E. Equation of State

In the original MOM, a third order, local in z, polynomial approximation is constructed to serve as the equation of state for seawater (Bryan & Cox, 1972). The polynomial fit closely approximates either the Knudsen or Unesco formula, and the evaluation of density from this approximation significantly reduces the computational effort over using the Knudsen and Unesco formulae directly.

The approximation is in potential temperature (θ) and salinity (S), and it is based on the deviations (anomalies) of salinity, potential temperature, and density (ρ) from some reference profiles. Consequently, it has the following form,

$$\delta\rho = c_1\delta\theta + c_2\delta S + c_3\delta\theta^2 + c_4\delta\theta\delta S + c_5\delta S^2 + c_6\delta\theta^3 + c_7\delta\theta\delta S^2 + c_8\delta\theta^2\delta S + c_9\delta S^3 \quad \text{(E1)}$$

where

$$\begin{aligned}\delta\rho &= \rho - \rho_r,\\ \delta\theta &= \theta - \theta_r,\\ \delta S &= S - S_r.\end{aligned}$$

Here ρ_r, θ_r, and S_r represent the reference profiles for density, potential temperature, and salinity, respectively. Also, ρ is in sigma units (kg/m^3), θ is in $°C$, and S is in model units defined as $0.001(S_p - 35)$, where S_p is salinity in parts per thousand. The nine coefficients actually contain the effects of pressure (depth) on density and, therefore, they vary with depth. Note that, in Eq. (E1), the influences of salinity, potential temperature and pressure on density are separated.

Even though the polynomial approximation uses potential temperature, the bounds (ranges) for the polynomial fit are given in terms of insitu temperature and salinity. This is due to the fact that the Knudsen and Unesco equations of state require insitu temperature to compute density. The bounds are specified at 33 depth levels from $0m$ to $8000m$ with $250m$ increments which are different than the actual model level depths. The original MOM first determines the model level, z_k, for which the coefficients are to be evaluated. Hence, the term *local approximation* is used. Then, the bounds of the polynomial fit for the nearest and shallowest depth level are utilized to compute the range averages of insitu temperature, potential temperature, and salinity which are used as reference profiles. Finally, the reference density profile is calculated, using either the Knudsen or Unesco equation of state with the reference insitu temperature and salinity profiles. Consequently, the original MOM reference profiles are not realistic. Once the reference profiles are ready, the anomalies are formed and a least squares algorithm is applied to find the nine coefficients.

In the present work, we replaced the range average reference profiles of the original MOM with more realistic profiles, corresponding to the annual mean distributions obtained from Levitus (1982). Levitus (1982) provides only the potential temperature and salinity distributions. Because the Knudsen and Unesco equations of state necessitate insitu temperature, a separate code was utilized to compute the reference insitu temperature profile as a function of salinity, potential temperature, and pressure (depth). The code uses the same subroutine as MOM to compute potential temperature and iterates on insitu temperature until the output potential temperature satisfies the annual mean value within a specified error ($= 10^{-12}$).

Next, one of the original MOM subroutines, *denscoef.F*, which computes the nine coefficients of the polynomial fit and prepares an include file, *dncoef.h*, was modified to incorporate the new reference profiles. The reference annual mean insitu temperature and salinity distributions were defined in data statements at 45 levels ($0m$ to $8000m$) with $50m$ increments down to $300m$, $100m$ increments down to $1500m$, and $250m$ increments for deeper levels. Wherever necessary, the linear interpolation was applied to obtain the required values. Because Levitus (1982) data were available only down to $5500m$, the values at $5500m$ were used for deeper levels. The reference potential temperature and density were then evaluated for each level, using the potential temperature and the Unesco equation of state subroutines with the reference insitu temperature and salinity profiles. The vertical distributions of reference salinity, potential temperature, and density are given in Fig. E1. For comparison purposes, the original MOM reference profiles (evaluated at the 33 levels) are also included in the figure.

