compromise value of $C_s = 0.225$.

The ridging mode is the sum of two distributions describing the ice participating in ridging $a(h)$ and the newly ridged ice $n(h)$, normalized to conserve area and volume:

$$w_r(h) = \frac{-a(h) + n(h)}{\int_0^\infty [a(h) - n(h)]dh}. \tag{220}$$

The ice participating in ridging is found by weighting $g(h)$ by a function $b(h)$ that is designed to make thinner ice more likely to ridge than thicker ice. The newly ridged ice is found by integrating $a(h)$ times the redistribution function $\gamma(h', h)$ over the range of ice thicknesses h' that can contribute to form newly ridged ice of thickness h. Hence,

$$a(h) = b(h)g(h)n(h) = \int_0^h \gamma(h', h)b(h')g(h')dh'. \tag{221}$$

Thorndike et al. (1975) argued that a plausible $b(h)$ might depend linearly on the cumulative thickness distribution $G(h) = \int_0^h g(h')dh'$ according to

$$b(G) = \frac{2}{G^*}\left[1 - \frac{G(h)}{G^*}\right] \tag{222}$$

when $G \le G^*$; otherwise 0 for $G > G^*$, for which G^* is the limiting fraction below which all ridging occurs and is assumed to be 15% as in Thorndike et al. (1975). The redistribution process is parameterized according to Hibler (1980), who constructed a rule for deriving $n(h)$ from $a(h)$ based on observations that constrain ice of thickness h participating in ridging to be linearly distributed between thicknesses $2h$ and $2\sqrt{Kh}$:

$$\gamma(h', h) = \frac{1}{2(K - h')} \tag{223}$$

when $2h' \le h \le 2\sqrt{Kh'}$; otherwise 0.

The mechanical redistribution function R_n is the integral of the continuous function (see Eq. 204) over the thickness limits of each category:

$$R_n = \int_{h_n^*}^{h_{n+1}^*} Rdh = \delta(h)\left[\dot{\epsilon}_I + |\dot{\epsilon}|\alpha_r(\theta)\right] + |\dot{\epsilon}|\alpha_r(\theta)W_n, \tag{224}$$

where

$$W_n = \int_{h_n^*}^{h_{n+1}^*} w_r(h)dh. \tag{225}$$

The W_n factors can be separated into two components, participation W_{an} (loss) and redistribution W_{nn} (gain),

$$W_n = -W_{an} + W_{nn}, \tag{226}$$

such that

$$W_{an} = \frac{1}{\omega}\int_{h_n^*}^{h_{n+1}^*} b(h)g(h)dh$$
$$W_{nn} = \frac{1}{\omega}\int_{h_n^*}^{h_{n+1}^*}\int_0^{h_{n+1}^*}\gamma(h',h)b(h')g(h')dh'dh \tag{227}$$

where ω normalizes W_n such that $\sum_{n=0}^{N} R_n = \dot{\epsilon}_I$. After substituting $g(h)dh = dG$ into the equation for W_{an}, we find

$$W_{an} = \frac{1}{\omega}\int_{\min(G^*,G_n)}^{\min(G^*,G_{n+1})} b(G)dG, \tag{228}$$

where $G_n = \sum_{p=0}^{n} A_p$, with $G_{-1} = 0$. Finally we express W_{an} as a function of the auxiliary function Y_n:

$$W_{an} = \frac{1}{\omega}(Y_n - Y_{n+1}), \quad \text{where} \quad Y_n = [1 - \frac{G_n}{G^*}]^2 \tag{229}$$

when $G_n \leq G^*$; otherwise 0 for $G_n > G^*$. In Eq. 227, W_{nn} is evaluated by first changing the order of integration and then expanding the outer integral into a sum of integrals over the categories:

$$W_{nn} = \frac{1}{\omega}\sum_{p=0}^{n+1}\int_{h_p^*}^{h_{p+1}^*}\int_{h_n^*}^{h_{n+1}^*}\gamma(h',h)dh\ b(h')g(h')dh'. \tag{230}$$

Taking $\int_{h_n^*}^{h_{n+1}^*}\gamma(h',h)dh$ outside of the remaining integral we have

$$W_{nn} = \frac{1}{\omega}\sum_{p=0}^{n+1}\Gamma_{pn+1}\int_{h_n^*}^{h_{n+1}^*} b(h')g(h')dh' = \frac{1}{\omega}\sum_{p=0}^{n+1} W_{ap}\Gamma_{pn+1}. \tag{231}$$