The vertical distributions of coefficients obtained using the original MOM bounds for the polynomial fit revealed significant non-smooth behavior. After numerous test runs, we found that the bounds of the fit had to be changed such that the level averages corresponded to the specified reference profile values. Consequently, the ranges of insitu temperature were expanded for $depth \leq 1400$ and contracted for deeper levels. The ranges of salinity values were also changed accordingly. However, we found that the ranges of salinity values did not significantly affect the distributions of coefficients except c_5. Therefore, we adjusted the bounds of salinity to obtain a smooth distribution for c_5. In Fig. E2, the present bounds of insitu temperature and salinity are presented. In this and the following figures, the distributions are given only down to $5500m$. The corresponding vertical distributions of the polynomial approximation coefficients reveal that the coefficients are not very different than the original MOM coefficients (evaluated at the 33 levels) with the exception of c_5 which extends to about -35 in the original MOM.

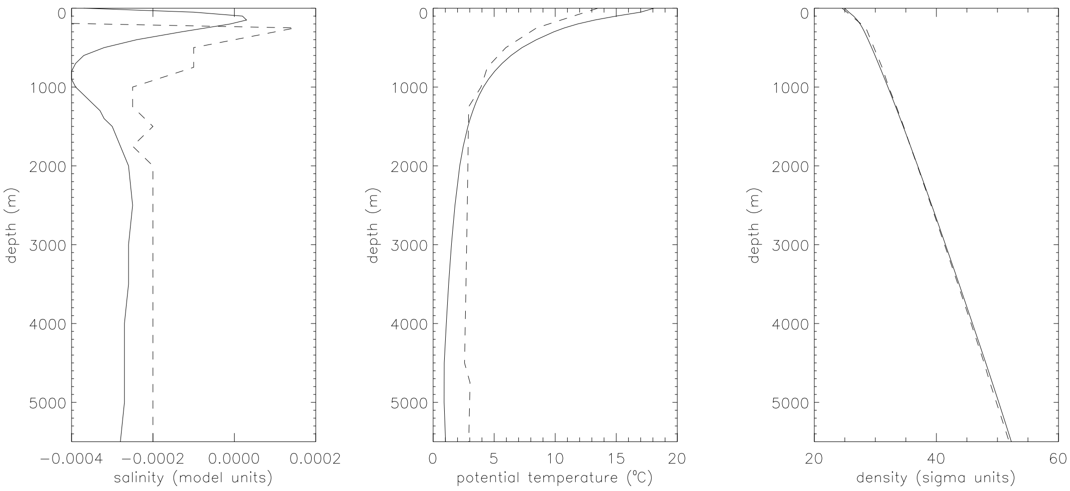
We next computed the root-mean-square (rms) error which was based on the difference of the values obtained from the current polynomial approximation and the Unesco equation of state for seawater. The rms error was calculated at the 45 depth levels, using 5000 equally spaced points over the ranges given in Fig. E2. The vertical distribution of the rms error is displayed in Fig. E3, revealing a maximum error of about 0.007 which occurs at the surface. The same figure also includes the corresponding distribution from the original MOM (evaluated at the 33 levels) for comparison purposes. Due to the wider ranges of temperature for $depth < 1250$, we observe slightly larger error values for the present results. For $depth \geq 1250$, the observed rms error is much less than the original MOM distributions, because the bounds of the present polynomial fit are narrower. In Fig. E4, the contour plot of the deviation of the present results from the Unesco equation of state at the surface is given, displaying a maximum error of about $0.018kg/m^3$. In practice the density anomaly cannot be determined to better than $0.05kg/m^3$ (Levitus & Isayev, 1992).

Therefore, the maximum error of the present results is much less than the accuracy of the equation of state for seawater.

Bryan, K. & M.D. Cox, 1972: An Approximate Equation of State for Numerical Models of Ocean Circulation, *J. Phys. Oceanogr.* **2**, 510–514.

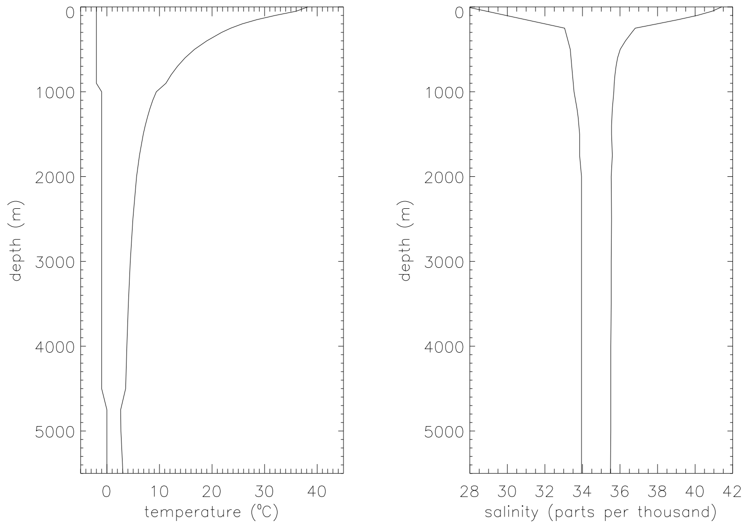
Levitus, S. 1982: Climatological Atlas of the World Ocean, Prof. Paper 13, US Department of Commerce, NOAA, US Government Printing Office, Washington, DC.

Levitus, S. & G. Isayev, 1992: Polynomial Approximation to the International Equation of State for Seawater, *J. Atm. Oce. Tech.* **9**, 705–708.



(a) (b) (c)

Figure E1. Vertical distributions of reference (a) salinity (model units), (b) potential temperature ($^{\circ}C$), and (c) density (sigma units) profiles. The solid and dashed lines represent the present and the original MOM profiles, respectively.



(a) (b)

Figure E2. Vertical distributions of bounds of (a) insitu temperature ($^{\circ}C$) and (b) salinity (parts per thousand) for polynomial fit.

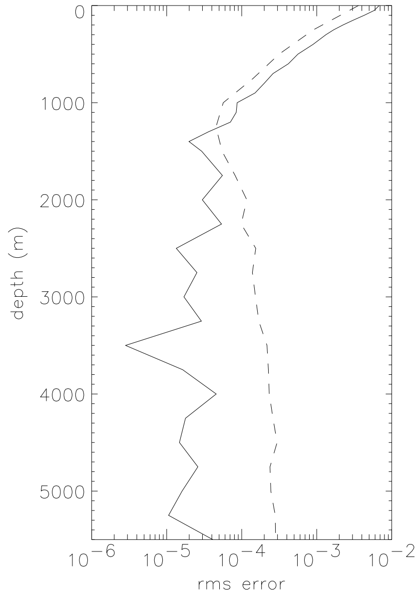


Figure E3. Vertical distributions of *rms* error. The solid and dashed lines represent the present and the original MOM profiles, respectively.

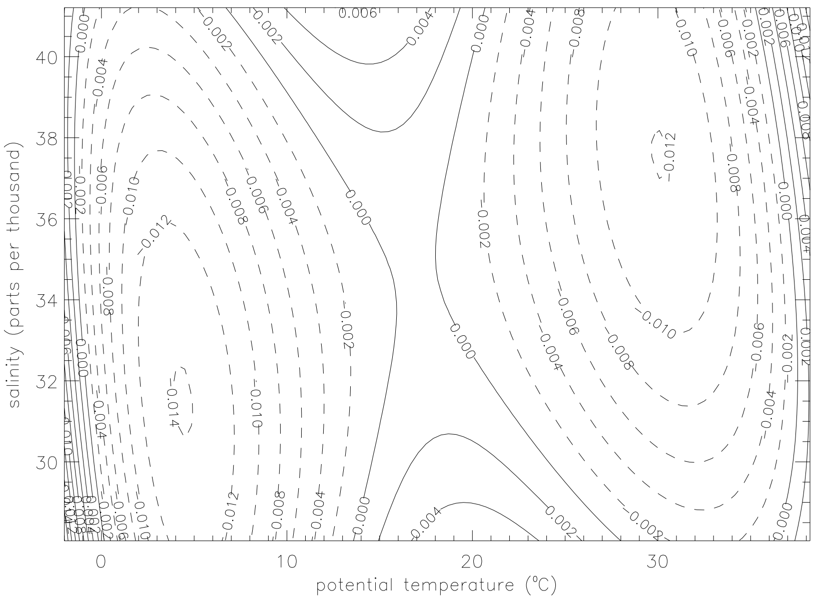


Figure E4. Contour plot of the deviation of the present results from the Unesco equation of state for seawater at the surface (CI= 0.002). The solid and dashed lines represent the positive and negative levels, respectively.