The discrete distributor Γ_{pn+1} is computed as in the method of Hibler (1980) from

$$\Gamma_{pn+1} = \int_{h_n^*}^{h_{n+1}^*}\gamma(h_p,h)dh = \frac{\min(2\sqrt{Kh_p}, h_{n+1}^*) - \max(2h_p, h_n^*)}{2(K-h_p)}, \tag{232}$$

when $2h_p < h_n^*$ or $2\sqrt{Kh_p} > h_{n+1}^*$; otherwise $\Gamma_{pn+1} = 0$. In general, ice from any category may participate in ridging provided that the cumulative distribution of ice up to that category is less than G^*. S_{MAn} (Eq. 201) can now be expressed as:

$$S_{MAn} = |\dot{\epsilon}|\alpha_r(\theta)\left[-W_{an} + \sum_{p=1}^{n+1} W_{ap}\Gamma_{pn+1}\right]. \tag{233}$$

Because the volume of the ice and snow is influenced by ridging, the volume of the ice and snow must be redistributed as well as the ice concentration. The ice participating in ridging reduces the volume in category n proportional to $h_n W_{an}$. The ice that ridges into category n increases the volume proportional to $\sum_{p=1}^{n+1}\tilde{h}_{pn+1}W_{an}\Gamma_{pn+1}$, which is summed over the p categories that ridge into category n. The thickness of this newly ridged ice $\tilde{h}_{pn+1}$ is uniquely determined by conservation of volume and the discrete distributor. For simplicity, we assume the snow thickness redistribution process is independent of the category that receives the newly ridged ice and snow. Hence, the snow thickness would be the same on top of each ridged ice category. The rate of change of the ice and snow volumes due to mechanical redistribution is

$$
\begin{aligned}
S_{MVn} &= |\dot{\epsilon}|\alpha_r(\theta)\left[-h_n W_{an} + \sum_{p=1}^{n+1} \tilde{h}_{pn+1} W_{ap} \Gamma_{pn+1}\right] \\
S_{MVsn} &= |\dot{\epsilon}|\alpha_r(\theta)\left[-h_{sn} W_{an} + \sum_{p=1}^{n+1} \tilde{h}_{spn+1} W_{ap} \Gamma_{pn+1}\right],
\end{aligned}
\tag{234}
$$

where

$$
\tilde{h}_{pn+1} = \frac{1}{2}\left[\max\left(2h_p, h_n^*\right) + \min\left(2\sqrt{Kh_p}, h_{n+1}^*\right)\right] \tilde{h}_{spn+1} = h_{sp}\frac{h_p + \sqrt{Kh_p}}{h_p}. \tag{235}
$$

Conservation of energy requires a redistribution of internal energy as well. We assume that mechanical redistribution does not mix energy vertically. Again it may be helpful to think of the vertical dimension broken into a fixed number of layers, where energy is redistributed layer by layer. Hence, redistribution transfers heat only from the upper layer of one category to another and so on. Consistent with the redistribution of ice and snow volume, we find

$$
S_{MEn} = |\dot{\epsilon}|\alpha_r(\theta)\left[-q_n h_n W_{an} + \sum_{p=1}^{n+1} q_p \tilde{h}_{pn+1} W_{ap} \Gamma_{pn+1}\right] \tag{236}
$$

$$
S_{MESn} = |\dot{\epsilon}|\alpha_r(\theta)\left[-q_s h_{sn} W_{an} + \sum_{p=1}^{n+1} q_{sp} \tilde{h}_{spn+1} W_{ap} \Gamma_{pn+1}\right]. \tag{237}
$$

The ridged ice will have the same vertical temperature profile as the ice which has participated in ridging.

Finally, the surface temperature is also affected by mechanical redistribution:

$$
S_{MTsn} = |\dot{\epsilon}|\alpha_r(\theta)\left[-T_{sn} W_{an} + \sum_{p=1}^{n+1} T_{sp} W_{ap} \Gamma_{pn+1}\right]. \tag{238}
$$

With the redistribution of surface temperature, the ridged ice will have the same surface temperature as the ice that participates in ridging.

Using the definitions above, the mechanical pressure P (see Eq. 163) is:

$$
P = ZC_{pe}\sum_{n=1}^{N}\left[-h_n^2 W_{an} + \sum_{p=1}^{n+1} \tilde{h}_{pn+1}^2 W_{ap} \Gamma_{pn+1}\right]. \tag{239}
$$

Based on the work of Hopkins and Hibler (1991) and Flato and Hibler (1995), we let $Z = 17$.

4.13 Output to the Coupler

Aggregate states and fluxes over the ice distribution are computed for exchange with the coupler and for history output. The general aggregate equation for an arbitrary field $(\{X_n\}, n = 1, ...N)$ is;

$$X = \frac{1}{A}\sum_{n=1}^{N} X_n A_n \tag{240}$$

where $A = \Sigma_{n=1}^{N} A_n$ (note that some history fields do not normalize by $1/A$; see section 6).

Table 3 lists the states and fluxes which are sent by the sea ice model to the coupler (see section 2.3). The ice area A follows from Eqs. 2 and 3. The surface temperature T_s comes from the vertical heat conduction solutions in sections 4.6.1-5 . The albedos are from Eq. 35 (direct and diffuse albedos are identical; see section 4.4).

The latent and sensible heat fluxes F_{LH} and F_{SH} are from Eqs. 44 and 43 respectively. The upwelling flux F_{LWUP} is from Eq. 40. The evaporative flux F_{EVAP} is F_{LH} without the latent heat of sublimation L_s in Eq. 44.

The atmosphere/ice stresses τ_{ax} and τ_{ay} are given by Eqs. 41 and 42 respectively. These stresses are computed on the T-grid in the ice model, and are rotated from the displaced pole grid onto the geographical latitude/longitude directions before being sent to the coupler. Similarly, the ocean/ice stresses τ_{ox} and τ_{oy}, which are computed on the U-grid from Eq. 157, are first bilinearly interpolated to the T-grid, and then reprojected onto the geographical latitude/longitude directions. In this manner, all vector fields exchanged with the coupler are on the T-grid and projected onto geographical latitude/longitude directions.

Specifically, the following U-grid to T-grid interpolation is required. Note that (u, v) represents a vector field for which the subscripts "g" and "dp" refers to geographic and displaced pole grids respectively, and the superscripts "t"and "u" to the T-grid and U-grid respectively.

$$\begin{aligned} (u^t_{dp})_{ij} &= \frac{1}{4}(A^u_{ij}(u^u_{dp})_{ij} + A^u_{i-1j}(u^u_{dp})_{i-1j} + A^u_{ij-1}(u^u_{dp})_{ij-1} + A^u_{i-1j-1}(u^u_{dp})_{i-1j-1})/A^t_{ij} \\ (v^t_{dp})_{ij} &= \frac{1}{4}(A^u_{ij}(v^u_{dp})_{ij} + A^u_{i-1j}(v^u_{dp})_{i-1j} + A^u_{ij-1}(v^u_{dp})_{ij-1} + A^u_{i-1j-1}(v^u_{dp})_{i-1j-1})/A^t_{ij} \end{aligned} \tag{241}$$

where A^u_{ij} is the U-grid box area, and A^t_{ij} is the T-grid box area.

Then, the vector components (u^t_{dp}, v^t_{dp}) are rotated back to the geographic grid orientation by:

$$\begin{aligned} (u^t_g)_{ij} &= (u^t_{dp})_{ij} cos(\chi^t_{ij}) - (v^t_{dp})_{ij} sin(\chi^t_{ij}) \\ (v^t_g)_{ij} &= (u^t_{dp})_{ij} sin(\chi^t_{ij}) + (v^t_{dp})_{ij} cos(\chi^t_{ij}) \end{aligned} \tag{242}$$

where (ij) are the longitude/latitude indices of the appropriate grid.

The shortwave flux transmitted to the ocean is:

$$F_{SWon} = I_{0vs} e^{-\kappa_{vs} h_n} + I_{0ni} e^{-\kappa_{ni} h_n} \tag{243}$$

(see Eq. 75 and following).

The heat flux to the ocean F_{Qio} (Eq. 136) represents that heat flux available $F_{Qoi} < 0$ that is actually used in ice melt. The heat used by the ice model includes basal and lateral melt.

For the water flux F_{Wo}, recall that three types of ice formation are distinguished: **frazil** (which forms directly in the ocean surface layer), **congelation** (which forms at ice base), and **snow-ice** (which forms by flooding of snow covered ice). Frazil ice is formed in the ocean model as previously described (see Section 4.3). The amount of frazil ice given to the ice model is implied in the freezing/melting potential ($F_{Qoi} > 0$). Therefore, there is no explicit water flux from the ocean to the ice model in this case. However, when the ice melts, the melt water is passed back to the ocean. Congelation ice forms in the ice model at ice base. Therefore, the water exchange with the ocean must include both formation and melt. Snow-ice is an internal transformation in the ice model itself, and need not be included in the water flux to the ocean.

The water flux to the ocean for category n is

$$F_{Won} = A_n F_{RN} + \{A_n(-\rho_i \delta h_t - \rho_i \delta h_b - \rho_s \delta h_s) + (R_{side}(\rho_i V + \rho_s V_s))_n\}/\Delta t \tag{244}$$

where $A_n F_{RN}$ is the flux due to rain on ice assumed to drain directly to the ocean, δh_t is the change in the top ice thickness due to surface melt and sublimation/condensation, δh_b is the change in the basal ice thickness due to congelation ice formation and melt, δh_s is the change in surface snow depth due to melting and evaporation, and $R_{side}(\rho_i V + \rho_s V_s)$ is the ice and snow amount melted by lateral thermodynamic processes (see section 4.8).

Presently, frazil ice formation occurs within the ocean, and therefore the amount of salinity in frazil ice must be known to the ocean. This value is the reference salinity S_i, which then becomes the constant salinity that all ice must have for salinity conservation in ocean-ice exchange. This requires that non-frazil ice formation and all ice melt have salinity exchanges with the ocean (see Eqs. 123-126). The resulting ocean-ice salinity flux for category n is

$$F_{S_{on}} = \{\rho_i S_i \delta V_{ice\ melt} - \rho_i S_i \delta V_{cong\ ice} - \rho_i S_i \delta V_{sub\ cond} - \rho_i S_i \delta V_{snow-ice}\}_n/\Delta t \tag{245}$$

where the melt of total ice volume $\delta V_{ice\ melt}$, congelation ice formation $\delta V_{cong\ ice}$, and snow-ice formation $\delta V_{snow-ice}$ are each positive, and the atmosphere-ice sublimation $\delta V_{sub\ cond}$ is negative for sublimation and positive for condensation. It is admitted that salinity exchange with the ocean for sublimation/condensation and for snow-ice formation does not seem very physical, but it is required by the present version.

Diagnostic fields of 2 m reference temperature T_{ref} and specific humidity Q_{ref} follow from Eqs. 67-68 and 70 respectively. The total absorbed shortwave flux in the ice/ocean system derives from Eq. 39.

5. Active Ice Only (AIO) Framework

The ice model can be run through the coupler but with other components prescribed in a framework called Active Ice Only (AIO). In AIO the ice model communicates via the coupler with other components in the same manner as it would in a fully coupled run. The user need only ensure that the other components are data models (i.e. prescribed), and that they provide the sea ice model with reasonable values of all fields received from the coupler (see Table 2). Because of the way ice and ocean are coupled, the ice-ocean heat exchange in the data ocean model is independent of ice state. Therefore the sea ice model has an option to run with a simple ocean mixed layer model that is part of the sea ice component.

The mixed layer prognostic variable is ocean temperature T_o, determined by the thermodynamic equation:

$$\rho_o c_o h_o \frac{\partial T_o}{\partial t} = F_{SW} + F_{LW} + F_{SH} + F_{LH} + F_{Qio} - F_{Qo}. \tag{246}$$

where ρ_o, c_o, h_o are the ocean mass density, heat capacity and mixed layer depth respectively, F_{SW} is the absorbed shortwave flux, F_{LW} is the absorbed longwave flux: $F_{LW} = F_{LWDN} - F_{LWUP}$, F_{SH},F_{LH} are the sensible and latent heat fluxes between ocean and atmosphere respectively, F_{Qio} is the available mixed layer heat flux used by the sea ice model as shown in section 4.8, and F_{Qo} is the ocean heat flux which represents seasonal mixed layer heat storage/release and lateral oceanic heat transport. Note that the fluxes include contributions from both open water and sea ice. The required input (presently monthly mean) data are the mixed layer depth, salinity (presently not used), ocean currents, sea surface slopes and ocean heat flux, as shown in Table 6. The ocean currents and tilt are used in the sea ice dynamic calculation.

All of these data are supplied on a monthly basis from either an observational and/or a model source. For model data, the mean monthly surface temperatures and surface heat fluxes from a model run can be used. F_{Qo} is computed from the monthly data as:

$$F_{Qo}^k = -\rho_o c_o h_o^k \left(\frac{T_o^{k+1} - T_o^{k-1}}{2\Delta t_k}\right) + F_{SW}^k + F_{LW}^k + F_{SH}^k + F_{LH}^k + F_{Qio}^k \tag{247}$$

for the k^{th} month (k = 1, 2, ..., 12), and the required data are shown, with Δt_k = mean month time. The monthly data (if necessary) must be spatially interpolated to the sea ice model grid, and be formatted as an external netCDF file, containing the monthly fields shown in Table 6. Once read in, these monthly data are linearly interpolated in time to the sea ice model time step (therefore allowing for seasonal variation only).

Table 6. Ocean Fields Required for Mixed Layer

Symbol	Description	Units
h_o	mixed layer depth	m
S_o	salinity	ppt
u_o	x direction surface velocity	m s^{-1}
v_o	y direction surface velocity	m s^{-1}
$(\nabla H_o)_x$	x direction surface slope	m m^{-1}
$(\nabla H_o)_y$	y direciton surface slope	m m^{-1}
F_{Qo}	ocean heat flux	W m^{-2}

For a time step m with initial temperature T_o^m, the mixed layer temperature forecast equation is:

$$T_o^{'m+1} = T_o^m + \Delta t \left(F_{SW}^m + F_{LW}^m + F_{SH}^m + F_{LH}^m + F_{Qio}^m \right) / \rho_o c_o h_o \tag{248}$$

where we first evaluate the exchange between the mixed layer and the atmosphere/ice above, in order to limit possible loss of mixed layer heat to the deep heat source when the mixed layer temperature is at freezing (see below). F_{SW}^m is computed from:

$$F_{SW}^m = F_{SWDN}^m (1 - \alpha_o)(1 - A^m) + F_{SW_o}^m A^m \tag{249}$$

where α_o is a constant ocean surface albedo, $F_{SW_o}^m$ is the shortwave flux that penetrates the ice to be absorbed in the underlying ocean, and A^m is the sea ice fraction. The fluxes F_{LWUP}^m, F_{SH}^m and F_{LH}^m are computed over the open ocean using the ocean mixed layer temperature and surface properties, and then weighted by the open ocean fraction.

If $T_o^{'m+1} < T_{of}$, where T_{of} is the freezing temperature of the ocean, and $F_{Qo} > 0$ (implying heat loss from the mixed layer), then the heat exchange is limited such that $F_{Qo} = 0$. In other cases, deep exchange is evaluated by:

$$T_o^{m+1} = T_o^{'m+1} - F_{Qo}^* \Delta t / \rho_o c_o h_o. \tag{250}$$

Frazil ice heat flux (>0) or melt potential (<0) is then evaluated as:

$$F_{Qoi} = \rho_o c_o h_o \frac{(T_{of} - T_o^{m+1})}{\Delta t} \tag{251}$$

as in the ocean model (see Section 4.3). If $T_o^{m+1} < T_{of}, T_o^{m+1} = T_{of}$, to ensure the mixed layer temperature always remains above freezing.

6. Output to History Files

We present the history fields typically written to history file(s) during an ice model integration in order to relate them to named variables used in this technical note. Averaged quantities are written to the history file at the end of the time stepping loop.

There are two groups of fields written to the history file: time-invariant and time dependent. The former are fields related to the horizontal grid, while the latter are prognostic and diagnostic fields from an ice model integration. Table 7 presents a list of all history fields; the first fields from tmask to ANGLET are time-invariant, while the rest are prognostic or diagnostic.

The field names and the equivalent variable name used in this document (in parentheses) are shown. Fields received from or sent to the coupler have (cpl) in their description. Note that some of the fields have non-standard units and names not necessarily consistent with this document.

In some cases, two values of a flux are written to the history file: the value sent to the coupler and a value not divided by the ice area A (see Eq. 240). The value sent to the coupler has been divided by ice area, since the coupler multiplies by ice area, while the value used by the ice model does not.

History file format is netCDF. The history file frequency (i.e. how often written) can be either instantaneous (i.e. every time step), daily, monthly, or annual. A sequence of history files is written during model execution at the desired frequency, and all but the instantaneous files are time averaged over the interval between writes (see the CSIM User's Guide on the CCSM web page for more information). There is one exception to time averaging: the normalized principal stress components (sig1 and sig2) are always instantaneous because time average stress states at geographically fixed points are not physically meaningful.

There are four tendency fields included in the history file: two ice volume and two ice area tendencies $[(\partial V/\partial t)_T, (\partial V/\partial t)_D]$ and $[(\partial A/\partial t)_T), (\partial A/\partial t)_D]$ respectively. These tendencies are purely diagnostic, and distinguish the effect of thermodynamic processes (vertical and lateral, designated by subscript T) from dynamic processes (advection and rafting/ridging, designated by subscript D). For example, the ice volume change due to thermodynamic processes is:

$$(\partial V/\partial t)_T = [V(after\ vertical,\ lateral\ thermodynamics) - V(before)]/dt \tag{252}$$

Table 7. History Fields

Name	Description	Units
tmask	T grid mask (0 = land, 1 = ocean)	
tarea (A^t)	area of T grid cells	m^2
uarea (A^u)	area of U grid cells	m^2
dxu	U cell grid width longitudinally through middle	m
dyu	U cell grid width latitudinally through middle	m
dxt	T cell grid width longitudinally through middle	m
dyt	T cell grid width latitudinally through middle	m
HTN	T cell width on north side	m
HTE	T cell width on east side	m
ANGLE (χ^u)	angle grid makes with lat line on U grid	radians
ANGLET (χ^t)	angle grid makes with lat line on T grid	radians
hi (V)	grid box mean ice thickness	m
hs (V_s)	grid box mean snow thickness	m
Tsfc (T_s)	snow/ice surface temperature (cpl)	°C
aice (A)	aggregate ice area (cpl)	%
u (u_i)	x direction ice velocity	cm s^{-1}
v (v_i)	y direction ice velocity	cm s^{-1}
fswdn (F_{SWDN})	down solar flux	W m^{-2}
flwdn (F_{LWDN})	down longwave flux	W m^{-2}
snow (F_{SNW})	snowfall rate (cpl)	cm day^{-1}
snow$_{ai}$ ($F_{SNW}A$)	snowfall rate	cm day^{-1}
rain (F_{RN})	rainfall rate (cpl)	cm day^{-1}
rain$_{ai}$ ($F_{RN}A$)	rainfall rate	cm day^{-1}
sst (T_o)	sea surface temperature	°C
sss (S_o)	sea surface salinity	ppt
uocn (u_o)	x direction ocean current	cm s^{-1}
vocn (v_o)	y direction ocean current	cm s^{-1}
frzmlt (F_{Qoi})	freezing/melting potential	W m^{-2}
fswabs (F_{SW})	snow/ice/ocn absorbed solar flux (cpl)	W m^{-2}
fswabs$_{ai}$ ($F_{SW}A$)	snow/ice/ocn absorbed solar flux	W m^{-2}
albsni (α_{bb})	snow-ice broad band albedo	%
alvdr (α_{vsdr})	visible direct albedo	%
alidr (α_{nidr})	near IR direct albedo	%

Table 7 continued. History Fields

Name	Description	Units
flat (F_{LH})	latent heat flux (cpl)	W m^{-2}
flat$_{ai}$ ($F_{LH}A$)	latent heat flux	W m^{-2}
fsens (F_{SH})	sensible heat flux (cpl)	W m^{-2}
fsens$_{ai}$ ($F_{SH}A$)	sensible heat flux	W m^{-2}
flwup (F_{LWUP})	upward longwave flux (cpl)	W m^{-2}
flwup$_{ai}$ ($F_{LWUP}A$)	upward longwave flux	W m^{-2}
evap (F_{EVAP})	evaporative water flux (cpl)	cm day^{-1}
evap$_{ai}$ ($F_{EVAP}A$)	evaporative water flux	cm day^{-1}
Tref (T_{REF})	2m reference temperature (cpl)	°C
Qref (T_{REF})	2m reference specific humidity (cpl)	g/kg
congel ($\Sigma(\delta h\|_{\text{basal}} > 0)_n A_n$)	congelation ice growth	cm day^{-1}
frazil ($F_{Qoi}/q_f > 0$)	frazil ice growth	cm day^{-1}
snoice ($\Sigma\|z_{int}\|_n A_n$)	snow-ice conversion	cm day^{-1}
meltt ($\Sigma(\delta h\|_{\text{melt}} < 0)_n A_n$)	top ice melt	cm day^{-1}
meltb ($\Sigma(\delta h\|_{\text{basal}} < 0)_n A_n$)	basal ice melt	cm day^{-1}
meltl (VR_{side})	lateral ice melt	cm day^{-1}
fresh (F_{Wo})	freshwater flux ice to ocean (cpl)	cm day^{-1}
fresh$_{ai}$ ($F_{Wo}A$)	freshwater flux ice to ocean	cm day^{-1}
fsalt (F_{So})	salt flux ice to ocean (cpl)	kg m^{-2} day^{-1}
fsalt$_{ai}$ ($F_{So}A$)	salt flux ice to ocean	kg m^{-2} day^{-1}
fhnet (F_{Qio})	heat flux ice to ocean (cpl)	W m^{-2}
fhnet$_{ai}$ ($F_{Qio}A$)	heat flux ice to ocean	W m^{-2}
fswthru (F_{SWo})	SW thru ice to ocean (cpl)	W m^{-2}
fswthru$_{ai}$ ($F_{SWo}A$)	SW thru ice to ocean	W m^{-2}
strairx (τ_{ax})	x direction atm/ice stress (cpl)	N m^{-2}
strairy (τ_{ay})	y direction atm/ice stress (cpl)	N m^{-2}
strtltx ($H_{ox} = \bar{m}g\partial H_o/\partial x$)	x direction sea surface tilt stress	N m^{-2}
strtlty ($H_{oy} = \bar{m}g\partial H_o/\partial y$)	y direction sea surface tilt stress	N m^{-2}
strcorx ($+\bar{m}fv_i$)	x direction coriolis stress	N m^{-2}
strcory ($-\bar{m}fu_i$)	y direction coriolis stress	N m^{-2}
strocnx (τ_{ox})	x direction ocean/ice stress (cpl)	N m^{-2}
strocny (τ_{oy})	y direction ocean/ice stress (cpl)	N m^{-2}

Table 7 continued. History Fields

Name	Description	Units
strintx $(\nabla \cdot \sigma)_x$	x direction div internal ice stress tensor	N m^{-2}
strinty $(\nabla \cdot \sigma)_y$	y direction div internal ice stress tensor	N m^{-2}
strength (P)	compressive ice strength	N m^{-1}
divu $(\dot{\epsilon}_I)$	strain rate (divergence)	% day^{-1}
shear $(\dot{\epsilon}_{II})$	strain rate (shear)	% day^{-1}
sig1 (σ_I/P)	normalized principal stress component 1	
sig2 (σ_{II}/P)	normalized principal stress component 2	
dvidtt $(\partial V/\partial t)_T$	ice volume tendency thermodynamics	cm day^{-1}
dvidtd $(\partial V/\partial t)_D$	ice volume tendency dynamics	cm day^{-1}
daidtt $(\partial A/\partial t)_T$	area tendency thermodynamics	% day^{-1}
daidtd $(\partial A/\partial t)_D$	area tendency dynamics	% day^{-1}
opening $(\dot{\epsilon}_I + \lvert\dot{\epsilon}\rvert C(\theta))$	lead opening rate	% day^{-1}
aice1 (A_1)	ice area (category 1)	%
aice2 (A_2)	ice area (category 2)	%
aice3 (A_3)	ice area (category 3)	%
aice4 (A_4)	ice area (cateogry 4)	%
aice5 (A_5)	ice area (category 5)	%

7. Summary

During 1999-2003, the CCSM Polar Climate Working Group made several recommendation for improvement to the sea ice model. Additionally, during 2003-2004 the broader CCSM community imposed requirements concerning numerical efficiency and code architecture. The resulting CCSM3 sea ice model, CSIM5, addresses these recommendations and requirements, and includes:

(1) a plastic rheology with an elliptical yield curve

(2) enhanced sea ice thermodynamics

(3) an ice thickness distribution

(4) elimination of spurious polar convergence near the north pole

(5) an ice model on same grid as the ocean model

(6) an efficient parallel and vector architecture

(7) an active ice only framework for testing

CSIM5 represents a significant improvement to the original sea ice model of CSM1.

Acknowledgments

Thanks are due to the numerous individals, both at NCAR and in the scientific community, who contributed to the development of the CCSM3 sea ice component, CSIM5.

List of Physical Constants

Symbol	Code Symbol	Description	Value
ρ_s	rhos	density of snow	330 kg m^{-3}
ρ_i	rhoi	density of ice	917 kg m^{-3}
ρ_o	rhow	density of seawater	1026 kg m^{-3}
C_p	cp_air	specific heat of atmosphere dry	1005 J kg^{-1} K^{-1}
C_{pwv}	cp_wv	specific heat of atmosphere water	1810 J kg^{-1} K^{-1}
c_s	cp_sno	specific heat of snow	0 J kg^{-1} K^{-1}
c_0	cp_ice	specific heat of fresh ice	2054 J kg^{-1} K^{-1}
c_o	cp_ocn	specific heat of ocean	4218 J kg^{-1} K^{-1}
z_i	iceruf	aerodynamic roughness of ice	5.0x10^{-4} m
z_{ref}	zref	reference height for bulk fluxes	10 m
$q_1(ice)$	qqqice	saturation specific humidity const	11637800
$q_2(ice)$	TTTice	saturation specific humidity const	5897.8
$q_1(ocean)$	qqqocn	saturation specific humidity const	627572.4
$q_2(ocean)$	TTTocn	saturation specific humidity const	5107.4
c_d	dragw	drag coefficient for water on ice	0.00536
h_{min}	hi_min	minimum ice thickness for cat 1	0.1 m
h_{smin}	hs_min	minimum snow depth for heat eqn	0.01 m or 0.00001 m
k_s	ksno	thermal conductivity of snow	0.31 W m^{-1} K^{-1}
k_{fi}	kice	thermal conductivity of fresh ice	2.0340 W m^{-1} K^{-1}
β	betak	thermal conductivity ice constant	0.1172 W m^{-1} ppt^{-1}
u_{min}	ustar_min	minimum ice/ocean friction velocity	0.001ms^{-1}
L_i	Lfresh	latent heat of fusion of ice	3.337x10^5 J kg^{-1}
L_s	Lsub	latent heat of sublimation	2.835x10^6 J kg^{-1}
L_v	Lvap	latent heat of vaporization	2.501x10^6 J kg^{-1}
T_f	Tffresh	freezing temperature of freshwater	273.15 K
T_{of}	Tf	freezing temperature of ocean	-1.8°C
T_{melt}	Timelt,Tsmelt	melting temperature of top surface	0 °C
ΔT_{err}	Tsf_errmax	maximum error tolerance	5×10^{-4} °C
μ	depressT	ocean freezing temperature constant	0.054 °C ppt^{-1}

Constants continued

Symbol	Code Symbol	Description	Value
S_o	ocn_ref_salinity	ocean frazil ice ref salinity	34.7 ppt
S_i	ice_ref_salinity	sea ice frazil ice ref salinity	4 ppt
g_e	gravit	gravitational acceleration	9.80616 m s^{-2}
σ_{sb}	stefan_boltzmann	Stefan-Boltzmann constant	5.67x10^{-8} W m^{-2} K^{-4}
ε	emissivity	snow/ice emissivity	0.95
κ_{vs}	kappav	ice SW visible extinction coefficient	1.4 m^{-1}
κ_{ni}	kappan	ice SW near-ir extinction coefficient	17.6 m^{-1}
α_o	albocn	ocean albedo	0.06
E_0	eyc	dynamic constant	0.36
C_s	—	fraction of shear used for ridging	0.225
G^*	gstar	accumulative ice fraction for ridging	0.15
K	cK	max ridged ice thickness constant	100 m
Z	Zfric	ratio E dissipated to potential E gain	17
C_{pe}	cpe	ice/ocn potential energy constant	450 N m^{-3}

Appendix

During the development of CSIM5, several physics options were incorporated into the sea ice model. Some of these relate to options not chosen as default, and others are possible candidates for future versions of CSIM. These physics options have been retained for the released version of CSIM5. In this appendix these options are briefly described.

It is useful to run CSIM5 in column mode, namely, as a **1D column model** with prescribed inputs (i.e. SHEBA data or other). This allows efficient testing and validation of the vertical sea ice physics. This option has not been thoroughly tested and is not supported.

An earlier method for evaluating thickness space transport is the **delta scheme** of Bitz (2000) and Bitz et al. (2001). This method represents the distribution function $g(h)$ as a sum of delta functions, one for each populated ice thickness category. This physics option has the limitation of underpopulating the ITD, resulting in jumps in properties across category boundaries. The linear remapping scheme in CSIM5 was chosen as default because it represents $g(h)$ in each category as a linear function, and allows for incremental transport between categories, yielding a much smoother representation.

The previous version of the sea ice model evaluated the ice strength using the method of Hibler (1979). This **ice strength parameterization** depends only on the mean ice thickness and ice fraction. The scheme in CSIM5 makes use of the ice participating in ridging, but is dependable only if the ITD is well resolved (i.e. has at least five categories). For fewer categories, it is preferable to use the Hibler (1979) expression.

Very high spatial resolution implementations of the EVP dynamics using a reasonable subcycling time step can produce simulations for which the elastic waves useful for regularization are not sufficiently damped. In this case, a physical option of **damping of elastic waves** is available, as in Hunke (2001) and Hunke and Dukowicz (2002). The elastic wave damping is enhanced by reducing the ice strength compared to the default in CSIM5. For the resolution and subcycling time steps used in CSIM5, this option is not necessary.

The CSIM5 ridging scheme by default places snow on ice participating in ridging into the ocean. If **snow into ocean** is set false, then snow cover is retained on ridged ice.

The previous horizontal advection scheme was MPDATA, and before that upwind. Both **upwind advection** and **MPDATA** are retained as options in CSIM5. These are useful for assessing differences among advection schemes.

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